Bipolar Vague Bidirectional Measure Based Models for Solving MADM-Problems

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Abstract: Bipolar vague sets are the extension of vague sets and are based totally on the concept of positive and negative preferences of information. Projection is a very important measure in decision making. In this work, we enhance a new technique called bidirectional projection measure for handling multi attribute decision making problem in the light of bipolar vague sets. Through the bidirectional projection measure among each alternative decision matrix and the ideal alternative matrix, all the alternatives can be ranked to select the best one.

KEYWORDS: Bipolar vague set, projection measure, bidirectional projection measure.

1. Introduction

The idea of fuzzy has invaded nearly all branches of Mathematics ever since the introduction of fuzzy sets by L. A. Zadeh [14].The ranking techniques of MCDM have recently attracted more attention in different fields. A series of well-known techniques have been constructed to solve MCDM problems under various fuzzy environments, such as projection model [12], VIKOR [9], TOPSIS [10], etc. Among them, projection measure has its advantage that it can consider both the distance and the included angle between evaluated alternatives. Yang et al. [7] develop projection method for material selection problem in fuzzy environment. Xu and Hu [11,13] developed two projection based models for MADM in intuitionistic fuzzy and interval valued intuitionistic fuzzy environment. Zeng et al. [8] provided weighted projection algorithm for intuitionistic fuzzy MADM problems and interval valued intuitionistic fuzzy MADM problems. Chen and Ye [2] developed the projection based model for solving Neutrosophic MADM problem and applied it to select clay-bricks in construction field. However, the projection measure has its flaws in few cases due to simply considering the single directional projection magnitude between the evaluated alternatives and not standardizing the measurement values within [0,1]. The bidirectional projection method has the superiority in considering not only the distance and included angle but also the bidirectional projection between each alternative and the ideal solution.

The notion of vague set was introduced by W. L. Gau and D. J. Buehrer [3] in 1993. The continuous subinterval states both the proof that is in favour of the item and additionally opposing it which can be marked via truth membership function and false membership function. The idea of vague sets started with interpreting the real life problems in a higher manner than fuzzy sets. K. M Lee [5,6] initiated another extension of fuzzy sets in which the range [0, 1] enlarge to [-1, 1] named bipolar valued fuzzy sets which offer grade property of an object and the counter property of the same object. S. Cicily et al. [1] introduced bipolar vague sets where vague sets which takes the value from the subset of interval [-1, 1] is known as bipolar vague set. Bipolar vague set is an extension of vague sets and are based totally on the concept of positive and negative choice of data.

2. PRELIMINARIES

Definition 2.1[5,6]: Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is $\mu_A^+ : X \to [0,1]$, and a negative membership function μ_A^-, that is $\mu_A^- : X \to [-1,0]$. For the sake of simplicity, we shall use the symbol $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.5[1]: Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form $A = \{\langle x, [t_A^+(x), 1 - f_A^+(x)], [-1 - f_A^-(x), t_A^-(x)]\rangle : x \in X\}$ where $[t_A^+, 1 - f_A^+] : X \to [0,1]$ and $[-1 - f_A^-, t_A^-] : X \to [-1,0]$ are the mapping such that $t_A^+ + f_A^+ \leq 1$ and $-1 \leq t_A^- + f_A^-$. The positive membership degree $[t_A^+(x), 1 - f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1 - f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1 - f_A^+]$ and $v_A^- = [-1 - f_A^-, t_A^-]$.

Definition 2.8 [1]: Let $A = < x, [t_A^+, 1 - f_A^+][-1 - f_A^-, t_A^-] >$ and $B = < x, [t_B^+, 1 - f_B^+][-1 - f_B^-, t_B^-] >$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

1. $A \cup B = \{(x, [t_{A\cup B}^+(x), 1 - f_{A\cup B}^+(x)], [-1 - f_{A\cup B}^-(x), t_{A\cup B}^-(x)]) / x \in X\}$ where

$t_{A\cup B}^+(x) = \max\{t_A^+(x), t_B^+(x)\}, \quad t_{A\cup B}^-(x) = \min\{t_A^-(x), t_B^-(x)\}$ and

$1 - f_{A\cup B}^+(x) = \max\{1 - f_A^+(x), 1 - f_B^+(x)\}$,

$-1 - f_{A\cup B}^-(x) = \min\{-1 - f_A^-(x), -1 - f_B^-(x)\}$.

2. $A \cap B = \{(x, [t_{A\cap B}^+(x), 1 - f_{A\cap B}^+(x)], [-1 - f_{A\cap B}^-(x), t_{A\cap B}^-(x)]) / x \in X\}$ where

$t_{A\cap B}^+(x) = \min\{t_A^+(x), t_B^+(x)\}, \quad t_{A\cap B}^-(x) = \max\{t_A^-(x), t_B^-(x)\}$ and

$1 - f_{A\cap B}^+(x) = \min\{1 - f_A^+(x), 1 - f_B^+(x)\}$,

$-1 - f_{A\cap B}^-(x) = \max\{-1 - f_A^-(x), -1 - f_B^-(x)\}$.

3. $\overline{A} = \{(x, [f_A^+(x), 1 - t_A^+(x)], [-1 - t_A^-(x), f_A^-(x)]) / x \in X\}$ for all $x \in X$.

Definition 2.6[1]: A bipolar vague set $A = [v_A^+, v_A^-]$ of a set U with $v_A^+ = 0$ implies that $t_A^+ = 0, 1 - f_A^+ = 0$ and $v_A^- = 0$ implies that $t_A^- = 0, -1 - f_A^- = 0$ for all $x \in U$ is called zero bipolar vague set and it is denoted by 0.

Definition 2.7[1]: A bipolar vague set $A = [v_A^+, v_A^-]$ of a set U with $v_A^+ = 1$ implies that $t_A^+ = 1, 1 - f_A^+ = 1$ and $v_A^- = -1$ implies that $t_A^- = -1, -1 - f_A^- = -1$ for all $x \in U$ is called unit bipolar vague set and it is denoted by 1.

3. BIDIRECTIONAL PROJECTION MEASURE

Definition 3.1: Assume that $X = \{x_1, x_2, ..., x_m\}$ be a finite universe of discourse and Q be a bipolar vague set in X, then modulus of Q is defined as follows:

$$\|Q\| = \sqrt{\sum_{j=1}^{m} \alpha_j^2} = \sqrt{\sum_{j=1}^{m} (t_{Q_j}^{+})^2 + (1 - f_{Q_j}^{+})^2 + (t_{Q_j}^{-})^2 + (1 - f_{Q_j}^{+})^2}$$

Where $\alpha_j = [t_{Q_j}^{+}, 1 - f_{Q_j}^{+}][-1 - f_{Q_j}^{-}, t_{Q_j}^{-}]$ j=1,2,…,m

Definition 3.2: Assume that $u = \{u_1, u_2, ..., u_m\}$ and $v = \{v_1, v_2, ..., v_m\}$ be two vectors, then the projection of vector u onto vector v can be defined as follows:

$$proj(u)_v = \|u\| \cos(u, v) = \sqrt{\sum_{j=1}^{m} u_j^2} \times \frac{\sum_{j=1}^{m} (u_j v_j)}{\sqrt{\sum_{j=1}^{m} u_j^2} \times \sqrt{\sum_{j=1}^{m} v_j^2}} = \frac{\sum_{j=1}^{m} (u_j v_j)}{\sqrt{\sum_{j=1}^{m} v_j^2}}$$

Where, $proj(u)_v$ represents that the closeness of u and v in magnitude.

Definition 3.3: Assume that $X = \{x_1, x_2, ..., x_m\}$ be a finite universe of discourse and R and S be any two bipolar vague sets in X, then

$proj\ (R)_s = \|R\| \cos(R, S) = \frac{1}{\|S\|}(R, S)$ is called the projection of R on S, where

$$\|R\| = \sqrt{\sum_{j=1}^{m} [(t_R^{+})^2(x_i) + (1 - f_R^{+})^2(x_i) + (t_R^{-})^2(x_i) + (-1 - f_R^{-})^2(x_i)]}$$

$$\|S\| = \sqrt{\sum_{j=1}^{m} [(t_S^{+})^2(x_i) + (1 - f_S^{+})^2(x_i) + (t_S^{-})^2(x_i) + (-1 - f_S^{-})^2(x_i)]} \text{ and}$$

$$R.S = \sum_{j=1}^{m} \begin{bmatrix} (t_R^{+})(x_i)(t_S^{+})(x_i) + (1 - f_R^{+})(x_i)(1 - f_S^{+})(x_i) + \\ (t_R^{-})(x_i)(t_S^{-})(x_i) + (-1 - f_R^{-})(x_i)(-1 - f_S^{-})(x_i) \end{bmatrix}$$

Example 3.4: Suppose that $R = <[0.4, 0.7][-0.4, -0.2]>$, $S = <[0.3, 0.5][-0.6, -0.3]>$ be two bipolar vague sets in X, then the projection of R on S is obtained as follows:

$$proj(R)_S = \frac{1}{\|S\|}(R, S) = \frac{(0.4)(0.3) + (0.7)(0.5) + (-0.4)(-0.6) + (-0.2)(-0.3)}{\sqrt{(0.3)^2 + (0.5)^2 + (-0.6)^2 + (-0.3)^2}}$$

= 0.866

The bigger value of $proj(R)_s$ reflects that R and S are close to each other.

Definition 3.5: Consider $R = <[t_R^{+}(x_i), 1 - f_R^{+}(x_i)][-1 - f_R^{-}(x_i), t_R^{-}(x_i)]>$ and

$S = <[t_S^{+}(x_i), 1 - f_S^{+}(x_i)][-1 - f_S^{-}(x_i), t_S^{-}(x_i)]>$ be any two bipolar vague sets in

$X = \{x_1, x_2, ..., x_m\}$, then the bidirectional projection measure between R and S is defined as follows:

$$B - proj(R, S) = \frac{1}{1 + \left| \frac{R.S}{\|R\|} - \frac{R.S}{\|S\|} \right|} = \frac{\|R\| \|S\|}{\|R\|\|S\| + \left| \|R\| - \|S\| \right| R.S}$$

Where $\|R\| = \sqrt{\sum_{j=1}^{m}[(t_R^+)^2(x_i) + (1 - f_R^+)^2(x_i) + (t_R^-)^2(x_i) + (-1 - f_R^-)^2(x_i)]}$

$\|S\| = \sqrt{\sum_{j=1}^{m}[(t_S^+)^2(x_i) + (1 - f_S^+)^2(x_i) + (t_S^-)^2(x_i) + (-1 - f_S^-)^2(x_i)]}$ and

$$R.S = \sum_{j=1}^{m}\begin{bmatrix}(t_R^+)^2(x_i)(t_S^+)^2(x_i) + (1 - f_R^+)^2(x_i)(1 - f_S^+)^2(x_i) + \\ (t_R^-)^2(x_i)(t_S^-)^2(x_i) + (-1 - f_R^-)^2(x_i)(-1 - f_S^-)^2(x_i)\end{bmatrix}$$

Theorem 3.6: Let $B - proj\ (R)_s$ be a bidirectional projection measure between any two bipolar vague sets R and S, then

1. $0 \le B - proj(R,S) \le 1$
2. $B - proj(R,S) = B - proj(S,R)$
3 $B - proj(R,S) = 1\ for\ R = S$

Proof: 1. For any two non-zero vector R and S,

$\dfrac{1}{1 + \dfrac{R.S}{\|R\|} - \dfrac{R.S}{\|S\|}} > 0$, $\dfrac{1}{1+x} > 0$, when $x > 0$

$\therefore\ B - proj(R,S) > 0$, for any two non-zero vectors R and S.

$B - proj(R,S) = 0$ if and only if either $\|R\| = 0$ or $\|S\| = 0$ i.e. when either $R =< [0,0][0,0] >$ or $S =< [0,0][0,0] >$ which is trivial case.

$\therefore\ B - proj(R,S) \ge 0$

For any two vectors R and S,

$\|R\|\ \|S\| + \big|\|R\| - \|S\|\big| R.S \ge \|R\|\ \|S\|$

$\|R\|\ \|S\| \le \|R\|\ \|S\| + \big|\|R\| - \|S\|\big| R.S$

$\therefore\ \dfrac{\|R\|\ \|S\|}{\|R\|\|S\| + \big|\|R\| - \|S\|\big| R.S} \le 1$

$\therefore\ B - proj\ (R,S) \le 1$

$\therefore\ 0 \le B - proj\ (R,S) \le 1.$

2. From Definition $R.\,S = S.\,R$ therefore,

$$B - proj(R,S) = \frac{\|R\|\ \|S\|}{\|R\|\|S\| + \big|\|R\| - \|S\|\big| R.S}$$

$$= \frac{\|S\|\,\|R\|}{\|S\|\|R\| + \big|\|S\| - \|R\|\big| S.R}$$

$$= B - proj(S,R).$$

3. Obviously, $B - proj\,(R,S) = 1$, only when $\|R\| = \|S\|$ i.e. when $t_R^+(x_i) = t_S^+(x_i)$,

$1 - f_R^+(x_i) = 1 - f_S^+(x_i)$, $-1 - f_R^-(x_i) = -1 - f_S^+(x_i)$, $t_R^-(x_i) = t_S^-(x_i)$. This completes the proof.

Example 3.7: Assume that $R = <[0.4, 0.7][-0.4, -0.2]>$, $S = <[0.3, 0.5][-0.6, -0.3]>$ be two bipolar vague sets in X, then the bidirectional projection measure between R on S is computed as follows:

$$B - proj\,(R,S) = \frac{(0.6245)(0.8367)}{(0.6245)(0.8367) + |0.6245 - 0.8367|(0.48)} = 0.8373.$$

Bidirectional projection based decision making methods for MADM problems with bipolar vague information

In this section, we develop bi-projection based decision making models to MADM problems with bipolar vague assessments. Consider $E = \{E_1, E_2, ..., E_m\}$ be a set of alternatives, $S = \{s_1, s_2, ..., s_n\}$ be a set of attributes under consideration and $W = (w_1, w_2, ..., w_n)^T$ be the weight vector of the attributes such that $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$. Now, we present algorithms for MADM problems involving bipolar vague information.

ALGORITHMN

Step 1: The rating of alternative $E = \{E_1, E_2, ..., E_m\}$ for the predefined attribute $S = \{s_1, s_2, ..., s_n\}$ presented by the decision maker in terms of bipolar vague values and the bipolar vague decision matrix is constructed as given below.

$$[q_{ij}]_{m\times n} = \begin{bmatrix} q_{11} & q_{12} \cdots q_{1n} \\ q_{21} & q_{22} \cdots q_{2n} \\ . & . \;\cdots\; . \\ . & . \;\cdots\; . \\ q_{m1} & q_{m1} \cdots q_{mn} \end{bmatrix}$$

Here, we have $q_{ij} = [r_{ij}^+, 1 - f_{ij}^+][-1 - f_{ij}^-, -t_{ij}^-]$ with $t_{ij}^+ + f_{ij}^+ \le 1$ and $-1 \le t_{ij}^- + f_{ij}^-$.

Step 2: We construct the bipolar vague weighted decision matrix by multiplying weights w_j of the attributes as follows:

$$[q_{ij}]_{m\times n} \otimes W_j = \begin{bmatrix} \rho_{11} & \rho_{12} \cdots \rho_{1n} \\ \rho_{21} & \rho_{22} \cdots \rho_{1n} \\ . & . \;\cdots\; . \\ . & . \;\cdots\; . \\ \rho_{m1} & \rho_{m2} \cdots \rho_{mn} \end{bmatrix}$$

Where $[q_{ij}]_{m\times n} \otimes W_j = [\rho_{ij}]_{m\times n}$

$$= <1-(1-t_{ij}^{+})^{w_j}, -(f_{ij}^{+})^{w_j}, -(-(-1-f_{ij}^{-})^{w_j}, -(-t_{ij}^{-})^{w_j}>$$

$$= <[\mu_{ij}^{+}, 1-\gamma_{ij}^{+}][-1-\gamma_{ij}^{-}, \mu_{ij}^{-}]> \text{ for } i=1,2,...,m,\ j=1,2,...,n$$

Step 3: We identify the bipolar vague positive ideal solution (BV-PIS) as follows:

The evaluation criteria can be categorized into two categories, benefit, and cost. Let G_1 be a collection of benefit criteria and G_2 be a collection of cost criteria. (BV-PIS) can be defined as follows.

$$\rho^{PIS} = <[e_{ij}^{+}, 1-g_{ij}^{+}][-1-g_{ij}^{-}, e_{ij}^{-}]> = <[\{\max_i(\mu_{ij}^{+})|j\in G_1\};\{\min_i(\mu_{ij}^{+})|j\in G_2],$$

$$[\{\max_i(1-\gamma_{ij}^{+})|j\in G_1\};\{\min_i(1-\gamma_{ij}^{+})|j\in G_2],\ [\{\min_i(\mu_{ij}^{-})|j\in G_1\};\{\max_i(\mu_{ij}^{-})|j\in G_2],$$

$$[\{\min_i(-1-\gamma_{ij}^{-})|j\in G_1\};\{\max_i(-1-\gamma_{ij}^{-})|j\in G_2]>.$$

Step 4: Determine the bidirectional projection measure between ρ^{PIS} and $Z^i = [\rho_{ij}]$ for all $i=1,2,...,m$, $j=1,2,...,n$ using the equation as given below.

$$B-proj\ (Z^i,\ \rho^{PIS}) = \frac{\|Z^i\|\ \|\rho^{PIS}\|}{\|Z^i\|\ \|\rho^{PIS}\| + \left|\ \|Z^i\| - \|\rho^{PIS}\|\ \right| \left|Z^i \rho^{PIS}\right|}$$

Where $\|Z^i\| = \sqrt{\sum_{j=1}^{m}[(\mu_{ij}^{+})^2 + (1-\gamma_{ij}^{+})^2 + (-1-\gamma_{ij}^{-})^2 + (\mu_{ij}^{-})^2]}$ for i=1,2,…,m.

$$\|\rho^{PIS}\| = \sqrt{\sum_{j=1}^{m}[(e_j^{+})^2 + (1-g_j^{+})^2 + (-1-g_j^{-})^2 + (e_j^{-})^2]} \quad \text{for } i=1,2,\ldots,m.$$

$$Z^i.\rho^{PIS} = \sum_{j=1}^{n}(\mu_{ij}^{+})(e_j^{+}) + (1-\gamma_{ij}^{+})(1-g_j^{+}) + (-1-\gamma_{ij}^{-})(-1-g_j^{-}) + (\mu_{ij}^{-})(e_j^{-}) \text{ for } i=1,2,\ldots,m.$$

Step 5: According to the bidirectional projection measure $B-proj\ (Z^i,\ \rho^{PIS})$ for i=1,2,…,m. the alternative are ranked and highest value of $B-proj\ (Z^i,\ \rho^{PIS})$ reflects the best option.

Case study:

Assume that a new modern logistic centre is required in a town. There are four location E_1, E_2, E_3, E_4 are taken into consideration in the decision making situation. Four attributes namely:

Distance to supplier(S_1):

It is also called internal traffic convenience, having great effect on the transportation cost.

Distance to customer(S_2):

It is also called external traffic convenience, affecting the distribution efficiency and the transportation cost directly.

Infrastructure condition (S_3):

They mainly include the reliability of the infrastructure and convenience of communication systems.

Natural condition (S_4):

They mainly include the meteorological conditions, the terrain conditions, and the hydrologic conditions are considered to evaluate the alternatives.

Assume the weight vector of the attribute is given by $w=\{w_1,w_2,w_3,w_4\}$ ={0.5,0.25,0.125,0.125}

The proposed bidirectional projection measure based decision making with bipolar vague information is presented in the following steps:

Step 1: Construct the bipolar vague decision matrix.

The bipolar vague decision matrix $[q_{ij}]_{m\times n}$ presented by the decision makers as given in Table 1.

Table 1

The bipolar vague decision matrix

	S_1	S_2	S_3	S_4
E	[0.3,0.6][-0.6,-0.2]	[0.2,0.8][-0.5,-0.3]	[0.4,0.7][-0.4,-0.3]	[0.3,0.5][-0.8,-0.4]
E	[0.7,0.9][-0.4,-0.3]	[0.5,0.6][-0.7,-0.4]	[0.5,0.8][-0.6,-0.2]	[0.4,0.8][-0.3,-0.2]
E	[0.6,0.8][-0.5,-0.4]	[0.3,0.8][-0.6,-0.4]	[0.2,0.7][-0.4,-0.1]	[0.5,0.9][-0.6,-0.4]
E	[0.3,0.7][-0.5,-0.2]	[0.7,0.8][-0.4,-0.3]	[0.5,0.7][-0.6,-0.3]	[0.4,0.7][-0.6,-0.5]

Step 2: Construction of weighted bipolar vague decision matrix.

The weighted decision matrix $[z_{ij}]_{m\times n}$ is obtained by multiplying weights of the attributes to the bipolar vague decision matrix as follows

TABLE 2

The weighted bipolar vague decision matrix

	S_1	S_2	S_3	S_4
E	[0.163,0.368] [-0.775,-0.447]	[0.054,0.331] [-0.841,-0.740]	[0.062,0.14] [-0.892,-0.860]	[0.044,0.083] [-0.972,-0.892]
E	[0.452,0.684] [-0.632,-0.548]	[0.159,0.205] [-0.915,-0.795]	[0.083,0.182] [-0.938,-0.818]	[0.062,0.182] [-0.860,-0.818]
E	[0.368,0.553] [-0.707,-0.632]	[0.085,0.331] [-0.880,-0.795]	[0.028,0.14] [-0.892,-0.750]	[0.083,0.25] [-0.938,-0.892]
E	[0.163,0.452] [-0.707,-0.447]	[0.26,0.331] [-0.795,-0.740]	[0.083,0.14] [-0.938,-0.860]	[0.062,0.14] [-0.938,-0.917]

Step 3: Selection of BV-PIS

The BV-PIS $\rho^{PIS} =<[e_j^+, 1-g_j^+][-1-g_j^-, e_j^-]>$, j=1,2,3,4 is computed as follows:

$<[e_1^+, 1-g_1^+][-1-g_1^-, e_1^-]>$ = <[0.452,0.684][-0.775,-0.632]>

$<[e_2^+, 1-g_2^+][-1-g_2^-, e_2^-]>$ = <[0.159,0.331][-0.915,-0.795]>

$<[e_3^+, 1-g_3^+][-1-g_3^-, e_3^-]>$ = <[0.083,0.182][-0.938,-0.860]>

$<[e_4^+, 1-g_4^+][-1-g_4^-, e_4^-]>$ = <[0.083,0.182][-0.972,-0.917]>

Step 4: Calculation of bidirectional projection measure

The bidirectional projection measure between The BV-PIS and each weighted decision matrix $[z_{ij}]$ can be determined as given as follows:

$B-proj\ (Z^1,\ \rho^{PIS})$ = 0.836

$B-proj\ (Z^2,\ \rho^{PIS})$ = 0.885

$B-proj\ (Z^3,\ \rho^{PIS})$ = 0.904

$B-proj\ (Z^4,\ \rho^{PIS})$ = 0.852.

Step 5: Rankingthe alternatives

we observe that $E_3 > E_2 > E_4 > E_1$. Hence, E_3 is the best alternative.

CONCLUSIONS

In this paper, we introduced a bidirectional projection measures between bipolar vague sets. Through these projections we have developed multi criteria decision making models under a bipolar vague environment. Finally, a logistic centre location problem has been solved to show the applicability of the proposed method. In future, the proposed methods can be extended to interval bipolar vague set environment.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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