# **Robust of Low Rank Matrix and Collaborative Representation for Face Recognition**

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**Abstract.**In this paper, we have introduced improvement Robustness method for face recognition, which combine of the low-rank with collaborative representation. The applications of this model are based on the truth that proposed method can effectively deal with the face recognition across different illumination and occlusion, as well as the nature of corrupted and occluded regions. The method is able to be applied directly on original face image neither does it require feature selection, nor does it need many training samples. Experiments have been performed on various benchmark face database. The proposed method outperforms many state-of-the-art methods.

## Introduction

" Face recognition has garnered considerable interest over the last three decades, and many research have been conducted on it. However, many obstacles remain. In this paper, we will present an improved method for addressing practical face recognition system challenges such as variable lighting, posture, expression, corruption, and occlusion, as well as make significant advancements in face recognition capabilities via improvements and minor tweaks to standard parameters. We shall cross-reference (suggest) many effective techniques for face recognition, low-rank matrix (LR) and collaborative representation (CRC) were used to produce the aforementioned method. Follows":

- 1. Some low-rank transformations in the images to interpret the context of face recognition.
- 2. To deal effectively with the proposed method is to recognize faces by different lighting and obstruction, and in addition to that work the nature of the damaged and unclear areas (covered).
- 3. In this method, we can apply it directly to the original face image and it does not require feature selection and does not require many training samples.
- 4. Finally, we obtained results in several experiments, quantities and qualities, which show the work of those methods proposed above.

The remainder of this chapter is organized as follows: Section 1 discusses our proposed method, which is Low rank matrix as RPCA. In Section 2 Collaborative representation

classifications (CRC), In Section 3 Contribution Low rank matrix and CRC for face recognition the experimental results are detailed, while the summary of this chapter left to Section 4. -Low-rank matrix as RPCA

Recently, efforts have been directed on developing transformations that allow for the decomposition of altered pictures into the sum of a low-rank matrix component and a sparse error matrix component. This section discusses an idealization of the robust PCA issue, "which is concerned with recovering a""low-rank matrix from severely damaged data". In [1] RPCA, the following decomposition is used:

$$X = D + E \tag{1}$$

Where  $X \in \mathbb{R}^{m \times n}$  is a predefined big data matrix, D is a low-rank matrix, and E is a sparse matrix. The simplest approach is to minimize the energy function by using the  $l_0$  -norm:  $(D) + \lambda \parallel E \parallel_0 \text{s.t.}$  X = D + E (2)

Where  $\lambda$ "is" an arbitrary balanced parameter, although this issue is NP-hard, a typical approach may include a combinatorial search". To make this easier to solve, the obvious approach is to fix the minimization with the  $l_1$ -norm, which generated an approximation convex problem:

$$\min_{D,E} \| D \|_* + \lambda \| E \|_1 \text{s.t.} \quad X = D + E$$
(3)

Where  $\|*\|_*$  denotes the nuclear norm (which is the singular value  $l_1$ -norm). Assuming these bare minimums, given that the order of the low-rank and sparsity matrices is limited by the following equation, the (Principal Component Pursuit (PCP)) solution recovers fully the low-rank and sparse matrices:

$$rank(D) \le \frac{P_r \max(n,m)}{(\mathcal{M} \log \min\{(n,m)\}^2)} \qquad \|E\|_0 \le P_E n, m \tag{4}$$

Where  $P_r$  rand  $P_E$  are "positive constants" and m and n denote the dimensions of the matrix D. For the sake of explanation  $\lambda$ , is fixed  $(1/\sqrt{\max(n,m)})$ . The optimization method for (Augmented Lagrange Multipliers (ALMs)) [2]. Was used to solve of the problem (4), because of their superior accuracy and speed when applied to real-world face pictures:'

#### Collaborative representation classifications (CRC)

We assume *n* unique bases" $X = [X_1, X_2, ..., X_n] \in \mathbb{R}^{m \times n}$ , which are drawn from N distinct classes, "where m denotes the dimension of each base". Class *i* contains  $n_i$  training pictures indicated by the symbol  $X_i$ . Xcan be rewritten as  $X = [X_1, X_2, ..., X_n]$ . "When a fresh test sample" y

 $\in R^m$  is encountered, SRC attempts to ""identify a sparse linear representation coefficient vector""  $\alpha \in R^n$  such that  $y = X\alpha$  may be represented. Calculate this approximation problem by minimizing the following issue"":

$$\hat{\alpha} = \arg \min_{\alpha} \{ \| y - X\alpha \|_{2}^{2} + \lambda \| \alpha \|_{1} \}$$
(5)

where  $\lambda$  is a scalar constant,  $\|\cdot\|_2$  is the  $l_2$  norm, and  $\|\cdot\|_1$  is the  $l_1$  norm. Numerous methods, such as basis pursuit[3] and Homotopy[4], may be utilized to solve the  $l_1$  norm minimization issue described above. The test sample y should be contained inside the space bounded by the proper class's training samples.

Once we have the solutions  $\hat{\alpha}$  (5), where  $\hat{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_N]$  and  $\alpha_i$  is the representation vector  $\hat{\alpha}$  for class *i*, the test sample y may be identified by the reconstruction error of each class, i.e.

$$identity(y) = \arg \min_{i} \{ \|y - X_i \hat{\alpha}_i\|_2 \}$$
(6)

Because various individuals' facial image may look similar, samples from uncorrelated classes may contribute to portraying the test sample y. CRC is a regularized least squares technique with a substantially reduced complexity that is defined as,

$$\hat{\rho} = \operatorname{Arg} \min_{\rho} \{ \| y - X\rho \|_2^2 + \lambda \| \rho \|_1 \}$$
(7)

Where  $\lambda$  is a balance factor. CRC can offer improvements in decreasing computational complexity by using the  $l_2$ -norm-based model. A closed-form solution:

$$\hat{\rho} = (X^T X + \lambda I)^{-1} X^T y \qquad (8)$$

May be obtained by solving (8), in which  $(X^T X + \lambda I)^{-1}X^T$  can be pre-calculated, resulting in rapid CRC computation speed. The regularized residuals  $e_i = ||y - X_i \hat{\rho}_i||_2 / ||\hat{\rho}_i||_{2^n}$  are utilized in the classification stage to categorize the test image y "using the discriminating information contained "in"  $||\hat{\rho}_i||_2$ , "where"  $\hat{\rho}_i$  is the coefficient vector associated with class *i*.

Finally, the test sample belongs to the class with the smallest regularized residual.

## Algorithm 1: Collaborative representation-based classification [7]

- 1. Normalize the columns of *X* to have unit  $l_2$ -norm.
- 2. Code *Y* over *X* by

*p̂*=*py* 

Where  $p = (X^T X + \lambda I)^{-1} X^T y$ 

3. Compute the regularized residuals

$$e_i = \parallel y - X_i \widehat{\rho_i} \parallel_2 / \parallel \widehat{\rho_i} \parallel_2$$

4. Output the identity of y

$$identity(y) = \arg \min_i(e_i)$$

#### Contribution Low rank matrix and CRC for face recognition

The suggested implementation of the method here in the presence of the creation of a new algorithm for facial recognition in two stages is presented in this section:

"First", "in the "presence of light fluctuations, pixel corruption, and continuous occlusion, lowrank matrix recovery was used. Set a test face image, in particular, for each subject we first put it with the training. Under various subjects, the appropriate low-rank matrix recovery has been obtained with regard to the test image for each subject's photos separately.

Second, for low-rank image, collaborative representation-based classification (CRC) is implemented. The process may be summarized as follows: algorithm 2.

#### Algorithm 2: Proposed method algorithm'

- 1. Input: A matrix of normalized training samples  $X = [X_1, X_2, ..., X_i] \in \mathbb{R}^{m \times n}$  at testing sample  $Y = [Y_1, Y_2, ..., Y_i] \in \mathbb{R}^{m \times n}$  for k classes.
- 2. For each subject i do
- 3. Using algorithm (ALM)[2] to perform RPCA on  $X_i$ ,  $Y_i$

$$min_{A,S} ||A_i||_* + \lambda ||S_i||_1 \quad s.t \quad X_i = A_i + S_i$$

 $min_{A,S} \|B_i\|_* + \lambda \|E_i\|_1$  s.t  $Y_i = B_i + E_i$ 

- 4. Low rank matrices  $A = A_i$  and  $B = B_i$ , sparse matrices  $S = S_i$  and  $E = E_i$
- 5. Normalize the columns of A to have unit  $l_2 norm$ .
- 6. Code over by  $\hat{\rho} = PB$  where  $P = (A^T A + \lambda \cdot I)^{-1} A^T$ .
- 7. Compute the regularized residuals
  - $e_i = \|B A_i \cdot \hat{\rho}_i\|_2 / \|\hat{\rho}_i\|_2.$
- 8. Output the identity of B as identity (B) =  $\arg min_i(e_i)$ ."

#### **Experimented result**

We have performed three different types of experiments to verify capabilities of the proposed approach. In the first one, Face Recognition task is executed. In the second one, Gender Recognition is implemented. Finally, the last experiment is carried out over a Facial expression. For all experiments there are three parameters in multi-block RCR: and (the Lagrange multiplier of the entropy constraint), it should be set those experiments on four of the most popular face

database benchmarks: (1) The Olivetti Research Laboratory (ORL) face database, (2) AR face database, (3) Extended Yale B face database, and (4) UMIST face database. The following sections go over the specifics of the experiments and their outcomes



FIGURE 1.Examples of the face databases

The results of our experiments on the ORL face database are presented in Table 1. The table shows the comparison of the our proposed method with low rank Collaborative Representation based Classification (LR-CRC) method, Sparse Representation based Classification (SRC) and Collaborative Representation based Classification (CRC) methods. We note that our proposed method achieves recognition rate better than the maximal recognition rates that have been achieved by the other methods in the same feature dimensions.

 Table 1 Recognition rate (%) of different methods on the ORL database and the associated dimension of feature.

SCR	CRC	LRC	RCR	LR-CRC
90.0	89.17	85.60	82.08 94.60	

The results of the experiments on AR face database are presented in Table 2. We report the comparison of our proposed method low rank for Collaborative Representation based Classification with support vector machine (SVM), SRC, CRC, LRC, RCR and multi-task joint sparse representation based classification (MTJSRC) methods.

**TABLE 2.** Recognition rate (%) of different methods on the AR database and the associated dimension of feature.

SVM	SRC	CRC	LRC	MDTJSR	RCR	
LR-CRC						
87.1	93.7	93.3	76.4	95.8	95.9	
96.13						

In Table 3, we present the results of our experiments on the Extended Yale B face database. By comparison, our proposed method which is Low Rank Collaborative Representation based Classification (LR-CRC) method with SVM, SRC, CRC, LRC, RCR and MTJSRC methods, The results of the experiments on the Extended Yale B face database suggest that the recognition rate in our proposed method is better than the highest recognition rate presented by the other methods in the same feature dimensions.

Dim	15	20
	25	
SVM	67.1 87.1	76.5
SRC	84.6 92.0	91.3
CRC	84.7 92.4	91.3
LRC	81.8 89.0	87.0
MDTJSR	87.2 93.6	91.5
RCR	87.2 93.6	93.3
LR-CRC	93.27 98.02	95.55

**TABLE 3.**Recognition rate (%) of different methods onExtended Yale B database and the associated dimension of feature.

The experimental results on the UMIST face database are shown in Table 4, which demonstrates comparison of our proposed method with Extreme Learning Machine (ELM) [6], Extreme Learning Machine with Sparse Representation based Classification (ELMSRC) methods were compared in [8] and CRC methods. It can be inferred from that the performance of our proposed method is better than performance of the other methods.

**TABLE4**. Recognition rate (%) of different methods on the UMIST database and the associated dimension of feature

dimension of readure					
SRC	CRC	ELM	ELM-SRC		
LR-CRC					
98.33	98.18	96.51	98.36		
99.27					

# CONCLUSION

We proposed an effective method for image feature extraction and classification that is carried out in two steps:

First, it is adopted RPCA to obtain low rank matrix in the presence of different challenges like as illumination variations, pixel corruption and contiguous occlusion.

Secondly, address the data as low rank matrix by using collaborative representation based classification. Our method outperforms compared a many state-of-the-art methods.

# RFERENCES

- 1. Cande's E. J., Li X., Ma Y., et al., Robust principal component analysis? Journal of the ACM (JACM), 2011, 58(3):11.
- 2. Lin Z., Chen M., Ma Y., The augmented lagrange multiplier method for exact recovery of corrupted low-rank matrices. arXiv preprint arXiv:1009.5055, 2010
- 3. Elad M., Figueiredo M. A., Ma Y., On the role of sparse and redundant representations in image processing. Proceedings of the IEEE, 2010, 98(6):972–982.
- Yang M., Zhang D., Wang S., Relaxed collaborative representation for pattern classification. in: Proceedings of Computer Vision and Pattern Recognition (CVPR), 2012 IEEE Conference on. IEEE, 2012, 2224–2231.
- 5. Yang M., Zhang L., Yang J., et al., Metaface learning for sparse representation based face recognition. in: Proceedings of Image Processing (ICIP), 2010 17th IEEE International Conference on. IEEE, 2010, 1601–1604.
- Huang G. B., Zhu Q. Y., Siew C. K., Extreme learning machine: a new learning scheme of feedforward neural networks. in: Proceedings of Neural Networks, 2004. Proceedings. 2004 IEEE International Joint Conference on. IEEE, 2004, 985–990.
- 7. Wright J., Yang A. Y., Ganesh A., et al., Robust face recognition via sparse representation. Pattern Analysis and Machine Intelligence, IEEE Transactions on, 2009, 31(2):210–227.

8. Luo M., Zhang K., A hybrid approach combining extreme learning machine and sparse representation for image classification. Engineering Applications of Artificial Intelligence, 2014, 27:228–235.