GENERALISATIONOF POWER-3 MEAN LABELING OF GRAPHS

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Abstract

Mean Labeling Graphs first introduced by Somasundram.S and Ponraj .R in the year 2003.Several authors began to investigate various methods of Mean labelling after then.In the picture filtering procedure, Power-3 mean labelling is very essential. The Power-3 Mean edges are generalized in this study so that the edges have separate labels.

Keywords :

Power-3 Mean,Power-3 Mean Labeling ;Power-3 Mean edge labels

1.Introduction

The following generalization of Power-3 Mean edges labels is obtained based on the notion of Power-3 Mean labelling and with reference to (Jussi Pahikkala 2010

A graph G with p vertices and q edges is called a Power -3 mean graph,if it is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $1,2, \ldots \ldots, q+1$ in such a way that in each edge $e = uv$ is labelled with $f(e = uv) = \left\lceil \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rceil$ or $\left\lfloor \left(\frac{x^3+y^3}{2}\right)^{\frac{1}{3}} \right\rfloor$. Then the edge labels are distinct. In this case f is called Power -3 Mean labeling of G and G is called a Power- 3 Mean Graph.

2.Important Results

Theorem2.1

If u and v are the vertex with labels i and $i + m$ of a graph $G$with Power -3 Mean labeling f then the edge labels are :

If m is even

$$f(u,v) = \begin{cases} i + {}^{m}/_{2} \ or \\ i + {}^{m}/_{2} + 1, \forall \ i < \dfrac{2}{3(m+m^2)} \ if \ l = 0 \\ i + {}^{m}/_{2} + l \ \ or \\ i + {}^{m}/_{2} + (l+1), \ \forall \frac{2l^3}{3(m+m^2)} \ \le i \le \frac{2(l+1)^3}{3(m+m^2)} \ if \ l = 1,2,3, \dots \dots \end{cases}$$

If m is odd

$$f(u,v) = \begin{cases} i + {}^{m-1}/_{2} \ \ or \\ i + {}^{m-1}/_{2} + 1, \forall \ i > \dfrac{3m^2+2m}{2} \ if \ l = 0 \\ i + {}^{m-1}/_{2} + l \ \ or \\ i + {}^{m-1}/_{2} + (l+1), \ \forall \frac{2l^3 - m - 3(m+m^2)}{3(m+m^2)} \ \le i \le \frac{2(l+1)^3 - m - 3(m+m^2}{2} \end{cases}$$

Proof:

Let $f(u) = i$ and $f(v) = i + m$,where $m = 1,2,3, \dots$. Be the vertex labels with Power-3 mean labeling f

Then ,$f(uv) = \left[\frac{i^3+(i+m)^3}{2}\right]^{\frac{1}{3}}$

$$= \left[\frac{\begin{matrix} i^3 + i^3 \\ +m^3 + 3i^2m + 3im^2 \end{matrix}}{2}\right]^{\frac{1}{3}}$$

$$\left[\frac{2i^3 + m^3 + 3i^2m + 3im^2}{2}\right]^{\frac{1}{3}}$$

$$= \left[i^3 + \frac{m^3 + 3mi^2 + 3im^2}{2}\right]^{\frac{1}{3}}$$

$$= (i^3)^{\frac{1}{3}} + \left(\frac{m^3}{2}\right)^{\frac{1}{3}} + \left[\frac{3(i^2m + im^2)}{2}\right]^{\frac{1}{3}}$$

$$f(uv) = i + \frac{m}{\sqrt[3]{2}} + R$$

Where $R = \left[\frac{3(i^2m+im^2)}{2}\right]^{\frac{1}{3}} \geq 0$

Suppose $R = 0$, then $\left[\frac{3(i^2m+im^2)}{2}\right]^{\frac{1}{3}} = 0$

$\Longrightarrow m = 0$, which is a contradiction .

Therefore ,$R > 0$

Consider ,the case when m is even .

In this case $i + \frac{m}{2}$ is an integer .

Then $f(uv) = i + \frac{m}{2} + l$ or $i + \frac{m}{2} + (l + 1)$;

Clearly $l \leq R \leq l + 1$

Let the integral part of R be denoted by l

Then $l = 0,1,2, \dots \dots$

That is $l \leq \left[\frac{3(i^2m+im^2)}{2}\right]^{\frac{1}{3}} < l + 1$

$$l^3 \leq \frac{3(i^2m + im^2)}{2} < ((l + 1)^3)$$

$2l^3 \leq \frac{3(i^2m+im^2)}{2}$ <$2(l + 1)^3)$

That is $l \leq \left[\frac{3(i^2m+im^2)}{2}\right]^{\frac{1}{3}} < l + 1$

$$l^3 \leq \frac{3(i^2m + im^2)}{2} < ((l+1)^3)$$

$2l^3 \leq \frac{3(i^2m+im^2)}{2}$ $<2(l+1)^3)$

On simplification ,$l \neq 0$ and $i < \frac{2l^3}{3lm+3m^2}$ if $l \neq 0$ and $i < \frac{2(l+1)^3}{3lm+3m^2}$

That is ,

$\frac{2l^3}{3(m+m^2)} \leq i \leq \frac{2(l+1)^3}{3(m+m^2)}$if $l \neq 0$

When $l = 0$, $0 \leq \left[\frac{3(im+im^2)}{2}\right]^{\frac{1}{3}} < 1$

$\Rightarrow m^2 \geq 0$ and $i < \frac{2}{3(m+m^2)}$

Therefore

$$f(u,v) = \begin{cases} i + {}^{m}/_{2} \ or \\ i + {}^{m}/_{2} + 1, \forall\ i < \dfrac{2}{3(m+m^2)}\ if\ l = 0 \\ i + {}^{m}/_{2} + l \ \ or \\ i + {}^{m}/_{2} + (l+1),\ \ \forall \frac{2l^3}{3(m+m^2)} \ \ \leq i \leq \frac{2(l+1)^3}{3(m+m^2)}\ \ if\ l = 1,2,3,\dots\dots \end{cases}$$

When m is odd ,m is not an integer ,but $\frac{m-1}{2}$ will be an integer .

$f(u,v) = i + \frac{m-1}{2}$+R,Where$R = \left(\frac{3(i^2m+im^2)+m+2i}{2}\right)^{\frac{1}{3}} \geq 0$

Now $R = 0{,}3(i^2m + im) + m + 2i = 0$

$\Rightarrow 2i = -m - 3(im + im^2)$, which is a Contradition .

Therefore ,$R > 0$

Then $f(uv) = i + \frac{m-1}{2} + l$ or $i - \frac{m-1}{2} + (l+1)$

Clearly $l \leq R \leq l + 1$

Let l be the integral part of R ,then $l = 0,1,2,3,\dots\dots$

That is ,$l \leq \left[\frac{3(i^2+im^2)+m+2i}{2}\right]^{\frac{1}{3}} < l+1$,Which implies

$2l^3 \leq 3(i^2m+im^2)+m+2i$ and $3(i^2m+im^2)+m+2i < 2(l+1)^3$

On simplification ,

$$\frac{2l^3-m-3(m+m^2)}{2} \leq i \leq \frac{2(l+1)^3-m-3(m+m^2)}{2} \quad if\ l > \frac{1}{2}, i.e) l \geq 1$$

Now Consider the case when $l = 0$

When $l = 0$,$0 \leq \left[\frac{3(i^2m+im^2)+m+2i}{2}\right]^{\frac{1}{3}}$ <1

$$\Rightarrow 3(i^2m+im^2)+m+2i \geq 0,$$

Which is true always and $i > \frac{2m+3m^2}{2}$

Therefore

$$f(u,v) = \begin{cases} i + {}^{m-1}/_2 \quad or \\ i + {}^{m-1}/_2 + 1, \forall\ i > \dfrac{3m^2+2m}{2} \ if\ l = 0 \\ i + {}^{m-1}/_2 + l \quad or \\ i + {}^{m-1}/_2 + (l+1),\ \forall \frac{2l^3-m-3(m+m^2)}{3(m+m^2)} \leq i \leq \frac{2(l+1)^3-m-3(m+m^2}{2}\ l = 1,2,3\ldots \end{cases}$$

Hence (I) &(II) provide Power-3 Mean edge labels

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The above result can be applied to get $f(im)$ for any i and m by putting $m-i$ in the place m.But the range for i turn out to be complicated .So consider $f(im)$ seperatively for $m = 0,1,2 \ldots\ldots$

$$f(im) = \left[\frac{i^3+m^3}{2}\right]^{\frac{1}{3}}$$

$$f(im) = \left[\frac{i^3+m^3}{2}\right]^{\frac{1}{3}}$$

$$= i + \frac{m}{\sqrt[3]{2}} + \left[\frac{3(im^2 + i^2m).}{2}\right]^{\frac{1}{3}}$$

$$= i + \frac{m}{\sqrt[3]{2}} + R$$

Where $R = \left[\frac{3(im^2+i^2m).}{2}\right]^{\frac{1}{3}}$

Let $l = [R]$ then $f(im) = i - m + l$ or $i - m + (l + 1)$.

$R = 0 \Rightarrow m = 0$.Hence$f(io) = i - 0 = i$

Hence ,let as assume $m > 0$

Clearly $l \leq R < l + 1$

$$\Rightarrow l < \left[\frac{3(im^2 + i^2m).}{2}\right]^{\frac{1}{3}} < l + 1$$

$$\frac{l^3}{3m + 3m^2} \leq i < \frac{2(l + 1)^3}{3m + 3m^2}$$

When $l = o$

$$0 \leq \left[\frac{3(im^2 + i^2m).}{2}\right]^{\frac{1}{3}} < 1$$

$$i > 3m + 3m^2$$

Hence

$f(im)$

$$= \begin{cases} i, & if\ m = o \\ i - \frac{m}{\sqrt[3]{2}}\ or\ i - \frac{m}{\sqrt[3]{2}} + 1, & \forall\, i > 3m + 3m^2, l = 0, m \neq 0 \\ i + \frac{m}{\sqrt[3]{2}} + 1\ or\ i + \frac{m}{\sqrt[3]{2}} + (l + 1), if\ \frac{l^3}{3m + 3m^2} \leq i \leq \frac{2(l + 1)}{3m + 3m^2} l = 1,2,3 \dots \dots and\ m \neq 0 \end{cases}$$

Applying this result for $m = 0,1,2,3 \dots \dots$distinct Power -3 Mean edge labels are obtained .

Result 2.2

$\lceil\ i + \frac{m-1}{2} \leq f(i\,i + m) \leq i + m\ \rfloor$

For we know

$$f(im+i)=\begin{cases} When\ m\ is\ odd \\ i+{}^{m}/_{2}\ or \\ i+{}^{m}/_{2}+1, \forall i<\frac{2}{3(m+m^2)}, if\ l=0 \\ i+{}^{m}/_{2}+l\ \ or \\ i+{}^{m}/_{2}+(l+1), \forall \frac{2l^3}{3(m+m^2)}\leq i\leq \frac{2(l+1^3)}{3(m+m^2)} \quad l=1,2,3\ldots.. \\ When\ m\ is\ odd \\ i+\frac{m-1}{2}\ \ or \\ i+\frac{m-1}{2}+1, \forall\ i>\frac{3m^2+2m}{2}, l=0 \\ i+\frac{m-1}{2}+l\ \ or \\ i+\frac{m-1}{2}+(l+1), if \frac{2l^3-m-3(m+m^2)}{2}\leq l\frac{2((l+1)^3)-m-3(m+m^2)}{2} \end{cases}$$

Clearly $i+\frac{m}{2}$ (m is even) and $i+\frac{m-1}{2}$ (m is odd) are the lower bound for $f(i\ im)$.

Combining both the cases

A lower bound for $f(i\ i+m)=\begin{cases} i+{}^{m}/_{2}, if\ m\ is\ even \\ i+\frac{m-1}{2}, if\ m\ is\ odd \end{cases}$

$\lceil\ i+\frac{m-1}{2}\rfloor, \forall m$

To find an upper bound

Claim :

$$\left[\frac{i^3+(i+m)^3}{2}\right]^{\frac{1}{3}}\leq i+m$$

Consider

$$\left[\frac{i^3+(i+m)^3}{2}\right]^{\frac{1}{3}}\leq i+m$$

$$\Leftrightarrow \frac{i^3+(i+m)^3}{2}\leq (i+m)^3$$

$$\Leftrightarrow i^3+(i+m)^3\leq 2(i+m)^3$$

$$\Leftrightarrow i^3 \leq 2(i+m)^3 - (i+m)^3$$

$$\Leftrightarrow i^3 \leq (i+m)^3$$

$$\Leftrightarrow i^3 \leq i^3 + 3i^2m + 3im^2 + m^3$$

$$\Leftrightarrow 3(i^2m + im^2)m^3 \geq 0$$

Which is true

Since $i \geq 0, m \geq 0$

Therefore $i + m$ is an upper bound

Hence $\lceil\ i + \frac{m-1}{2} \leq f(i\,i+m) \leq i + m\ \rfloor$

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