Lict Subdivision Connected Domination in Graphs

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Abstract

For some graph G, the subdivision of lict graph $n[S(G)]$ of a graph G ,whose $V\{n[S(G)]\} = E[S(G)] \cup C[S(G)]$,in which two vertices are adjacent if and only if subsequent elements of $S(G)$ are adjacent or incident. A dominating set D_{Sc} is called connected dominating set of $n[S(G)]$, if D_{Sc} is also connected and its minimal cardinality is represented by $\gamma_{nsc}(G)$. we are going to relate $\gamma_{nsc}(G)$ with the elements and other standard components of G. Throughout this paper we impose the relation with different domination parameters.

Keywords: Lict Graph, Lict Subdivision Graph, Domination Number, Connected Domination Number.

AMS classification: 05C69, 05C70

1 . Introduction

The graphs considered here are simple, finite, non-trivial and undirected. Commanly p and q are the vertices and edges of graph G. Any unknown terms or notation in this paper can be seen in Harary [4] and V.R Kulli[6]. The study of domination in graphs was introduced by Ore [14] and Berge [2], and also it is been discussed by S.L. Mitchell and S.T.Hedetniemi[7]. The maximal / minimal degree of a vertex in G is $\Delta(G)/\delta(G)$. A vertex v is known as a cutvertex C if G gets disconnected after removing it . For some real number $x, \lceil x \rceil$ defines a least integer not less that x and $\lfloor x \rfloor$ represents the highest integer than x.

For a graph $G = (V, E), S \subseteq V$ is a vertex cover if all edge is incident on a vertex in S and its minimal cardinality of the set S is called the vertex covering number of G, denoted as $\alpha_0(G)$. Similarly a subset is called a edge covering of G if every vertex of G is incident with at least one edge. The minimal cardinality of the subset is known as edge covering number of G that is $\alpha_1(G)$. A subset S of vertex set/edge set of the graph G is called independent set if no two vertices/edges of S are adjacent in G. The vertex/edge independence number $\beta_0(G)/\beta_1(G)$ of a graph G is the maximum cardinality of an independent set of vertices/edges .

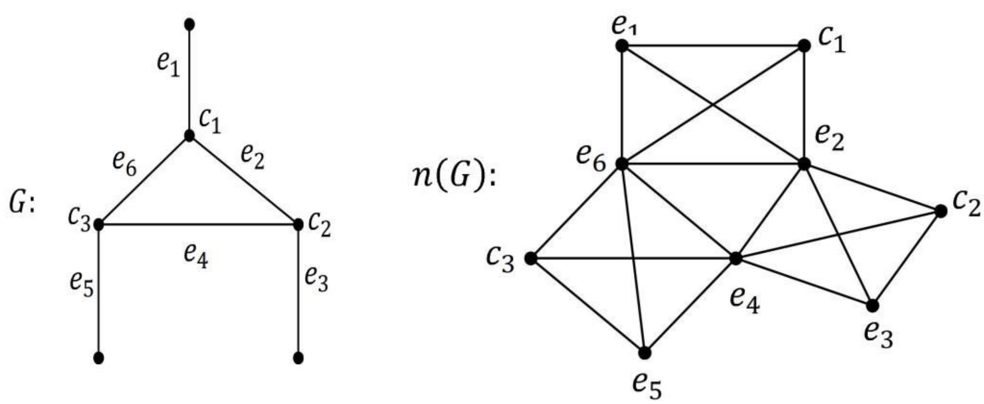
Let us know some basic definition from the domination theory.

A set D of a graph $G = (V, E)$is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set in G.

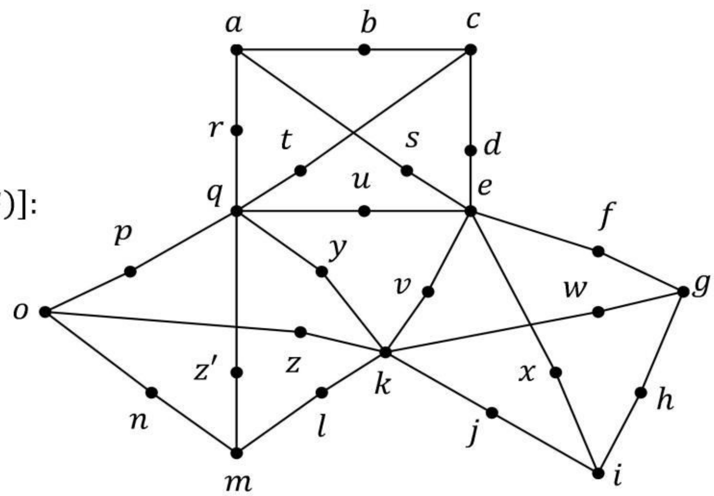
If the induced subgraph $\langle D \rangle$ is nonisolated vertex set, than D is said to be total dominating set, whose minimal cardinality will results to $\gamma_t(G)$ which is known as the total domination number. Introduction to this was given by Cockayne , Dawas and Hedethemi in [3] .

The edge set $F' \subseteq E$ is an edge dominating set of G if each $e \in E - F'$,where $e \in E(G)$ is adjacent to at least one edge in $F'$and is denoted by $\gamma'(G)$ of a graph G is the minimum cardinality. Currently wide domination parameters can be referred from V.R Kulli[6].

In figure1 we can see the connected domination number in lict subdivision graphs G.



$n[S(G)]$:



$$D_{sc} = \{a, b, c, d, e, f, i, k, l, m, n, o, t, v, x\} \quad : \quad \gamma_{nsc}(G) = 15$$

Figure 1

In this paper we establish the results on $\gamma_{nsc}(G)$ with the other standard domination parameter of G.

2 . Results:

To prove our further results the following theorems are helpful.

Theorem A[1] : For any connected (p, q) graph $G, \gamma'(G) \leq \left\lfloor \frac{p}{2} \right\rfloor$.

Now we establish the relation between $\gamma_{nsc}(G)$ with diameter of G.

3. Main Results:

Theorem 1: For any connected (p, q) graph G,

$$\gamma_{nsc}(G) \leq \left\lceil \frac{diam(G)+1}{3} \right\rceil .$$

Proof: Let $E = \{e_1, e_2, e_3, \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, e_{j-1}, e_j\}$ be the edge set of G and $C = \{c_1, c_2, c_3, \dots\dots\dots\dots\dots\dots\dots, c_{j-1}, c_j\}$ is cutvertices set of G. In $n(G), V[n(G)] = E(G) \cup C(G)$. Let $u_1, u_2, u_3, \dots\dots\dots\dots, u_n, n \leq j$ be the vertices of $n[S(G)]$ inserted on the edges of G, such that two vertices v_i and u_j of $n[S(G)$ are at a distance two if and only if they are adjacent in $n(G)$, where $u_i, u_j \in V[n(G)]$ gives the induced subgraph on $n[S(G)]$ isomorphic to $n(G)$. Now let $D_s = \{u_1', u_2', u_3', \dots\dots\dots\dots\dots\dots\dots, u_{k-1}', u_k'\}$ be the minimal dominating set of $n[S(G)]$ and let $\{v_1', v_2', v_3', \dots\dots\dots\dots\dots\dots\dots.\dots\dots, v_{k-2}', v_{k-1}'\}$ be the neighborhood vertices of D_s ,such that $\{u_1', u_2', u_3', \dots\dots\dots\dots\dots\dots\dots\dots\dots, u_k'\} \cup$

$\{v_1', v_2', v_3', \dots\dots\dots\dots\dots\dots\dots, v_{k-1}'\}$ forms vertex connectivity in $n[S(G)]$ which is also gives minimum connected dominating set D_{sc}, with the result $|D_{sc}| = \gamma_{nsc}(G)$ .Now let us take into account an arbitrary path of length which is $diam(G)$. This path which constitutes to diameter of G includes atmost two edges from the induced subgraph $\langle n[V]\rangle$ for each $v \in D_{sc}$. Moreover D_{sc} is connected $\gamma - set$. The diameteral length consists atmost $\gamma_{nc}[S(G)]-1$ edges connecting the neighborhood vertices of D_{sc}.

Thus we get $2\,\gamma_{nsc}(G) + \gamma_{nsc}(G) - 1 \geq diam(G)$

$$3\,\gamma_{nsc}(G) - 1 \geq diam(G)$$

$$\gamma_{nsc}(G) \geq \frac{diamG+1}{3} .$$

Theorem 2: For any connected (p, q) graph G,

$$\gamma_{nsc}(G) - diam(G) \leq 2p - \alpha_0(G).$$

Proof: Let $E = \{e_1, e_2, e_3, \dots\dots\dots\dots\dots, e_{j-1}, e_j\} \subseteq E(G)$ be the minimal set of edges which contributes to the farest distance between any two distinct vertices $u, v \in V(G)$, such that $dist(u, v) = diam(G)$. Let $B = \{v_1, v_2, v_3, \dots\dots\dots\dots\dots\dots\dots, v_{j-1}, v_j\}$ be the minimal number of vertices that covers all edges ,than $|B| = \alpha_0(G)$. Now let $C = \{c_1, c_2, c_3, \dots\dots\dots, c_{n-1}, c_n\}$ be the set of non end cutvertices and then $C' \subseteq C$ forms a $\gamma -$ set of G. Further let $V = \{u_1, u_2, u_3, \dots\dots\dots\dots\dots\dots\dots\dots, u_{j-1}, u_j\}$ be the vertex set of $n(G) = E(G) \cup C(G)$. Let $V = \{u_1u_1', u_2u_2', u_3u_3', \dots\dots\dots\dots, u_{j-1}, u_{j-1}'u_ju_j'\}$ be the vertex subset of $V\{n[S(G)]\}$ formed by inserting a vertex $u_i \in V\{n[S(G)]\}$ of degree atmost two between every edge of $n[S(G)]$ such that $\langle v\rangle \subseteq V\{n[S(G)]\}$. Now let $D_s = \{u_1', u_2', u_3', \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, u_{k-1}', u_k'\}, k \leq j$ be the vertex set in lict subdivision ,such that $D_s \subseteq V\{n[S(G)]\}$ which is minimal dominating set of $n[S(G)]$. Let $F' = \{e_1, e_2, e_3, \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, e_{i-1}, e_i\}$; $\forall\, i \leq j$ be a minimum independent set of edges in G that also corresponds to edge dominating set of G. Than for the corresponding set F', let$D' \subseteq V\{n[S(G)]\}$ forms a vertex dominating set of $n[S(G)]$.

If $D_{sc} = \{v_1', v_2', v_3', \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, v_{i-1}', v_i'\}$ be the vertex set in $n[S(G)]$ such that $D_{sc} \subseteq V\{n[S(G)]\} - D'$ and $D_s \in N(D)$ Now consider $D_s \subseteq D_{sc}$ such that $< D' \cup D_s >$is minimal connected dominating set, which gives $|D' \cup D_s| - diam(G) \leq 2p + |\beta|$. Consequently it follows that

$$\gamma_{nsc}(G) - diam(G) \leq 2p - \alpha_0(G).$$

The below theorem gives the connection of $\gamma_{nsc}(G)$ with total domination and $\delta(G)$.

Theorem 3: For any connected (p,q) graph G,

$$\gamma_{nsc}(G) - \gamma_t(G) \leq 2p + \delta(G) + 1$$

Proof: Suppose T be the γ_t −set by the minimality, for any vertex $v \in T$ the subgraph $\langle s - v \rangle$ contains an isolated vertex. Let $S_1 = \{v : v \in T\}$ and v_1 be the set of all end vertices in $\langle T_1 \rangle, B = T_1 - v_1$,further let C be the minimum set of vertices of $T - T_1$ such that each vertex v_i is adjacent to few of cutvertices , clearly $|C| = |v_1|$. Now let $T' = T - \{v_1 \cup C\}$ and every $u_i v_i \in \langle T_1 \rangle, 1 \leq i \leq k,$ so that T' forms a minimal total dominating set. Thus $|T'| = \gamma_t(G)$. Now by the discription of $n(G)$, let $F = \{v_1, v_2, v_3, \dots\dots\dots..., v_i\} \subseteq V[n(G)]$ or $\forall\, v_i \in e_i \subseteq E(G) \cup C(G)$ be the set vertices matching to the edges that are incident to the elements of T' in $n(G)$ and the set of cutvertices adjacent to $V(G)$. Let $F' = \{v_1 v_1', v_2 v_2', v_3 v_3', \dots\dots\dots, v_i v_i'\}$ be the set of vertices formed by inserting a vertex of degree atmost two between every adjacent vertices such that $|F'| = V\{n[S(G)]\}$. Now let $D_s = \{v_1, v_2, v_3, \dots\dots\dots\dots\dots.\dots., v_l\} \subseteq F'$, such that for every $v_k \in D_s$ thus there exists exactly one neighbour $v_{k+1} \in D_s$,$1 \leq i \leq l$ be the vertex set that contributes all the vertices in $n[S(G)]$ such that $D_s \subseteq V\{n[S(G)]\} - F'$ and $D_s \in N(F')$ which is minimal dominating set of $n[S(G)]$. Now let $D_s{}'$ be the set of vertices with property $D_s{}' \subseteq D_s$ than $\langle D_s \cup D_s{}' \rangle$ is the minimum connected dominating set of $n[S(G)]$ with the result $|D_s \cup D_s{}'| = \gamma_{ncs}(G)$. Since for any graph G, there exists at least one vertex $w \in V(G)$, so that $\deg(w) = \delta(G) \leq 1$. Clearly we conclude by above description as, $|D_s \cup D_s{}'| \cup |T'| \leq 2|V(G)| + \deg(w) + 1$

$$\gamma_{nsc}(G) - \gamma_t(G) \leq 2p + \delta(G) + 1.$$

Theorem 4 : For any connected (p,q) graph G,

$$\gamma_{nsc}(G) \leq 4p - \beta_0(G).$$

Proof : Suppose D_s be the vertex set of $n[S(G)$ where $D_s \subseteq V[n(G)] \subseteq V\{n[S(G)]\}$ such that $D_s \subseteq V[n(G)] - D'$ nd $D_s \in N(D')$. Now consider $D_{sc} \subseteq D_s$ such that $\langle D' \cup D_{sc} \rangle$ is the minimal connected dominating set which gives $|D' \cup D_{sc}| = \gamma_{nsc}(G)$ Let $B = \{v_1, v_2, v_3, \dots\dots\dots\dots\dots., v_i\}$; $\forall\, i \leq n \in V(G)$ be the maximum independent set of vertex set so that no vertices of G are adjacent to each other , hence $|\beta| = \beta_0(G)$. Finally we can conclude that

$$|D' \cup D_{sc}| \leq 4|V(G)| - |B|$$
$$\gamma_{nsc}(G) \leq 4p - \beta_0(G) \;.$$

Theorem 5: For any connected (p, q) graph G,

$$\gamma_{nsc}(G) \geq \Delta(G) \;.$$

Proof: Let D_{sc} be a γ_{nsc} –set of $n[S(G)]$. For every vertex $V \in D_{sc}$, let $D_s = \{u \,/\, u \in D_{sc} and\ u\ is\ adjacent\ to\ v\}$. Since v is a cut vertex $D_s \neq \phi$ and for any cut vertex $w \in v - D_{sc}$, u is not adjacent to w thus any domination set of $n[S(G)]$ must contain either v or w.

Hence , $\gamma_{nsc}(G) \geq |v - D_{sc}|$ or

$$D - \gamma(G) \leq \gamma_{nsc}(G)$$

Since $\gamma(G) \leq p - D(G)$

$$\Delta(G) \leq \gamma_{nsc}(G).$$

Theorem 6: For any connected graph G which is not trivial,

$$\gamma_{nsc}(G) \geq p + \gamma'(G).$$

Proof: Suppose $V(G) = p$, now let $E' = \{e_1, e_2, e_3, \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots, e_{k-1}, e_k\}$ be the edge set which covers all the vertices of G, so that $E' = \alpha_1(G)$ and $E' \subseteq E$. Let $B = E(G) - E'$ is a set with least number of end edges in G,such that $|B| = \beta_1$. Suppose $F' = \{e_1, e_2, e_3, \dots\dots\dots\dots, e_j\}$ be the minimum independent dominating set of G. Than F' itself is a dominating edge set of G, this corresponding edges of F' forms a vertices set $D_s = \{v_1, v_2, v_3, \dots\dots\dots\dots\dots, v_j\}$; $\forall\, v_j \in V\{n[S(G)]\}$ which is dominating set in $n[S(G)]$. If $D' = V\{n[S(G)]\} - D_s$ and let $D'' \in N(D_s)$ and $D' \subset D''$ in $n[S(G)]$. Then $\langle D_s \subset D' \rangle$ is $\gamma_c -$ set of subdivision $n(G)$,

Thus $|D_s \cup D'| \leq |\beta| + |E'| + |F'|$

$$\gamma_{nsc}(G) \geq \alpha_1 + \beta_1 + \gamma'(G)$$
$$\gamma_{nsc}(G) \geq p + \gamma'(G).$$

Theorem 7: For any connected (p, q) graph G,

$$\gamma_{nsc}(G) \geq \gamma_t(G) + \Delta(G).$$

Proof: If D be γ-set of G , if $T \subseteq V$ be a minimum set of vertices, that are adjacent to D so that $(D \cup T)$ produces total dominating set of G. Now let$D = \{v_1, v_2, \dots\dots, v_{j-1}, v_j\}$ be the dominating set of $n[S(G)]$ and $D' = \{u_1, u_2, u_3, \dots\dots\dots\dots, u_{k-1}, u_k\}$; $\forall\, k \leq j$ be the vertex set of $n[S(G)]$, which declares that $D_s \in N(D')$ and if we take $D'' \subset D'$ then the $< D'' \cup D_s >$ is the minimal connected dominating set of $n[S(G)]$. If suppose $V = \{u_1, u_2, u_3, \dots \dots \dots \dots, u_i\}$ is the vertex set with degree larger than one in G, then there is existence of atleast one vertex $v \in G$ with the highest degree $\Delta(G)$.

Clearly, $|D'' \cup D_s| \geq |D \cup T| + \Delta(G)$

Thus it follows that $\gamma_{nsc}(G) \geq \gamma_t(G) + \Delta(G)$

Consequently the result follows.

Theorem 8: For any (p, q) tree T,

$$\gamma_{nsc}(G) \leq q - \delta'(T) + \left\lceil \frac{q}{\Delta'(T)+1} \right\rceil.$$

Proof: Let $e \in E(T)$ with $\deg(e) = \delta'(T)$. Now by definition of $n(T), E \cup C = u \in V[n(G)]$, where c is set of cut vertices in T. Now let $n[S(G)]$ be subdivision graph of graph G obtained from $n(G)$ by adding a new vertex of $v \in \Delta = 2$ on each edge of $n(G)$. Let D_s be the minimal γ_{ns} −set of $n[S(G)]$ and $V_1 = V\{n[S(G)]\} - D_s$, let D' be the another vertex set adjacent to all the vertices of D_s , so that $D' \subset D_s$ and $D' \in N(D_s)$ the union of both gives a dominating connected set of $n[S(G)]$ where $\langle D' \cup D_s \rangle \in \gamma_{nsc}$ − set. If $\delta(T) \leq 2$, than $V\{n[S(G)]\} \geq 2$ and $q - \gamma_{nsc}(T) \geq 2$. Clearly $\gamma_{nsc}(T) \leq (q-2) \cup [q - \delta'(T)]$. Suppose $\delta'(T) \leq 2$, than for any edge $e' \in N(e), e' = w \in N(u)$ in $n[S(T)]$ and $D' \cup D_s \subseteq V\{n[S(G)]\} - N(u) \cup \{w\}$, hence $\gamma_{nsc}(T) \leq \{q - [\delta'(T) + 1]\}$, thus in any case $\gamma_{nsc}(T) \leq q - \delta'(T)$. Now since all components in $V\{n[S(G)]\}$ is adjoining atleast 1- component $'v'$ in $D' \cup D_s$ and $|V\{n[S(G)]\} - (D' \cup D_s)| \leq \sum degv_i$ and the fact that for any tree T, there exists one or more than one edge $e \in E(T)$, such that $\deg(e) = \Delta'(T)$, so we have

$$q - \gamma_{nsc}(T) = |V\{n[S(G)]\} - (D' \cup D_s)|$$

$$\leq \sum degv_i \leq \gamma_{nsc}(T) \,.\Delta'(T) + 1 \,; v \in D' \cup D_s$$

Therefore $\gamma_{nsc}(G) \leq q - \delta'(T) + \left\lceil \frac{q}{\Delta'(T)+1} \right\rceil$

Theorem 9: For any connected (p, q) graph G,

$$\gamma_{nsc}(G) \geq \left\lceil \frac{p}{2} \right\rceil + q \,.$$

Proof: Let $E = e_i$ for all $e_i\,; 1 \le i \le n$ belongs to an edge set of G and $C = \{c_1, c_2, c_3, \dots\dots\dots\dots, c_n\}$ be the cutvertex set in G, wherein $V[n(G)] = E \cup C$. Let $F = \{e_1', e_2', e_3', \dots\dots\dots\dots\dots\dots\dots\dots\dots.., e'_n\} \in E \cup C$ the edge set on $n(G)$. Le $F' = \{u_1'v_1', u_2'v_2', u_3'v_3', \dots\dots\dots\dots\dots, u_n'v_n'\}$ denotes edge set that are adjacent to the edges of F that belongs to $E\{n[S(G)]\}$. $D_s = \{v_1', v_2', v_3', \dots\dots\dots\dots, v_n'\}$ be the minimal dominating set of lict subdivision of graph G, where $v_i' \in V\{n[S(G)]\}\,; \forall\, 1 \le i \le n$ such that $E(G) \cup V[n(G)] \subseteq V\{n[S(G)]\}$ which gives $D_{sc} \subseteq V\{n[S(G)]\}$ which gives $|D_{sc}| = V\{G\}$ of lict subdivision graph.. If for some $v_i \in S$ such that $v_i \notin D_s$ in $n[S(G)]$ than $D_{sc} = (E \cup C) \cup F'$ otherwise $D_{sc} = F'$ further let $D' = \{v_1, v_2, v_3, \dots\dots\dots\dots\dots., v_n\}$ for some $v_i \in V$ for subdivision of $n(G)$ $, D' \in N(D_s)$ and $D' \subseteq V\{n[S(G)]\} - D_{sc}$. Now consider $D'' \subset D'$ such that $(D_s \cup D'')$ is the minimum connected γ–set of $n[S(G)]$. Further by theorem[A], clearly it follows that

$$|D_{sc} \cup D''| \ge \left\lceil \frac{p}{2} \right\rceil + q,$$

and thus we have $$\gamma_{nsc}(G) \ge \left\lceil \frac{p}{2} \right\rceil + q$$

Consequently the result follows.

Conclusion:

Here we discuss and establish the results on connected domination number of Lict subdivision graphs. Also we derive few relations between connected Lict subdivision domination number and some other standard parameters. Also we extend this results in future.

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