Solutionof Fuzzy Quadratic Programming Problems

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Abstract:

This paper presents a computational procedure for finding the optimal solution of the quadratic programming problem (QPP) with fuzzy parameters in the constraint.

The suggested algorithm transform the fuzzy QPP into ordinary one without fuzzy parameters which treated as the decision variables based on α -level set concept and then solved using Wolfe procedure.

Keywords: Multi – criteria decision making, quadratic programming problem (QPP), Fuzzy parameters.

1. Introduction

In [1] a computational procedure is done for finding the minimum of a quadratic function of variables subject to linear inequality constraints. In [2], [3] Dubois and Prade state the fuzzy parameters in the objective function and the constraints are characterized by fuzzy numbers. They also introduce the α -cut of the fuzzy numbers. In this paper we shall discuss the quadratic programming problem (QPP) with fuzzy parameters in the constraints.. The plan of this paper is as follows: in section 2, we formulate quadratic programming problem (QPP) with fuzzy parameters in the constraints in section 3, the transformation to an equivalent non-fuzzy QPPis presented. In section 4, we suggest the solution procedure to QPP. In section 5, a numerical example is included to clarify the problem of the article. Finally section 6 contains the conclusion.

2. Problem formulation:

Consider the following QPP with fuzzy parameters in the constraints

Where p_1

$$MinF(p,x)=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\times i\times j+\sum_{i=1}^{n} p_i\times i$$

$$subject\ to\ M=\{x\in R^n:\sum_{j=1}^{n}a_{rj}\times j\le b_r, r=1,..,m\}$$

$Where\ x\in R\ is\ n\times 1 vector, a_{rj}\ (r=1,..,m,\ j=1,..,n) be\ m\times n$ coefficix of coefficient matrix of left-hand side of constraints, c_{ij} (i=1,..,n, j=1,.., n), C is positive semi-definite symmetric matrix, p_i (i=1,.., n) are arbitrary real numbers and b_r (r=1,.., m) are fuzzy parameters.

3. The transformation to an equivalent non-fuzzy QPP.

The fuzzy numbers b_r is a convex fuzzy subset of the real line whose membership functions $\mu_{b_r}(b_r)$ which defined by Dubois [2, 3], as follows:

1-A continuous mapping from E to closed interval [0, 1]

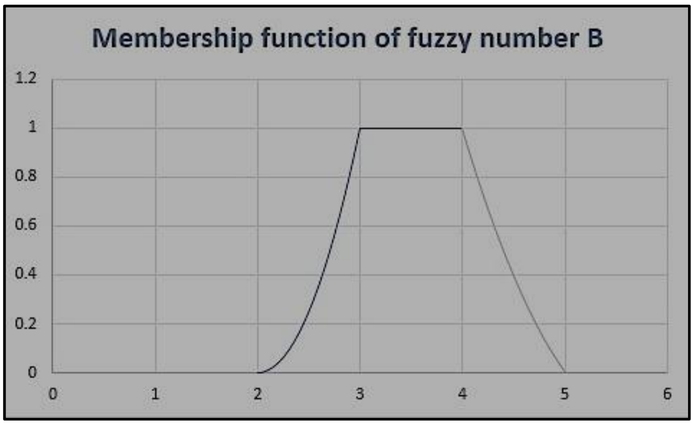
2- $\mu_b^-(b) = 0 \ \forall b \in [-\infty, b_1]$

3- Strictly increasing on [b_1, b_2]

4- $\mu_b^-(b) = 1 \ \forall b \in [b_2, b_3]$

5- - Strictly increasing on [b_3, b_4]

6- $\mu_b^-(b) = 0 \ \forall b \in [b_4, \infty]$



The α -level set of the fuzzy number b is defined as an ordinary $L_\alpha(b)$ for which the degree of its membership functions exceeds the level α .

$$L_\alpha(b_r) = [b / \mu_{b\ r}^-(b_r) \geq \alpha, r = 1,..,m]$$

The problem P_1 can be reformulated as a non-fuzzy QPP using certain degree α

$$(p_2): MinF(p,x) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \times i \times j + \sum_{i=1}^{n} p_i \times i$$

$$s.t. b_r \in L_\alpha(b_r), r = 1,..., m.$$

The parameters b_r , r=1, m are treated as decision variables. So the decision space of the problem P_2 *becomes* R^{n+m} .

The problem P_2 can take another equivalent form which formulated as follows:

$$(p_3): MinF(p,x) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \times i \times j + \sum_{i=1}^{n} p_i \times i$$

$$s.t. l_r \leq b_r \leq L_r , r = 1,..., m.$$

Where l_r *and* L_r the lower and upper bounds of α -cut or α -level set of fuzzy parameters b_r .

The above problem (P_3) is an ordinary quadratic programming problem QPP.

In order to ensure the success of the computational procedure of Wolfe [1] to determine the optimal solution of problem (P_3), the objective function F (x, p) must be convex, that is the matrix C must be positive semi-definite. Under the assumption of semi-definiteness on matrix C, any local minimum for the problem (P_3) will be the global solution.

4- The solution procedure:

Suppose that $u_r \geq 0, v_r \geq 0, w_r \geq 0, d_r \geq 0,$ r=1,…,m and $h_\alpha \geq 0, \alpha = 1,2,..,n$ are the lagrangian multipliers, the lagrangian functions for problem (P_3) can take the following form:

$$f(x,b,u,v,w,h,d) = \left\{ \begin{array}{l} \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \times i \times j + \sum_{i=1}^{n} p_i \times i + \\ w_r(l_r - b_r) - h_\alpha x_\alpha - d_r b_r, r = 1,..,m, \alpha = 1,..,n. \end{array} \right\}$$

The necessary and sufficient conditions to find a solution (x^-, b^-) for the problem (P_3) can summarize in the following Kuhn-Tucker.

Theorem

$(x^-, b^-) \in R^{n-m}$ Solves the problem P_3

if then there exists

$$u_r \geq 0, v_r \geq 0, w_r \geq 0, d_r \geq 0, r = 1,2,..,m \ and \ h_\alpha \geq 0, \alpha = 1,..,n.$$

such that

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \bar{x} j + P_\alpha + \sum_{r=1}^{m} u_r ar_\alpha - h_\alpha = 0$$

$$\left\{ \sum_{\alpha=1}^{n} a_{r\alpha} \bar{x}_\alpha - \bar{b}_r \bar{\leq} = 0 \right\}$$

$$\bar{b}_r - L_r \leq 0, l_r - \bar{b}_r \leq 0, \ u_r[\sum_{\alpha=1}^{n} a_{r\alpha} \bar{x}_\alpha - \bar{b}_r] = 0$$

$$v_r[\bar{b}_r - L_r] = 0, w_r[l_r - b_r] = 0, h_\alpha \bar{x}_\alpha = 0, d_r \bar{b}_r = 0$$

$$u_r \geq 0, v_r \geq 0, w_r \geq 0, d_r \geq 0, r = 1,2,..,m \ and \ h_\alpha \geq 0, \alpha = 1,..,n..$$

Returning to the quadratic programming problem, the conditions that (n+m) vector $\bar{x}, \bar{b}$ solves the problem P_3 may be written as linear system as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} \bar{x} j + P_\alpha + \sum_{r=1}^{m} u_r ar_\alpha - h_\alpha = 0, \ \alpha = 1,2,..,n.$$

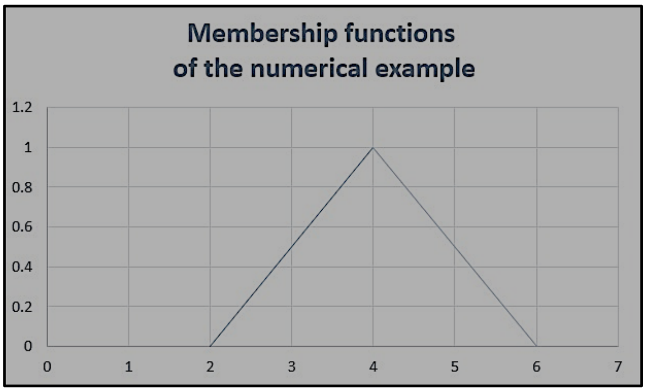
$$-u_r + v_r - w_r - d_r = 0, \ r = 1,2,..,m$$

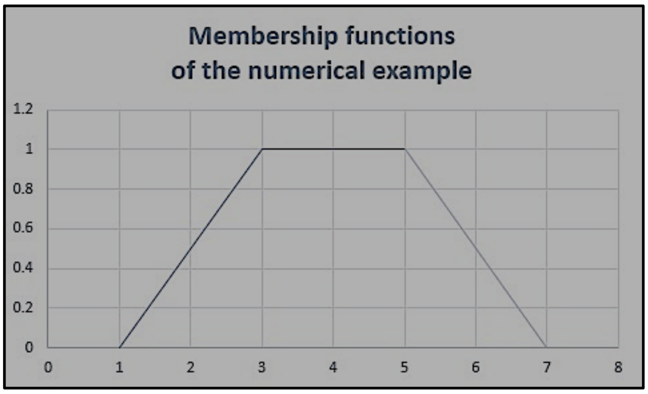
$$x \geq 0, b \geq 0, u \geq 0, v \geq 0, w \geq 0, d \geq 0, h \geq 0$$

The above linear system has (n+m) equations in (2n+4m) non-negative variables for which the simplex method can be used to explore the basic solutions (Wolfe algorithm). [1].

Numerical Example.

$$\min \quad x^2+y^2+2x-2y$$
$$subject\ to\ \{-x-y\le b_1, -y\le b_2, x\ \&\ y\ge 0\}$$
$$Where\ b_1, b_2\ are\ fuzzy\ parameters$$
$$\mu(b_1)\ and\ \mu(b_2)\ are\ shown\ in\ figures$$





Taking $\alpha = 0.5$, the equivalent non-fuzzy problem takes the form

$$\min \quad x^2+y^2+2x-2y$$
$$subject\ to: -x-y\le b_1, -y\le b_2, 3\le b_1\le 5, 2\le b_2\le 6$$
$$Where\ x, y, b_1, b_2\ge 0.$$

Using Wolfe algorithm the optimal solution is $(x=0, y=1, b_1=3, b_2=2)$.

6. Conclusions

In this paper, we deal with quadratic programming problem with fuzzy parameters in the constraint. Through the use of α -level set concept, a new equivalent formulation of this problem is done and a computational procedure based on Wolfe algorithm can be applied to solve the problem, also we show how to overcome the difficulties in treating with the non-linearity of the problem. The

Advantages of the approach are differs from the others methods in computational step, easier Than the other method that can be solved algebraically, the final solution can be obtained Rapidly and implemented in various types of non-linear programming.

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