

Results on a Chromatic Number of a Bi-polar Fuzzy Complete Bipartite Graphs and Labelling of Tri-Polar Fuzzy Graphs

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Abstract: The ultimate objective of a piece of research work is to present the labelling of vertices in 3-PFG and labelling of distances in 3-PFG. Also, we characterize some of its properties. Later, we define the vertex and edge chromatic number BF-Complete Bipartite graph. Further we illustrated an example for BFRGS which represents a Route Network system.

Keywords: 3-polar-Fuzzy Graphs, Regular Graph, Labelling Graph, Chromatic number, Complete Bipartite graph.

1. Introduction

The conventional investigation of SG starts in the mid-20th century. SGs are fundamental logarithmic models in numerous parts of designing formal dialects, in coding, Finite State Machine, automaton. The significant part of graph theory in computer presentations is the improvement of calculations in graphs. A graph structure is a helpful tool in cracking the combinatory problems in diverse areas of computer science containing clustering, image capturing, data mining, image segmentation, networking, and computational intelligence systems. (Zadeh. L. A.,1965) presented the fuzzy theory. Further, (Rosenfeld., 1971) applied it to the classical theory of subgroups. Later, (Bhargavi. Y., 2020) and (Mordeson J. N2003) developed the classical theory of fuzzy graphs, SGs. (Akram. M., 2011) presented several new ideas including bipolar Fuzzy-Graphs. Furthermore, quite a few authors (Bhargavi. Y 2020; Bhargavi. Y 2020; Bhargavi. Y 2020; Loganathan, J., 2017; Murali Krishna Rao. M. (2015); Ranjeeth.S, (2020); Ragamayi.S. (2019); Ragamayi.S. (2015) done on Fuzzy and vague structures of SGs and Nearrings. Later (Akram. M., 2011) proved outstanding results on applications of graph theory and Labelling. Accepting the above examination as starting point, in this Research article present the labelling of vertices in 3-PFG and labelling of distances in 3-PFG. Also, we characterize some of its properties. Later, we define the vertex and edge chromatic number BF- Complete Bipartite graph. Further we illustrated an example for BFRGS which represents a Route Network system. Moreover, we discussed about Cartesian product of two BFRGS.

Notations

- 1) BF represents Bipolar Fuzzy.
- 2) BFG represents Bipolar Fuzzy-Graph.
- 3) F-Graph represents Fuzzy-Graph
- 4) C-Graph represents Crisp-Graph
- 5) BFGS represents Bipolar Fuzzy Graph of Semi-group.
- 6) BF-IGS represents bipolar fuzzy ideal graph of Semi-group.
- 7) BF-IS represents bipolar fuzzy ideal of a Semi-group
- 8) SG represents Semi-group.
- 9) FS represents Fuzzy Subset.
- 10) BFRG represents Bipolar Fuzzy Regular Graph
- 11) BFRGS represents Bipolar Fuzzy Regular Graph of a Semi-group
- 12) 3-PFG represents 3-polar fuzzy graph or tri-polar fuzzy graph
- 13) 3-PFP represents 3-polar fuzzy Pathor tri-polar fuzzy path

2. Preliminaries

Definition 2.1 (Ragamayi, S, 2020) A pair (V, E) is a graph if $V \neq \emptyset$ and E is a set of un-ordered pairs of elements of V .

Definition 2.2 (Zadeh. L. A.,1965) A non-empty set A is said to be a fuzzy subset if a mapping $g: A \rightarrow [0,1]$.

Definition 2.3 (Akram. M., 2011) A finite fuzzy subset, V is a mapping $\mu: V \rightarrow [0, 1]$ which assigns to each element $x \in V$ a degree of membership $0 \leq \mu \leq 1$, and a fuzzy subset of $V \times V$ is a mapping $\rho: V \times V \rightarrow [0, 1]$ which assigns to each pair $(x, y) \in V \times V$ a degree of membership $0 \leq \rho(x, y) \leq 1$.

Definition 2.4 (Akram. M., 2011) Let $A = (\mu_{AP}, \mu_{AN})$. A BF relation on $X \neq \emptyset$ is defined as $A: X \times X \rightarrow [0,1] \times [-1,0]$ where $\mu_{AP}(p,q) \in [0,1]$ and $\mu_{AN}(p,q) \in [-1,0]$.

Definition 2.5 (Akram. M., 2011) If $A = (\mu_{AP}, \mu_{AN})$ is a BF set on an underlying set V and $B = (\mu_{BP}, \mu_{BN})$ is a BF set in V^2 for which $\mu_{BP}(mn) \leq \min\{\mu_{AP}(m), \mu_{AN}(n)\}, \forall mn \in V^2$ and $\mu_{BN}(mn) \geq \max\{\mu_{AN}(m), \mu_{AN}(n)\}, \forall m, n \in V^2$, and $\mu_{BP}(mn) = \mu_{BN}(mn) = 0, \forall m, n \in (V^2 - E)$, then $G = (V, A, B)$ is termed a BFG of the graph $G = (V, E)$.

Definition 2.6 (Akram. M., 2011) The order of a BFG, $G = (V, A, B)$ is represented by $O(G) = (OP(G), ON(G))$ and is distinguished as $OP(G) = \sum_{a_1 \in V} \mu_P(a_1) \in V$ and $ON(G) = \sum_{a_1 \in V} \mu_N(a_1)$. The size of a BFG, $G = (V, A, B)$ is denoted by $S(G) = (SP(G), SN(G))$ and is distinguished as $SP(G) = \sum_{a_1 a_2 \in \overline{V^2}} \mu_{BP}(a_1, a_2)$ and $SN(G) = \sum_{a_1 a_2 \in \overline{V^2}} \mu_{BN}(a_1, a_2)$.

Definition 2.7 (Loganathan, J., 2017) The open neighbourhood degree of a vertex ‘m’ in a BFG, G is distinguished as $\text{deg}(m) = (\text{deg } P(m), \text{deg } N(m))$, where $\text{deg } P(m) = \sum_{mn \in \overline{V^2}} \mu_{BP}(mn)$ and $\text{deg } N(m) = \sum_{mn \in \overline{V^2}} \mu_{BN}(mn)$.

Definition 2.8 (Loganathan, J., 2017) A FS, ξ of a Semi-group, T is known as a fuzzy-sub-SG of T if $\xi(mn) \geq \min\{\xi(m), \xi(n)\} \forall m, n \in T$.

Definition 2.9 (Ragamayi, S, 2020) A FS, ξ of a semi-group T is known as a FIOF of T if $\xi(mn) \geq \max\{\xi(m), \xi(n)\} \forall m, n \in T$.

Definition 2.10 (Loganathan, J., 2017) If $\xi(mn) \leq \min\{\xi(m), \xi(n)\}, \forall m, n \in T$ then the ξ , a FS of a SG, T is known as an anti-fuzzy ideal of T .

Definition 2. (Ragamayi, S, 2020) A relation $\sigma: T \times T \rightarrow [0,1]$ is known as a fuzzy relation on a FS, ξ of T if $\sigma(m, n) \leq \min\{\xi(m), \xi(n)\}, \forall m, n \in T$.

Definition 2.12 (Ragamayi, S, 2020) If $\sigma(m, n) \leq \min\{\mu(m), \mu(n)\} \forall \{m, n\} \in T$ then $G = (\mu, \sigma)$ is known to be a F-Graphon vertex set $V \neq \emptyset$ where μ and σ are FS on V and $V \times V$ correspondingly.

Definition 2.13 The bipolar fuzzy vertex chromatic number of complete bipolar fuzzy graph $G = (A, B)$ is (n, n) , where n is the number of vertices of G .

Definition 2.14 The edge chromatic number of complete bipolar fuzzy graph $G = (A, B)$ on n vertices is (n, n) , if n is odd and is $(n-1, n-1)$, if n is even.

3. Vertex and Edge Chromatic Number of a Bipolar Fuzzy of a Complete Bipartite Graph

In this segment, we propose the concept of BFG signifying a Route network representing a regular Graph of a SG as a generalization of BFG and Regular Graph and C-Graph. Here, we work on simple graphs having limited number of routes (Edges), Nodes(vertices). Also, we describe Vertex and Edge Chromatic Number of a Bipolar Fuzzy of a Complete Bipartite Graph $K_{1,3}$ and $K_{2,3}$ through examples. Moreover, we discussed about Cartesian product of two BFRGS.

Definition 3.1 Let $Gr(V, A, B)$ be a Regular graph signifying a Route network system. Let (V) be a commutative SG with finite vertices. If $A = (\mu_{AP}, \mu_{AN})$ is a BF set on V , $B = (\sigma_{BP}, \sigma_{BN})$ is a BF set in B where $\mu_{AP}: V \rightarrow [0,1], \mu_{AN}: V \rightarrow [-1,0], \sigma_{BP}: V \times V \rightarrow [0,1]$ for which $\sigma_B^P(xy) \leq \min\{\mu_A^P(x), \mu_A^P(y)\}, \forall \{x, y\} \in V \times V = V^2$ and $\sigma_{BN}: V \times V \rightarrow [-1,0]$ for which $\sigma_B^N(xy) \leq \max\{\mu_A^N(x), \mu_A^N(y)\}, \forall \{x, y\} \in V \times V = V^2$ and $\sigma_B^P(xy) = \sigma_B^N(xy) = 0, \forall \{x, y\} \in V^2 - E$, then $G = (V, A, B)$ is called a BFRGS and is symbolized by $Gr = (V, A, B, \mu, \sigma)$.

Definition 3.2 Let $Gr(V_1, A, B, \mu, \sigma)$ be a BFRGS.

- (1) The order of a BFRGS, $Gr(V_1, A, B, \mu, \sigma)$ is designated by $O(G) = (OP(G), ON(G))$ and is distinguished as $OP(G) = \sum_{a \in V} \mu_P(a)$ and $ON(G) = \sum_{a \in V} \mu_N(a)$.
- (2) The size of a BFRGS, $Gr(V_1, A, B, \mu, \sigma)$ is designated by $S(G) = (SP(G), SN(G))$ and is distinguished as $SP(G) = \sum_{a_1 a_2 \in \overline{V^2}} \sigma_{BP}(a_1 a_2)$ and $SN(G) = \sum_{a_1 a_2 \in \overline{V^2}} \sigma_{BN}(a_1 a_2)$.
- (3) The open neighbourhood degree of a vertex V of BFRGS (V_1, A, B, μ, σ) is defined as $D(a) = (D^P(a), D^N(a))$, were

$$D^P(a) = \sum_{ab \in \overline{V^2}} \sigma_B^P(ab) \text{ and } D^N(a) = \sum_{ab \in \overline{V^2}} \sigma_B^N(ab) \text{ --- (1)}$$

Example 3.3 Let $V_1 = \{K, L, M, N\}$. The ‘.’ is a binary operation on V_1 is defined by

.	(4) K	(5) L	(6) M	(7) N
K	(8) K	(9) L	(10) M	(11) N

(12) L	(13) L	(14) M	(15) N	(16) K
(17) M	(18) M	(19) N	(20) K	(21) L
N	(22) N	(23) K	(24) L	(25) M

A Route Network, $Gr(V_1, E_1)$ is taken with route set, $E_1 = \{(K, L), (K, M), (K, N), (L, N), (L, M), (M, N)\}$ where (V_1, \cdot) is a finite junctions of SG. Then

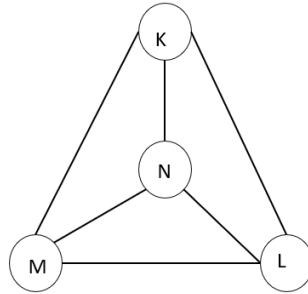


Figure 1. A Regular Network graph (k_4)

Let a FS, $\mu_A^P: V_1 \rightarrow [0,1]$ be a Positive membership degree of A, which is distinguished for every $p \in V_1$ and $\{p, q\} \in E_1$

$$\mu_A^P(p) = \begin{cases} 0.7 & \text{if } p = K \\ 0.6 & \text{if } p = L \\ 0.4 & \text{if } p = M \\ 0.1 & \text{if } p = N \end{cases} \text{ and } \sigma_B^P(pq) = \begin{cases} 0.9 & \text{if } \overline{pq} = \overline{KL} \\ 0.7 & \text{if } \overline{pq} = \overline{KM} \\ 0.6 & \text{if } \overline{pq} = \overline{KN} \\ 0.5 & \text{if } \overline{pq} = \overline{LN} \\ 0.4 & \text{if } \overline{pq} = \overline{LM} \\ 0.2 & \text{if } \overline{pq} = \overline{MN} \end{cases} \text{----- (2)}$$

$$\mu_A^N(p) = \begin{cases} -0.4 & \text{if } p = K \\ -0.3 & \text{if } p = L \\ -0.2 & \text{if } p = M \\ -0.2 & \text{if } p = N \end{cases} \text{ and } \sigma_B^N(pq) = \begin{cases} -0.5 & \text{if } \overline{pq} = \overline{KL} \\ -0.5 & \text{if } \overline{pq} = \overline{KM} \\ -0.4 & \text{if } \overline{pq} = \overline{KN} \\ -0.3 & \text{if } \overline{pq} = \overline{LN} \\ -0.3 & \text{if } \overline{pq} = \overline{LM} \\ -0.3 & \text{if } \overline{pq} = \overline{MN} \end{cases} \text{----- (3)}$$

Then from Definition 3.1, $Gr(V_1, A, B, \mu, \sigma)$ is a BFRGS of V_1 .

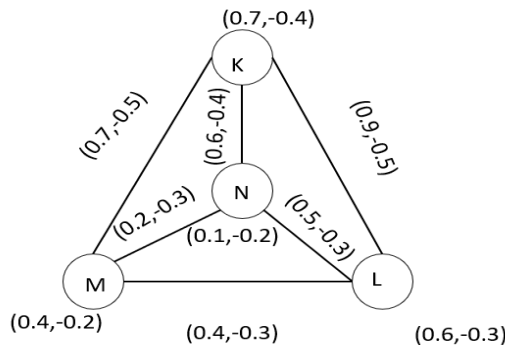


Figure 2. A Bi-polar fuzzy Route Network of a Regular graph (K_4)

Since $Gr(V_1, E_1) = k_4$ which is a complete Regular graph, and from definition 3.2, we can also say that the BFRGS, $Gr(V_1, A, B, \mu, \sigma)$ is an anti- BF-IGS, V_1 .

1. The order of a BFRGS, $Gr(V, A, B, \mu, \sigma)$ is designated by $O(G) = (OP(G), ON(G))$ which is distinguished as

$$OP(G) = \sum_{a \in V} \mu^P(a) = \mu^P(K) + \mu^P(L) + \mu^P(M) + \mu^P(N) = 0.7 + 0.6 + 0.4 + 0.1 = 1.8 \text{ and}$$

$$\begin{aligned}
 \text{ON}(G) &= \sum_{a \in V} \mu^N(a) \\
 &= \mu^N(K) + \mu^N(L) + \mu^N(M) + \mu^N(N) \\
 &= -0.4 - 0.3 - 0.2 - 0.2 \\
 &= -1.1
 \end{aligned}$$

2. The size of a BFRGS, $Gr(V, A, B, \mu, \sigma)$ is designated by $S(G) = (SP(G), SN(G))$ which is distinguished as

$$\begin{aligned}
 SP(G) &= \sum_{a_1 a_2 \in \overline{V^2}} \sigma_B^P(a_1 a_2) = \sigma_B^P(KL) + \sigma_B^P(KM) + \sigma_B^P(KN) + \sigma_B^P(LN) + \sigma_B^P(LM) + \sigma_B^P(MN) \\
 &= 0.9 + 0.7 + 0.6 + 0.5 + 0.4 + 0.2 \\
 &= 3.3
 \end{aligned}$$

$$\begin{aligned}
 SN(G) &= \sum_{a_1 a_2 \in \overline{V^2}} \sigma_B^N(a_1 a_2) = \sigma_B^N(KL) + \sigma_B^N(KM) + \sigma_B^N(KN) + \sigma_B^N(LN) + \sigma_B^N(LM) + \sigma_B^N(MN) \\
 &= -0.5 - 0.5 - 0.4 - 0.3 - 0.3 - 0.3 \\
 &= -2.3
 \end{aligned}$$

The open neighborhood degree of a vertex V of BFRGS (V, A, B, μ, σ) is distinguished as $D(a) = (D^P(a), D^N(a)), \forall 'a'$ in V , where

$$D^P(K) = \sigma_B^P(KL) + \sigma_B^P(KM) + \sigma_B^P(KN) = 0.9 + 0.7 + 0.6 = 2.2$$

$$D^P(L) = \sigma_B^P(LK) + \sigma_B^P(LN) + \sigma_B^P(LM) = 0.9 + 0.5 + 0.4 = 1.8$$

$$D^P(M) = \sigma_B^P(MK) + \sigma_B^P(ML) + \sigma_B^P(MN) = 0.7 + 0.4 + 0.2 = 1.3$$

$$D^P(N) = \sigma_B^P(NK) + \sigma_B^P(NM) + \sigma_B^P(NL) = 0.6 + 0.2 + 0.5 = 1.3 \text{ and}$$

$$D^N(K) = \sigma_B^N(KL) + \sigma_B^N(KM) + \sigma_B^N(KN) = -0.5 - 0.5 - 0.4 = -1.4$$

$$D^N(L) = \sigma_B^N(LK) + \sigma_B^N(LN) + \sigma_B^N(LM) = -0.5 - 0.3 - 0.3 = -1.1$$

$$D^N(M) = \sigma_B^N(MK) + \sigma_B^N(ML) + \sigma_B^N(MN) = -0.5 - 0.3 - 0.3 = -1.1$$

$$D^N(N) = \sigma_B^N(NK) + \sigma_B^N(NM) + \sigma_B^N(NL) = -0.4 - 0.3 - 0.3 = -1.0$$

The neighborhood of each vertex is 3 {i.e., $N(x) = 3 \forall x \in V$ }, Since $G = (V, E) = K_4$, which is a complete Regular graph.

Definition 3.4: A BFRGS of $Gr(V_1, A_1, B_1, \mu, \sigma)$ is stated to be semi strong if $\mu_{B_1}^P(kp) = \min\{\mu_{A_1}^P(k), \mu_{A_1}^P(p)\}$ or $\mu_{B_1}^N(kp) = \max\{\mu_{A_1}^N(k), \mu_{A_1}^N(p)\}, \forall k, p \in E$.

Theorem 3.5: If $Gr_1 \times Gr_2$ is strong BFRGS, then at least Gr_1 or Gr_2 must be semi- strong.

Proof. Suppose that Gr_1 and Gr_2 are not semi-strong BFRGS.

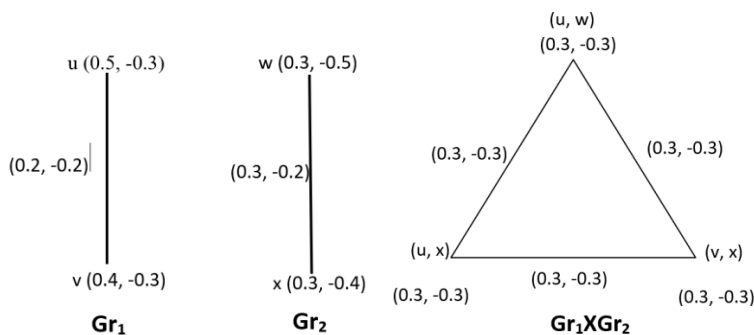
Then $\exists k_1 p_1 \in E_1; k_2 p_2 \in E_2 \exists$

$$\mu_{B_1}^P(k_1 p_1) < \min\{\mu_{A_1}^P(k_1), \mu_{A_1}^P(p_1)\} \text{ or}$$

$$\mu_{B_1}^N(k_1 p_1) > \max\{\mu_{A_1}^N(k_1), \mu_{A_1}^N(p_1)\} \text{ and}$$

$$\mu_{B_1}^P(k_2 p_2) < \min\{\mu_{A_1}^P(k_2), \mu_{A_1}^P(p_2)\} \text{ or}$$

$$\mu_{B_1}^N(k_2 p_2) > \max\{\mu_{A_1}^N(k_2), \mu_{A_1}^N(p_2)\}$$



Let $E = \{(x, k_2)(x, p_2): x \in V_1, k_2 p_2 \in E_2\} \cup \{(k_1, z)(p_1, z): k_1 p_1 \in E_1, z \in V_2\}$.

Let $\{(x, k_2)(x, p_2)\} \in E$.

Then we have,

$$\begin{aligned} \mu_{B_1}^P \times \mu_{B_2}^P((x, k_2)(x, p_2)) &= \min \{ \mu_{A_1}^P(x), \mu_{B_2}^P(k_2 p_2) \} < \min \{ \mu_{A_1}^P(x), \mu_{A_2}^P(k_2), \mu_{A_2}^P(p_2) \} \\ &= \min \{ \min \{ \mu_{A_1}^P(x), \mu_{A_2}^P(k_2) \}, \min \{ \mu_{A_1}^P(x), \mu_{A_2}^P(p_2) \} \} \\ &= \min \{ (\mu_{A_1}^P \times \mu_{A_2}^P)(x, k_2), (\mu_{A_1}^P \times \mu_{A_2}^P)(x, p_2) \}. \end{aligned}$$

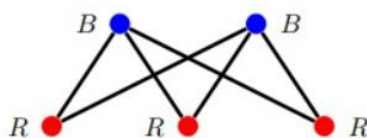
Definition 3.6 The vertex chromatic number of BFGS, $Gr(V_1, A_1, B_1, \mu, \sigma)$ is (n, n) and is denoted by $|\chi(Gr)| = (\chi^P(Gr), \chi^N(Gr)) = (n, n)$ where n is the number of vertices of Gr . And The fuzzy value of colouring of a vertex in BFGS is $(\mathbb{C}^P, \mathbb{C}^N)$ where \mathbb{C}^P , and \mathbb{C}^N are the fuzzy values of providing and not providing certain color to the vertex.

Definition 3.7 The edge chromatic number of BFGS, $Gr(V_1, A_1, B_1, \mu, \sigma)$ on n vertices is (n, n) , if n is odd and is $(n-1, n-1)$, if n is even and is denoted by

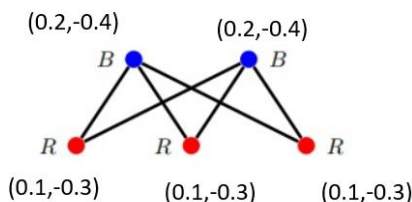
$$|\tau(Gr)| = (\tau^P(Gr), \tau^N(Gr)) = \begin{cases} (n, n) & \text{if } n \text{ is odd} \\ (n-1, n-1) & \text{if } n \text{ is even} \end{cases} \text{----- (4)}$$

And The fuzzy value of colouring of a edge in BFGS is $(\mathfrak{D}^P, \mathfrak{D}^N)$ where \mathfrak{D}^P , and \mathfrak{D}^N are the fuzzy values of providing and not providing certain color to the edge.

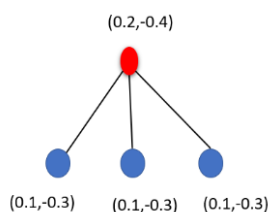
Example 3.8 The vertex chromatic number of a BFGS of a Complete Bipartite Graph is at most $(2,2)$. Consider a complete bipartite graph $K_{2,3}$.



$$\mu^P(V_i) = \begin{cases} 0.2 & \text{if } V_i = \text{Blue} \\ 0.1 & \text{if } V_i = \text{Red} \end{cases} \text{ and } \mu^N(V_i) = \begin{cases} -0.4 & \text{if } V_i = \text{Blue} \\ -0.3 & \text{if } V_i = \text{Red} \end{cases} \forall i = 1, 2, 3 \dots \text{----- (5)}$$



Also, For a complete bipartite graph $K_{1,3}$



Since no two adjacent colours should fill with same color, Hence, the vertex chromatic number of a BFGS of a Complete Bipartite Graph is $(2,2)$.

4. Labelling of Tri-polar Fuzzy of Graph

Definition 4.1 An edge AB is called a 3-polar fuzzy bridge of G if its removal reduces the strength of connectedness between some other pair of nodes in G .

Definition 4.2 In an edge AB , the node B is stated to be 3-polar fuzzy cut node of G if its removal reduces the strength of connectedness between some other pair of nodes in G .

Definition 4.3 In an edge AB , the node A is stated to be 3-polar fuzzy end node of G if it has exactly one strong neighbour in G .

Definition 4.4 In an edge AB of a 3-PFG is called strong edge if its weight is as great as the strength of connectedness of its 3-polar fuzzy end nodes.

Definition 4.5 An 3-PFP, $P = U - V$ is a sequence of distinct vertices $U = u_1, u_2, \dots, u_n = V, \forall j; \exists$ at least one $i \in P, P_i \circ (x_j \cdot x_{j+1}) > 0$.

Definition 4.6 A 3-PFP is strong if all its arcs are strong. A 3-PFP $x - y$ is said to be strongest 3-PFP if its strength equals to its connectedness.

Definition 4.7 A 3-polar fuzzy weakest arc is an arc having least degree of membership.

Example 4.8 Consider a 3-PFG, G as shown in Figure 3. it is familiar to see

1. x_2x_5, x_1x_2, x_2x_4 are 3-polar fuzzy bridges.
2. x_2 is 3-polar fuzzy cut node.
3. x_1, x_5, x_4 are 3-polar fuzzy end nodes of G.
4. x_1x_2, x_2x_5, x_2x_4 are 3-polar fuzzy strong arcs.
5. $x_1 - x_2 - x_5, x_1 - x_2 - x_4$ are 3-PFP.
6. $x_1 - x_2 - x_5, x_1 - x_2 - x_4, x_4 - x_2 - x_5$ are strongest 3-PFP.

In Figure 4, The 3-polar fuzzy weakest arc is x_1x_2 .

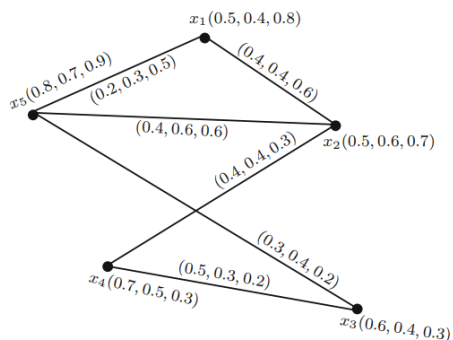


Figure 3. 3-polar Fuzzy Graph

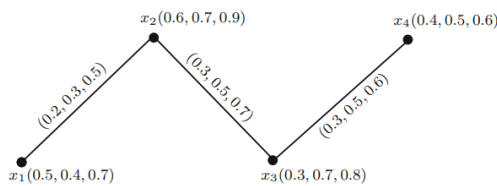


Figure 4. 3-polar Fuzzy Path

Definition 4.9 A graph $Gr_p^w = (\mathbb{C}_p^w, \mathcal{D}_p^w)$ is stated to be a 3-polar fuzzy labelling graph, if $\mathbb{C}_p^w : V \rightarrow [0,1]^3$ and $\mathcal{D}_p^w : V \times V \rightarrow [0,1]^3$ are bijective such that the membership values of vertices and edges are distinct and also $P_i \circ \mathcal{D}_p^w(ab) < P_i \circ \mathbb{C}_p^w(a) \wedge P_i \circ \mathbb{C}_p^w(b) \forall a, b \in V, 1 \leq i \leq 3$.

Example 4.10 Consider a 3-polar fuzzy labelling graph as shown in Figure 3, Labelling of 3-polar fuzzy set \mathbb{C}_p^w is

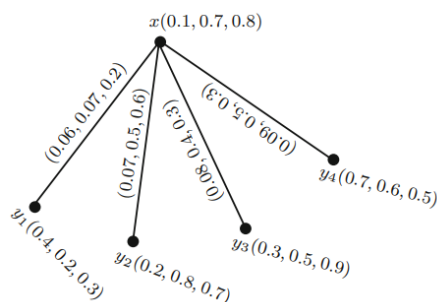
\mathbb{C}_p^w	x_1	x_2	x_3	x_4	x_5
$P_1 \circ \mathbb{C}_p^w$	0.5	0.5	0.6	0.7	0.8
$P_2 \circ \mathbb{C}_p^w$	0.4	0.6	0.4	0.5	0.7
$P_3 \circ \mathbb{C}_p^w$	0.8	0.7	0.3	0.3	0.9

Labelling of 3-polar fuzzy relation \mathcal{D}_p^w is,

\mathcal{D}_p^w	x_1x_2	x_1x_5	x_5x_2	x_5x_3	x_4x_2	x_3x_4
$P_1 \circ \mathcal{D}_p^w$	0.4	0.8	0.4	0.3	0.4	0.5
$P_2 \circ \mathcal{D}_p^w$	0.4	0.7	0.6	0.4	0.4	0.3
$P_3 \circ \mathcal{D}_p^w$	0.6	0.9	0.6	0.2	0.3	0.2

Definition 4.11 A star in 3-PFG can be defined as having two 3-polar fuzzy node sets V and E with $|V| = 1$ and $|E| > 1, \exists, P_i \circ \mathcal{D}_p^w(ab_j) > 0$ and $P_i \circ \mathcal{D}_p^w(b_jb_{j+1}) = 0, 1 \leq j \leq n$. It is denoted by $Sp(1, n)$.

Example 4.12 A 3-polar fuzzy star given below



5. Conclusion

In this article, we presented the concept of BFRG symptomatic of a Route network system on SG. Also, the notion of BFRGS and the concept of 3-PFP and 3-PFG is characterized through some examples.

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