Quadruple Series Equation InvolvingGeneralized- BatemanK- Functions

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Abstract

Formal solution four series equation involving generalized Bateman k Functions is given is this paper. Finally, the solution of four series equation is reduceto Freadholm integrals equation of second time in one independent which can be solve by some numerical method.

INTRODUCTION

Srivastava[1] presented formal solution of dual series equations involving Generalized Bateman K- Functions. Later on Dwivedi and Trivedi[2] gave the solutions of triple series equations. We reduce the triple series equations to Fredholm integral equations of second kind which can be solved by some numerical methods. This paper shall be devoted in finding the solution of four series equations involving Generalised Bateman K-Functions. Our methods is similar to that of Cooke[3] used in solving triple equations of Bessel functions. The analysis given here is purely formal and no attempt is made to justify the various limiting processes

Some useful result

In the course of the analysis we shall use the following result The orthogonality relation for Generalied Bateman K- Function is

(1) $\int_0^\infty x^{-2\propto-2\sigma}$ (x)$K^{2(\propto+\sigma)}$ (x)dx = $\frac{K^{2\propto+2\sigma}\Gamma(\propto+\sigma)}{\Gamma(2\propto+\sigma+n+1)}\delta$ mn

Where $\propto+\sigma+1>0, \sigma+1\leq$0 and δmn is the Kronecker delta.

We obtained in Dwivedi and Trivedi the value of the serics given below :

(2) S(r,x) = $\sum_{n=0}^{\infty}\frac{\Gamma(2\nu+\sigma+n+1)}{2^{2\beta}+2\sigma\Gamma(\mathrm{n}+\sigma)}K_{2(n+\beta)}^{2(\beta+\sigma)}(r)K_{2(n+\beta)}^{2(\propto+\sigma)}(x)$s

(3) = $\frac{c^{-x}2^{2\propto-2v}}{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}\int_0^t E(\xi)\,(x-\xi)^{2\propto-2y-1}(\gamma-\xi)^{ZB-ZV-1}d\xi$

= = $\frac{c^{-x}2^{2\propto-2v}}{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}S_t(r,x)$

E(ε)=$e^{2\varepsilon}\varepsilon^{2v+2\sigma+1}$ $, t=$min (r,x).

If f(x) and f'(x) are continuous in a $\leq$x$\leq$ b and if o<$\sigma<1$ then the solutions of the Abel integral equations

(4) f(x) = $\int_a^x\frac{F(y)}{(x-y)^\sigma}dy$

and

(5) f(x) = $\int_x^b\frac{F(y)}{(y-x)^\sigma}dy$

are given by

(6) $F(y)=\frac{Sin\sigma\ \pi}{\pi}\frac{d}{dy}\int_a^y \frac{F(x)dx}{(y-x)^{1-\sigma}}$

and

(7) $F(y)= -\frac{Sin\sigma\ \pi}{\pi}\frac{d}{dy}\int_a^b \frac{F(x)dx}{(x-y)^{1-\sigma}}$

respectively.

Solution of four series equations

We shall now solve four series equations involving Generalized Bateman K-Functions defined as below :

(8) $\sum_{n=0}^{\infty}\frac{An}{\Gamma(2\beta+\sigma+n+1)}K_{2(n+\propto)}^{2(\propto+\sigma)}(x) = f_1(x), \quad 0\leq x \leq a,$

(9) $\sum_{n=0}^{\infty}\frac{An}{\Gamma(2\gamma+\sigma+n+1)}K_{2(n+\beta)}^{2(\beta+\sigma)}(x) = f_2(x), \quad a<x<b,$

(10) $\sum_{n=0}^{\infty}\frac{An}{\Gamma(2\beta+\sigma+n+1)}K_{2(n+\propto)}^{2(\propto+\sigma)}(x) = f_3(x), \quad b<x<c,$

(11) $\sum_{n=0}^{\infty}\frac{An}{\Gamma(2v+\sigma+n+1)}K_{2(n+\beta)}^{2(\beta+\sigma)}(x) = f_4(x), \quad c<x<\infty,$

here the unknown coefficients An are to be determined

Let us suppose

(12) $\sum_{n=0}^{\infty}\frac{An}{\Gamma(2v+\sigma+n+1)}K_{2(n+\beta)}^{2(\beta+\sigma)}(x) = h(x), \quad 0\leq x \leq a,$

$= g(g) \quad b<x<c$

Making use of the equation (1) we get from (9), (11) and (12)

(13) $An =\frac{\Gamma(2v+\sigma+n+1)\Gamma(2\beta+\sigma+n+1)}{2^{2\beta+2\sigma}\Gamma(n-\sigma)}[\int_0^a x^{-2\beta-2\sigma-1}K_{2(n+\beta)}^{2(\beta+\sigma)}(x)h(x)dx$
$+\int_0^b x^{-2\beta-2\sigma-1}K_{2(n+\beta)}^{2(\beta+\sigma)}(x)f_2(x)dx+\int_b^c x^{-2\beta-2\sigma-1}K_{2(n+\beta)}^{2(\beta+\sigma)}(x)g(x)dx$
$+\int_c^{\infty} x^{-2\beta-2\sigma-1}K_{2(n+\beta)}^{2(\beta+\sigma)}(x)f_4(x)dx]$

Substituting this expression for An from (13) in (8) and (10) and inter changing the order of integration we obtain the equations

(14) $\int_0^a r^{-2\beta-2\sigma-1}S(r,x)\,h(r)dr + \int_b^c r^{-2\beta-2\sigma-1}S(r,x)\,g(r)dr = M(x),\ 0\leq x<a,$

(15) $\int_0^a r^{-2\beta-2\sigma-1}S(r,x)\,h(r)dr + \int_b^c r^{-2\beta-2\sigma-1}S(r,x)\,g(r)dr = N(x),\ b<x<c,$

Where

(16) $M(x)= f_1(x)-\int_a^b r^{-2\beta-2\sigma-1}S(r,x)\,f_2(x)dx-\int_c^{\infty} r^{-2\beta-2\sigma-1}S(r,x)\,f_4(x)dx$

(17) $N(x)= f_3(x)-\int_a^b r^{-2\beta-2\sigma-1}S(r,x)\,f_2(x)dx-\int_c^{\infty} r^{-2\beta-2\sigma-1}S(r,x)\,f_4(x)dx$

(14) can be written with the help of (3) as

(18) $\int_0^x \frac{E(\xi)d\xi}{(x-\xi)^{1-2\propto+2v}}\int_\xi^a \frac{h(r)r^{-2\beta-2\sigma-1}}{(r-\xi)^{1-2\beta+2v}}dr=\frac{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}{e^{-x}2^{2\propto-2v}}M(x)$

$$-\int_b^c g(r)r^{-2\beta-2\sigma-1}dr\int_0^x \frac{E(\xi)d\xi}{(x-\xi)^{1-2\propto+2v}(r-\xi)^{1-2\beta+2v}}$$

With the help of (6), we find from (18) that

$$E(\xi)\int_\xi^a \frac{h(r)r^{-2\beta-2\sigma-1}}{(r-\xi)^{1-2\beta+2v}}dr = \frac{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}{2^{2\propto-2v}}\frac{\sin(1+2\propto+2v)\pi}{\pi}$$

$$\frac{d}{d\xi}\int_0^\xi \frac{M(x)dx}{e^{-x}(\xi-x)^{2\propto-2v}}-\frac{\sin(1+2\propto+2v)\pi}{\pi}\frac{d}{d\xi}\int_0^\xi \frac{dx}{(\xi-x)^{2\propto-2v}} -x$$

$[\int_b^c g(r)r^{-2\beta-2\sigma-1}dr \int_0^x \frac{E(t)dt}{(x-t)^{1-2\propto+2v}(r-t)^{1-2\beta+2v}}]$, 0< $\xi < a,$

The second integral of (19) can be written in the form

(20) - $\frac{\sin(1-2\propto+2v)\pi}{\pi} \int_b^c g(r)r^{-2\beta-2\sigma-1}dr$ -$\frac{d}{d\xi}\int_0^\xi \frac{E(t)dt}{(r-t)^{1-2\beta+2v}}$

$$\int_t^\xi \frac{dx}{(\xi-t)^{2\propto-2v}(x-t)^{1-2\propto+2v}}$$

= - $\int_b^c \frac{g(r)E(\xi)r^{-2\beta-2\sigma-1}}{(r-\xi)^{1-2\beta+2v}}dr,$

Since $\int_t^\xi \frac{dx}{(\xi-t)^{2\propto-2v}(x-t)^{1-2\propto+2v}} = \frac{\pi}{\sin(1+2\propto+2v)\pi}$

Hence

(21) $\int_\xi^a \frac{h(r)r^{-2\beta-2\sigma-1}\,dr}{(r-\xi)^{1-2\beta+2v}} = \frac{M_1(\xi)}{E(\xi)}$ - $\int_b^c \frac{g(r)r^{-2\beta-2\sigma-1}\,dr}{b(r-\xi)^{1-2\beta+2v}}$

Where

(22) $M_1(\xi) = \frac{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}{2^{2\propto-2v}}\frac{\sin(1+2\propto+2v)\pi}{\pi}\frac{d}{d\xi}\int_0^\xi \frac{M(x)dx}{e^{-x}(\xi-r)^{2\propto-2v}}$

Using (7) we obtain from (21)

(23)$h(r)r^{-2\beta-2\sigma-1}$ = - $\frac{\sin(1-2\beta+2v)\pi}{\pi}\frac{d}{dr}\int_r^a \frac{M_1(\xi)d\xi}{E(\xi)(\xi-r)^{2\beta-2v}}$

+ $\frac{\sin(1-2\beta+2v)\pi}{\pi}\frac{d}{dr}\int_r^a \frac{d\xi}{(\xi-r)^{2\beta+2v}}\int_b^c \frac{g(S)S^{-2\beta-2\sigma-1}\,dS}{(S-\xi)^{1-2\beta+2v}}$

New using the result

(24) - $\frac{d}{dr}\int_r^a \frac{d\xi}{(\xi-r)^{1-\beta}(S-\xi)^{\beta}}$ = - $\frac{(a-r)^{\beta-1}}{(S-a)^{\beta-1}(S-r)}$, 0<r<a,

We find that

(25) h(r) = M_2(r) =$\frac{\sin(1-2\beta+2v)\pi}{\pi(a-r)^{2\beta-2v}}r^{2\beta+2\sigma-1}$ ×

$\int_b^c \frac{(s-a)g(S)S^{-2\beta-2\sigma-1}\,ds}{(S-r)}$, 0<r<a,

Where

(26) M_2(r) = - $\frac{\sin(1-2\beta+2v)\pi}{\pi}r^{2\beta+2\sigma-1}\frac{d}{dr}\int_r^a \frac{M_1(\xi)d\xi}{E(\xi)(\xi-r)^{2\beta-2v}}$

Using (15) and (3), We find that

(27) $\int_b^x \frac{E(\xi)G(\xi)\,d\xi}{(x-\xi)^{1-2\propto+2v}} = \frac{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}{e^{-x}2^{2\propto-2v}}$ N(x)-

$$\int_a^b h(r)\,r^{-2\beta-2\sigma-1}dr\int_0^r \frac{E(\xi)G(\xi)\,d\xi}{(x-\xi)^{1-2\propto+2v}(r-\xi)^{1-2\beta+2v}}$$

$\int_0^b \frac{E(\xi)\,d\xi}{(x-\xi)^{1-2\propto+2v}}\int_0^b \frac{g(r)r^{-2\beta-2\sigma-1}\,dr}{(x-\xi)^{1-2\beta+2v}}$, b<x<c,

Where

(28) G(ξ) = $\int_\xi^c \frac{r^{2\beta-2\sigma-1}g(r)\,dr}{(r-\xi)^{1-2\beta+2v}}$ - .

From (28) and (7), we find that

(29) $r^{-2\beta-2\sigma-1}$ g(r) = - $\frac{\sin(1-2\beta+2v)\pi}{\pi}\frac{d}{dr}\int_r^c \frac{G(\xi)\,d\xi}{(\xi-r)^{2\beta-2v}}$

The equation 27 has the form (4), then using the equation (27) can be written in the form

(30) E(ξ) G(ξ) = $N_1(\xi)$ +I_1+ I_2, b< ξ<c,

Where

(31) $N_1(\xi) = \frac{\sin(1-2\beta+2v)\pi}{\pi} \frac{\Gamma(2\propto-2v)\Gamma(2\beta-2v)}{2^{2\propto-2v}} \frac{d}{d\xi}\int_b^\xi \frac{N(x)dx}{(\xi-x)^{2\propto-2v}}$,

(32) $I_1 = -\frac{\sin(1-2\beta+2v)\pi}{\pi} \frac{d}{d\xi}\int_b^\xi \frac{N(x)dx}{(\xi-x)^{2\propto-2v}}\int_o^a h(r)\, r^{-2\beta-2\sigma-1}dr$

$$\times \int_b^c \frac{dt}{(x-t)^{1-2\propto+2v}(r-t)^{1-2\beta+2v}}$$

(33) $I_2 = -\frac{\sin(1-2\propto+2v)\pi}{\pi} \frac{d}{d\xi}\int_b^\xi \frac{dx}{(\xi-x)^{2\propto-2v}}\int_0^b \frac{E(t)dt}{(x-t)^{1-2\propto+2v}}$

$$\times \int_0^r \frac{g(r)r^{-2\beta-2\sigma-1}}{(r-t)^{1-2\beta+2v}}$$

After some manipulation, we get

(34) $I_1 = -\frac{\sin(1-2\propto+2v)\pi}{\pi(\xi-b)^{2\propto-2v}}\int_0^a \frac{(b-t)^{2\propto-2v}E(t)dt}{(\xi-t)}\int_t^a \frac{h(r)^{-2\beta-2\sigma-1}dr}{(r-t)^{1-2\beta+2v}}$

putting the value of last integrate in the above equation, from (21), we obtain

(35) $I_1 = M_1(\xi) + \frac{\sin(1-2\propto+2v)\pi}{\pi(\xi-b)^{2\propto-2v}}\int_b^c g(r)r^{-2\beta-2\sigma-1}dr$ x

$$\int_0^a \frac{E(t)(b-t)^{2\propto-2v}dr}{(\xi-t)(r-t)^{1-2\beta+2v}}$$

where$M_3(\xi)$= - $\frac{\sin(1-2\propto+2v)\pi}{\pi(\xi-b)^{2\propto-2v}}\int_0^a \frac{M_1 t(b-t)^{2\propto-2v}}{(\xi-t)}dt$

After some manipulation, (33) can be written in the form

(36) I_3= - $\frac{\sin(1-2\propto+2v)\pi}{\pi(\xi-b)^{2\propto-2v}}\int_b^c g(r)r^{-2\beta-2\sigma-1}dr\int_a^b \frac{E(t)(b-t)^{2\propto-2v}dt}{(\xi-t)(r-t)^{1-2\beta+2v}}$

Hence

(37) $I_{1+}I_2 = M_3(\xi)$= - $\frac{\sin(1-2\propto+2v)\pi}{\pi(\xi-b)^{2\propto-2v}}\int_b^c g(r)r^{-2\beta-2\sigma-1}dr \times$

$$\int_a^b \frac{E(t)(b-t)^{2\propto-2v}dt}{(\xi-t)(r-t)^{1-2\beta+2v}}$$

$= M_3(\xi)$ - $\frac{\sin(1-2\propto+2v)\pi}{\pi(\xi-b)^{2\propto-2v}}\int_a^b \frac{n(t)(b-t)^{2\propto-2v}}{(\xi-t)}R\, dt,$

where

(38) R= $\int_b^c \frac{g(r)r^{-2\beta-2\sigma-1}dr}{(r-t)^{1-2\beta+2v}}$

By using a method similar to that of Cooke[2] we can easily show that

(39) R= $\int_b^c \frac{g(r)r^{-2\beta-2\sigma-1}dr}{(r-t)^{1-2\beta+2v}}(b-t)^{2\beta-2v}\int_b^c \frac{G(r)dr}{(r-b)^{2\beta-2v}(r-t)}$

Using equation (37) and (39) the equation (30) ca n be written in the form

(40) $n(\xi)\, G(\xi) = M_3(\xi) + N_1(\xi) + \int_b^c G(r)K(r,\xi)dr, \qquad b < \xi < c,$

where

(41) $K(r,\xi) = \frac{\sin(1-2\beta+2v)\pi \ \sin(1-2\propto+2v)\pi}{\pi^2\{(\xi-b)^{2\propto-2v}(r-b)^{2\beta-2v}\}} - \int_a^b \frac{E(t)(b-t)^{2\propto+2\beta-4v}dt}{(r-t)(\xi-t)}$

Equation (40) is a Fredholm integral equation of the second kind. There is a standard equation. From this equation we can determine G(S). Knowing G(S), g(u) and h(u) can be easily obtained from (29) and (25) hence the coefficients Δ_n can be determined.

References

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