Optimize investment portfolios Using statistical methods and Fuzzy approach

Raheleh Zamini
Department of Mathematics, Faculty of Mathematical Sciences and Computer, Kharazmi University, Tehran, Iran.
rahelehzamini@yahoo.com

Abstract
The aim of this research is the optimal portfolio selection using the Fuzzy Analytic Hierarchy Process (FAHP). This research is applied in terms of objective and descriptive-analytical in terms of data collection and processing. The assessment models of strategic programs such as the FAHP are utilized to conduct this research. The studied population consists of all companies listed on Tehran Stock Exchange during 2014-2019 and 7 companies are considered as the samples. The descriptive statistics including the demographic data of statistical sample such as the tables of frequency distribution, descriptive diagram, etc, are utilized for data analysis in this research and also the inferential statistics by FAHP is used for weighting the options. According to the results, the prioritization of portfolio selection criteria is as follows: Expected return, liquidity and risk criterion. Furthermore, the prioritization of studied listed companies is as follows: Esfahan's Mobarakeh Steel Company, Pipe and Machine Manufacturing Company of Iran, Borujerd textile Company, Rolling Mill & Steel Production Co., IRAN Merinos Co, Sadid Industrial Group, and Tous Wool Weaving Co.
Keywords: Optimal portfolio selection, expected return, liquidity rating, risk criterion, Fuzzy Analytic Hierarchy Process (FAHP), Tehran Stock Exchange

1- Introduction

The concept of risk is changed with development of scientific research in recent years. If the securities are risky, the investor's main concern is to determine the portfolio with maximum utility. This means selecting the optimal portfolio from the possible portfolios and is called the optimal portfolio selection issue. The investors are seeking to select the portfolio with the lowest risk and highest return; in other words, the best portfolio in order to help the development of capital market (Raei, 2008).

On the one hand, the investment development attracts the inefficient capitals and directing them to productive economic sectors, and on the other hand, the investments are in industries with the lower risk and higher profits according to the investors' orientation (based on the risk and return) and this will lead to the efficient allocation of resources. The optimal allocation of resources is among the most important economic functions of capital markets. The stock exchange, as an important index of economic development in countries, plays the considerable role in development of investments and their efficient allocation. On the other hand, the economic and cultural barriers, governmental laws and regulations and the

uncertainty in stock market inhibit the growth of investments in stock exchange. Due to the chaotic situation in Tehran Stock Exchange in recent years, the numerous studies are conducted to overcome the existing problems (Azar, 2012). The portfolio selection theory was developed by Markowitz in 1952. Markowitz's theory is based on the risk optimization and portfolio return with several financial assets. The optimal portfolio selection issue was among the most important issues in modern affairs from the 1950s to 1952s. (Gordon, 2001)

For optimal portfolio selection, Markowitz introduced "mean- variance" model and Weatherstone introduced the "value at risk" model. The mean is as a criterion of return and the variance as a criterion of risk in "mean- variance" model; furthermore, the standard deviation and variance are the criteria of risk assessment assuming the normal distribution of rate of return (Raei, 2008). According to the importance of mean-variance model, the analysis of variance is the basis of equilibrium model extraction at various times and provides the capital asset pricing model (CAPM), Sharpe model, Lintner model, black and white model, and two-factor model. (W.F. Sharpe, 1999) The main task of optimal portfolio selection is to allocate the funds between different securities in a way that the risk and portfolio return are optimized. In general, the portfolio optimization is the process of portfolio analysis and management of existing assets in portfolio in a way that the maximum return is obtained for a specified level (desired level) of risk. (Keshtkar, 2008)

The FAHP methodology is established based on the concept of fuzzy set theory proposed by Professor Lotfizadeh in 1965 (Celik, 2009). The Fuzzy Analytic Hierarchy Process (FAHP) develops Saaty's AHP through its combination with fuzzy set theory. After creating the hierarchical structure for the issue which should be solved, the relative fuzzy scales are utilized in fuzzy AHP to show the relative importance of factors corresponding to the criteria. Therefore, a fuzzy judgment matrix is constructed; the final rates of options are provided by fuzzy numbers, and then the optimal option is obtained through ranking the fuzzy numbers by specific Algebraic operators (Doran, 2008). Thus, the main question of this study is raised as follows: How can we choose the optimal portfolio selection by FAHP?

1-1- Theoretical principles of research

In Markowitz's optimal portfolio selection theory, he assumes that all investors make choices based on both risk and return criteria. However, numerous studies have criticized ignoring investors' other preferences in Markowitz's model. Normally, the investor simultaneously follows the preferences and conflicting goals such as the return, risk and liquidity in portfolio selection issue. (Eslami, 2008) The optimal portfolio selection is one of the most important issues in financial literature and is responsible for maximizing the return, minimizing the investment risk, and considering other preferences. (Yahyazadehfar, 2011)

Chang (2012) investigated the optimal portfolio selection of projects in an article. He focused on solving this problem that the organizations are faced with limitations in applying the capital resources. Therefore, a model based on the data envelopment analysis, knapsack formulation (clarification) and fuzzy set theory is utilized to solve this problem. Using this model and artificial bee colony algorithm in artificial intelligence, a comparative process is done for optimization in issues with uncertainty in the industry.

Carrasco (2010) investigated the optimal portfolio selection through regularization. In this study, he studied four regularization techniques, namely, ridge, spectral cut-off,

Landweber-Fridman and LARS Lasso for stabilizing the covariance matrix. All four techniques have regulatory parameters which should be selected.

Patari (2010) investigated the potential of using the data envelopment analysis as a criterion for portfolio selection. The portfolio included a comprehensive sample of Finnish non-financial stocks were based on the data envelopment analysis criteria. The portfolio performance was evaluated based on the average return and adjustment criteria of performance risk. Furthermore, the effect of storage period was evaluated changing from one year to five years. The results of this research indicate that the effectiveness of data envelopment analysis leads to the optimization of decision making in portfolio selection.

In another research, Chen (2008) has introduced a new model for portfolio selection, which can make balance between the risk and return according to the investors' preferences, using the multi-purpose programming technique. The researcher has utilized the historical data of 10 stock markets for experimental test of his model. The results of this research indicate the ability of new model to solve the problems of traditional model for portfolio selection. The validity and adequacy of model are confirmed in this study.

Professor Lotfizadeh (1965) introduced the fuzzy set theory for issues without the clear statement terms in order to define the criteria values. In fact, when there is an uncertainty in human decisions and it cannot be converted into the definitive mathematical numbers, the results are slightly misleading and thus the fuzzy set is provided to solve these types of issues and has been utilized in various fields of sciences during the past four decades (Momeni, 2006).

Shams (2013) compared the effectiveness of momentum strategy criteria in optimal portfolio selection. Jegadeesh and Titman's method is utilized to evaluate the efficiency of criteria in optimal portfolio selection.

Kazemi (2013) developed a new model for optimal portfolio selection using Markowitz method and its modification by cosine model and genetic algorithm solving. This research is designed based on optimal portfolio selection using the genetic algorithm based on Markowitz model and specific method of convergence for assessing the risk and return in order to minimize the risk of proper investment and optimize the portfolio considering the rate of expected return. With regard to the financial management knowledge and research on operation, this research seeks to provide a model of risk and return evaluation for proper investment in optimal portfolio.

Jabbari (2011) evaluated the portfolio performance and selection from stock investment funds. This study seeks to determine the appropriate decision making model for investment. Therefore, the effective criteria are initially extracted for evaluating the performance of joint investment funds according to the review of research literature. Then, the importance of each criterion (Sharp, trainer, Jensen and Sortino) are measured using the Shannon entropy.

2- Research methodology

This research is applied in terms of objective and descriptive-analytical in terms of data collection and processing. The assessment models of strategic programs such as the FAHP are utilized to conduct this research. The studied population consists of all companies listed on Tehran Stock Exchange during 2014-2019 and 7 companies are considered as the samples. The required sample is selected from the companies with above-mentioned conditions by

simple random sampling and the raw data of target companies is collected. Afterwards, the portfolios are built, so that the companies of each portfolio are not from a specific industry. Furthermore, the managers and senior managers' views in companies listed on stock exchange are utilized to investigate the importance of optimal portfolio selection. The statistical sample size is calculated through Cochran formula as follows:

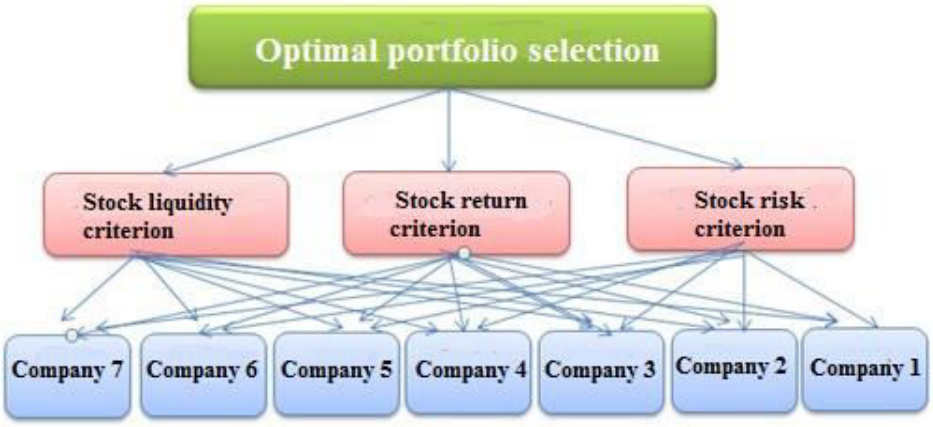
$$n = \frac{Z^2_{\alpha/2} P(1-P)}{\varepsilon^2}$$

In the above formula, $Z_{(\frac{\alpha}{2})}$ is the normal value of change corresponding to the confidence level (1-α). In this study, the confidence level is considered equal to 90% or 1.645, and P is the ratio of sampling which is typically considered equal to 0.5. Furthermore, ε is the authorized error which will be determined according to the results of pre-test; however, its maximum value is equal to 0.05, thus the sample size is obtained as follows:

$$= \frac{1.645^2 \times 0.5 \times 0.5}{0.05^2} = 270.66$$

The sample size is considered equal to 270.

The research model is presented as follows:



3- Research results

The research data and its analysis in addition to ranking and prioritizing the companies listed on Stock Exchange and indices of each factor are investigated by fuzzy AHP method as follows.

3-1- Descriptive statistics of sample demographic data

3-1-1- Respondents' status in terms of gender

Table 1- Frequency and frequency percentage of sample members based on the gender group

Gender	No.	Percentage
Women	119	44.1
Men	151	55.9
Sum	270	100

3-1-2- Respondents' status in terms of age group

The number and percentage of respondents in terms of age group are as follows:

Table 2- Frequency and frequency percentage of sample members based on the age group

No.	Age	Total	Percentage
1	20 to 30 years	1	0.4
2	30 to 40 years	84	31.1
3	40 to 50 years	122	45.2
4	50 years and above	63	23.3
5	Sum	270	100

3-1-3- Respondents' status in terms of educational level

The number and percentage of respondents in terms of educational levels are as follows:

Table 3- Frequency and frequency percentage of sample members based on the educational levels

No.	Educational level	Total	Percentage
1	Bachelor	58	21.5
2	Master	183	67.8
3	Ph.D.	29	10.7
4	Sum	270	100

3-1-4- Respondents' status in terms of work experience

The number and percentage of respondents in terms of work experience are as follows:

Table 4: Frequency and frequency percentage of sample members based on the work experience

No.	Work experience	Total	Percentage
1	Under 10 years	92	34.1
2	10 to 20 years	97	35.9
3	20 years and above	81	30
4	Sum	270	100

3-1-5- Descriptive statistics of research variables

The following table represents the descriptive statistics for mean, median, maximum, minimum, standard deviation, kurtosis and skewness according to each research variable:

Table 5: Descriptive statistics of research variables

	Liquidity rating	**Risk**	**Expected return**
Mean	218.1429	13.29429	4.022857
Median	238	12.92	3.46
Maximum	339	27.85	14.21
Minimum	21	6	-1.02
Standard Deviation	114.5316	7.282469	4.831831
Skewness	-0.57079	1.111976	1.443034

Kurtosis	2.246338	3.442454	4.093238
Jarque-Bera statistics	0.545766	1.499671	2.777996
Probability	0.761182	0.472444	0.249325
Total	0.1527	93.06	28.16

3-1-6- Investigating the data flow during 2014-2019

According to the data of expected return variable, the data flow diagram will be as follows:

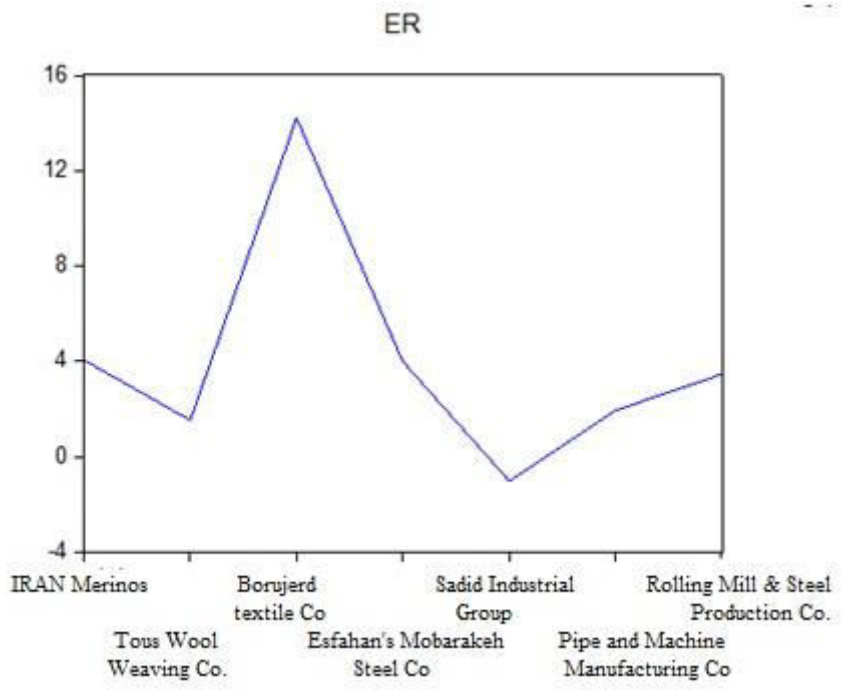


Figure 1 – Investigating the expected return data flow in companies

Given the chart above, the maximum expected return belongs to Borujerd textile Company.

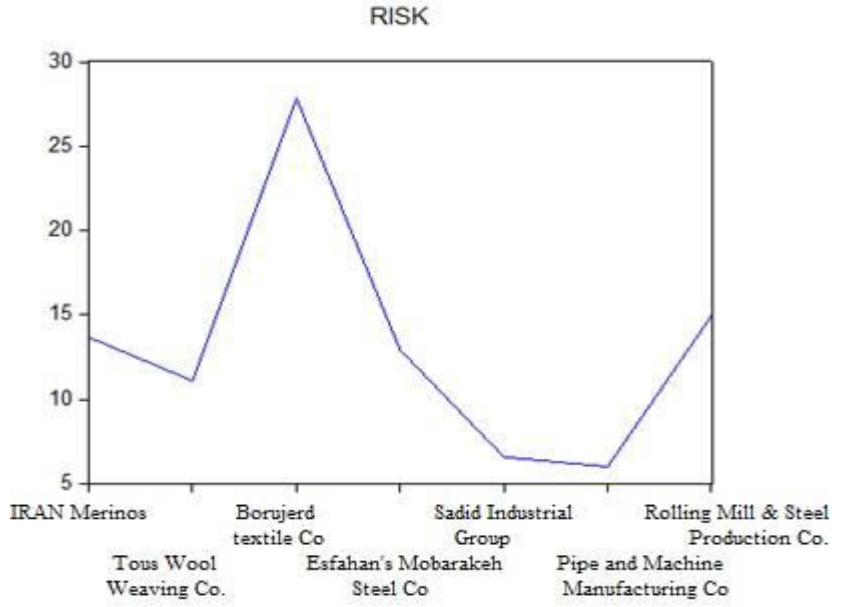


Figure 2- Investigating the risk data flow in companies

According to the above chart, the minimum risk belongs to Pipe and Machine Manufacturing Company.

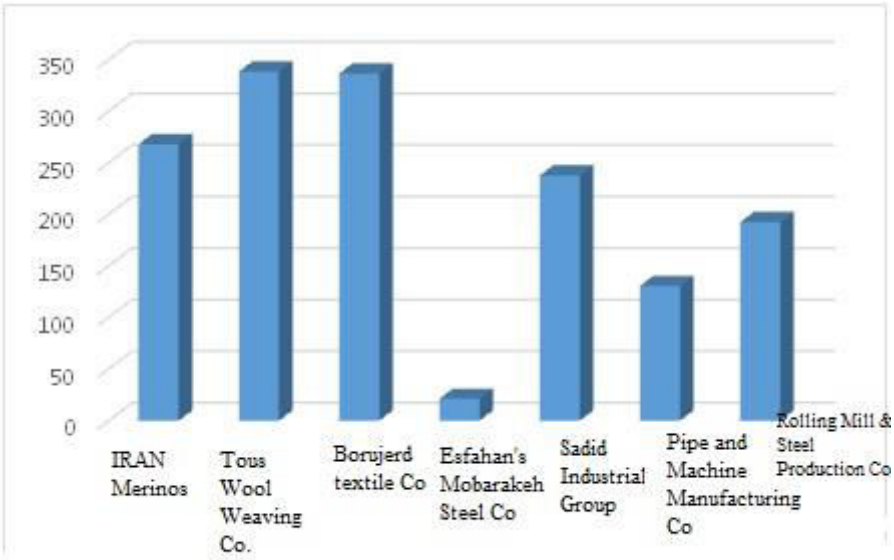


Figure 3- Investigating the prioritization of companies in liquidity rating

According to the findings above, the maximum liquidity rating belongs to Tous Wool Weaving Co. and the minimum rank to Esfahan's Mobarakeh Steel Company.

3-2- Inferential statistics

3-2-1 Ranking the criteria of optimal portfolio selection using the FAHP

Here, we seek to rank and determine the importance factor of optimal portfolio selection criteria by Fuzzy Analytic Hierarchy Process (FAHP). The criteria valuation is done by pair-wise comparison and granting the ranks which are triangular fuzzy numbers and indicate the priority or importance between these two criteria. Therefore, the decision maker compares the indices and utilizes the triangular fuzzy numbers for pair-wise comparisons. The SK value, which is a triangular fuzzy number, is calculated for each row of pair-wise comparison matrix prepared according to the above-mentioned method. After completing the tables for preferences of factors by respondents, the coefficients of each pair-wise comparison matrix are initially calculated (sk). The Sk value is a triangular number and calculated as follows:

$$S_K = \sum_{i=1}^{n} M_{kj} * \left[\sum_{i=1}^{m} \sum_{i=1}^{n} M_{ij} \right]^{-1}$$

K indicates the number of row, and i and j refer to the options and criteria, respectively.

270 respondents responded to the tables of weighting the options of questionnaire. The pair-wise comparisons of total group should be integrated for final prioritization of options; the geometric averaging is one of the best ways in this regard. In other words, table 6 is measured for each respondent.

Table 6 - Row sum of indices

Main factors	Row sum of main factors
Risk criterion	(1.83, 2.06, 2.5)
Liquidity rating	(2.5, 3.16, 4)
Expected return	(4, 5, 6)
Sum	(8.33, 10.22, 12.5)

1. S_k Calculation: The S_k value is calculated for each row of pairwise comparison matrix as prepared above:

$$S_1 = (1.83, 2.06, 2.5) \times (0.08, 0.97847, 0.120048) = (0.146, 0.202, 0.300)$$
$$S_2 = (2.5, 3.16, 4) \times (0.08, 0.97847, 0.120048) = (0.2, 0.309, 0.480)$$
$$S_3 = (4, 5, 6) \times (0.08, 0.97847, 0.120048) = (0.32, 0.489, 0.72)$$

2. Measuring the magnitude of S_i compared to each other

$V(S_1 \geq S_2)$ =0.48
$V(S_1 \geq S_3)$ =0.41

$V(S_2 \geq S_1) = 1$
$V(S_2 \geq S_3)$ =0.47

$V(S_3 \geq S_1) = 1$
$V(S_3 \geq S_2) = 1$

3. Measuring the weights of indices in pairwise comparison matrix

$$w'_{(x_1)} = Min\{V(S_1 \geq S_2, S_3)\} = Min\{0.48, 0.41\} = 0.41$$
$$w'_{(x_2)} = Min\{V(S_2 \geq S_1, S_3)\} = Min\{1, 0.47\} = 0.47$$
$$w'_{(x_3)} = Min\{V(S_3 \geq S_1, S_2)\} = Min\{1,1\} = 1$$

Ultimately, the non-normalized weight vector of indices will be as follows:

$$w' = [w'_{(x_1)}, w'_{(x_2)}, w'_{(x_3)}] = [0.41, 0.47, 1]$$

4. Normalizing the weight vector obtained from the third step and measuring the weight vector of criteria

$$\sum w'_{(x_i)} = 1.88$$
$$w = [0.21, 0.26, 0.53]$$

Therefore, the final weight and prioritization of four main factors affecting the rural road problems are according to the following table from the perspective of one of the respondents and through the FAHP:

Table 7 - Prioritization of main factors using the FAHP

Index (criterion)	Weight	Priority
Risk criterion	0.21	3
Liquidity rating	0.26	2
Expected return	0.53	1

It is found that the sum of importance coefficients is equal to 1 which indicates the full accuracy of calculations. The output chart of Expert Choice Software for final prioritization of main factors is as follows for all respondents:

Figure 4- The final prioritization of main factors for all respondents

According to the software output, the expected return is prioritized with the weight of 0.674, the liquidity rating with the weight of 0.226, and the risk criterion with 0.101 respectively. Considering that the inconsistency rate is lower than 0.1, thus the reliability of data is confirmed.

3-2-2- Pairwise comparison matrix and prioritization of companies according to the main factors

Prioritization of companies based on the first criterion or risk criterion

The degree of importance for companies based on the risk criterion is as follows:

Table 8- Degree of important for companies based on the risk criterion

Risk	IRAN Merinos Co.	Tous Wool Weaving Co.	Borujerd textile Co.	Esfahan's Mobarakeh Steel Co.	Sadid Industrial Group	Pipe and Machine Manufacturing Co.	Rolling Mill & Steel Production Co.
IRAN Merinos Co.	1	1.2	5	1.2	1.6	1.8	2
Tous Wool Weaving Co.	2	1	6	2	1.5	1.5	3
Borujerd textile Co.	1.5	1.6	1	1.4	1.6	1.8	1.3
Esfahan's Mobarakeh Steel Co.	2	1.2	4	1	1.5	1.5	2
Sadid Industrial Group	6	5	6	5	1	1.2	4
Pipe and Machine Manufacturing Co.	8	5	8	5	2	1	4
Rolling Mill & Steel Production Co.	1.2	1.3	3	1.3	1.4	1.4	1

The values of table above indicate that according to the mean responses, IRAN Merinos Co. has 0.5 of risk criterion higher than Tous Wool Weaving Co. for investment, and

Borujerd textile Company has the weight of 5 times higher than IRAN Merinos Co., and Esfahan's Mobarakeh Steel Company is 5 times higher than IRAN Merinos Co., and so on.
The output chart of Expert Choice Software for final prioritization of companies based on the risk criterion is as follows:

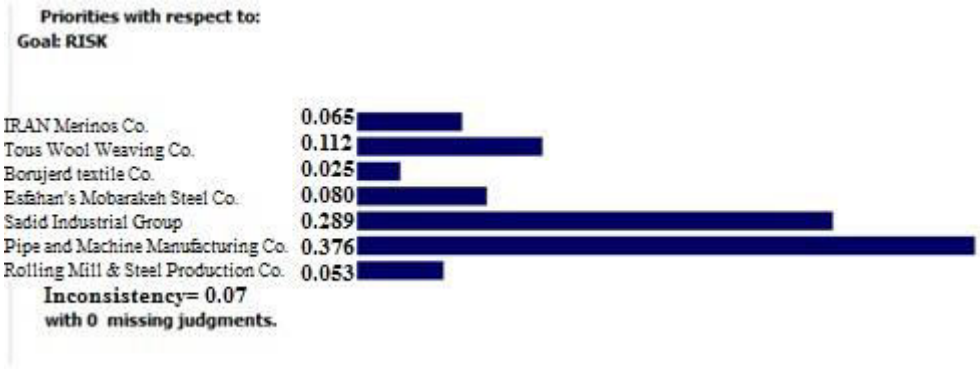


Figure 5 - Final prioritization of companies based on the risk criterion

According to the software output, the inconsistency rate is equal to 0.07, and since the inconsistency rate is lower than 0.1, the reliability of data above is confirmed. Therefore, the final weight and prioritization of companies based on the risk criterion by FAHP are respectively according to the following table:

Table 9- Prioritizing the important degree of companies based on the risk criterion

Company	Weight	Priority
IRAN Merinos Co.	0.065	5
Tous Wool Weaving Co.	0.112	3
Borujerd textile Co.	0.025	7
Esfahan's Mobarakeh Steel Co.	0.08	4
Sadid Industrial Group	0.289	2
Pipe and Machine Manufacturing Co.	0.376	1
Rolling Mill & Steel Production Co.	0.053	6

Prioritization of companies based on the second criterion or liquidity rating

The degree of importance for companies based on the liquidity rating is as follows:

Table 10**-** The degree of importance for companies based on the liquidity criterion

Risks	IRAN Merinos Co.	Tous Wool Weaving Co.	Borujerd textile Co.	Esfahan's Mobarakeh Steel Co.	Sadid Industrial Group	Pipe and Machine Manufacturing Co.	Rolling Mill & Steel Production Co.
IRAN Merinos Co.	1	3	3	1.5	1	1.3	1.2
Tous Wool Weaving Co.	1.3	1	1	1.6	1.5	1.4	1.3
Borujerd textile	1.3	1	1	1.6	1.3	1.3	1.2

Co.							
Esfahan's Mobarakeh Steel Co.	5	6	6	1	8	6	7
Sadid Industrial Group	1	5	3	1.8	1	1.3	1.2
Pipe and Machine Manufacturing Co.	3	4	3	1.6	3	1	2
Rolling Mill & Steel Production Co.	2	3	2	1.7	2	1.2	1

The output chart of Expert Choice Software for final prioritization of companies based on the liquidity criterion is as follows:

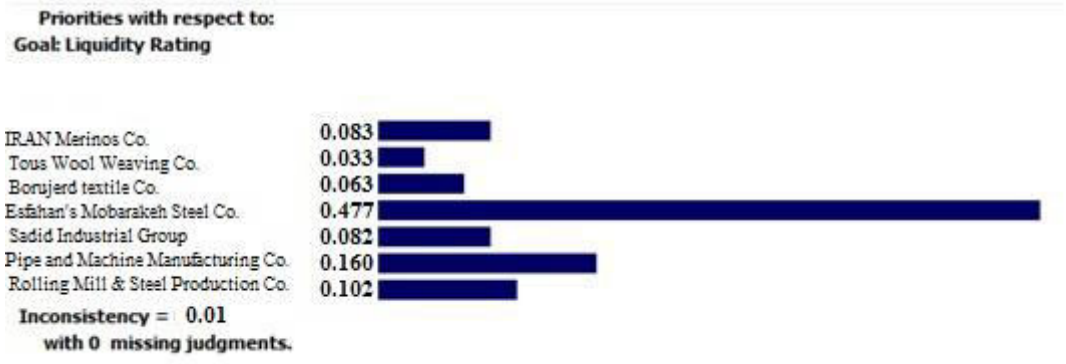


Figure 6- Final prioritization of companies based on the liquidity criterion

According to the software output, the inconsistency rate is equal to 0.01, and since the inconsistency rate is lower than 0.1, the reliability of data above is confirmed. Therefore, the final weight and prioritization of companies based on the liquidity criterion by FAHP are respectively according to the following table:

Table 11- Prioritization of importance degree for companies based on the liquidity criterion

Company	Weight	Priority
IRAN Merinos Co.	0.083	4
Tous Wool Weaving Co.	0.033	7
Borujerd textile Co.	0.063	6
Esfahan's Mobarakeh Steel Co.	0.477	1
Sadid Industrial Group	0.082	5
Pipe and Machine Manufacturing Co.	0.16	2
Rolling Mill & Steel Production Co.	0.102	3

Prioritization of companies based on the third criterion or expected return

The degree of importance for companies based on the expected return criterion is as follows:

Table 12- Degree of importance for companies based on the expected return

Risks	IRAN Merinos Co.	Tous Wool Weaving Co.	Borujerd textile Co.	Esfahan's Mobarakeh Steel Co.	Sadid Industrial Group	Pipe and Machine Manufacturing Co.	Rolling Mill & Steel Production Co.
IRAN Merinos Co.	1	3	1.6	2	5	1.3	2
Tous Wool Weaving Co.	1.3	1	1.7	1.4	2	1	1.2
Borujerd textile Co.	6	7	1	4	6	4	3
Esfahan's Mobarakeh Steel Co.	1.2	4	4	1	4	3	2
Sadid Industrial Group	1.5	1.2	1.6	1.4	1	1.3	1.4
Pipe and Machine Manufacturing Co.	3	1	1.4	1.3	3	1	3
Rolling Mill & Steel Production Co.	1.2	2	1.3	1.2	4	1.3	1

The output chart of Expert Choice Software for final prioritization of companies based on the expected return criterion is as follows:

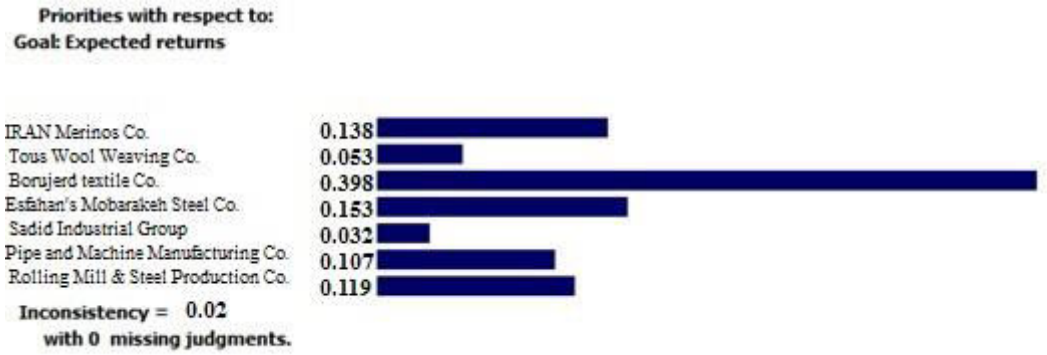


Figure 7- Final prioritization of companies based on the expected return criterion

According to the software output, the inconsistency rate is equal to 0.02, and since the inconsistency rate is lower than 0.1, the reliability of data above is confirmed. Therefore, the final weight and prioritization of companies based on the expected return criterion by FAHP are respectively according to the following table:

Table 13- Prioritizing the degree of importance for companies based on the expected return criterion

Company	**Weight**	**Priority**
IRAN Merinos Co.	0.138	3
Tous Wool Weaving Co.	0.053	6
Borujerd textile Co.	0.398	1
Esfahan's Mobarakeh Steel Co.	0.153	2
Sadid Industrial Group	0.032	7
Pipe and Machine Manufacturing Co.	0.107	5

Rolling Mill & Steel Production Co.	0.119	4

3-2-3- Final prioritization of companies for optimal portfolio selection

The following table summarizes the performed calculations in previous section:

Table 14- Prioritization of companies based on each of three criteria

Companies	Risk	Liquidity rating	Expected return
IRAN Merinos Co.	0.065	0.083	0.138
Tous Wool Weaving Co.	0.112	0.033	0.053
Borujerd textile Co.	0.025	0.063	0.398
Esfahan's Mobarakeh Steel Co.	0.08	0.477	0.153
Sadid Industrial Group	0.289	0.082	0.032
Pipe and Machine Manufacturing Co.	0.376	0.16	0.107
Rolling Mill & Steel Production Co.	0.053	0.102	0.119

Furthermore, the final weights of main factors are as follows:

Table 15- Prioritization of main factors using the FAHP

Index (criterion)	Weight
Risk criterion	0.101
Liquidity rating	0.674
Expected return	0.226

Now, the final weight of companies is obtained based on three mentioned criteria by integrating and multiplying the obtained matrix by final matrix of corporate prioritization based on the criteria.

$$W=\begin{vmatrix} 0.065 & 0.083 & 0.138 \\ 0.112 & 0.033 & 0.053 \\ 0.025 & 0.063 & 0.398 \\ 0.08 & 0.477 & 0.153 \\ 0.289 & 0.082 & 0.032 \\ 0.376 & 0.16 & 0.107 \\ 0.053 & 0.102 & 0.119 \end{vmatrix} * \begin{vmatrix} 0.101 \\ 0.674 \\ 0.226 \end{vmatrix} = \begin{vmatrix} 0.094 \\ 0.046 \\ 0.135 \\ 0.364 \\ 0.092 \\ 0.170 \\ 0.10 \end{vmatrix}$$

Ultimately, the final weight and rate are as follows:

Table 16- Prioritization and final weights of companies based on the optimal portfolio selection

Company	**Total weight**	**rating**
IRAN Merinos Co.	0.094	5
Tous Wool Weaving Co.	0.046	7
Borujerd textile Co.	0.135	3
Esfahan's Mobarakeh Steel	0.364	1

Co.		
Sadid Industrial Group	0.092	6
Pipe and Machine Manufacturing Co.	0.170	2
Rolling Mill & Steel Production Co.	0.10	4

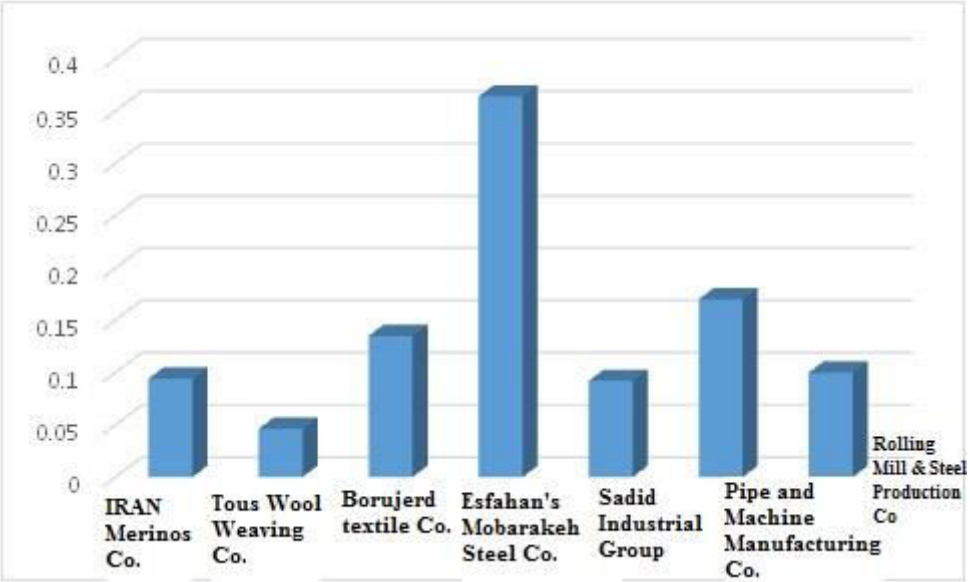


Figure 8- Bar chart of prioritizing the companies based on the optimal portfolio selection

4- Conclusion

The importance of portfolio creation is associated with its application in selecting the proper combination of portfolio according to the investors' type of risk-taking and their expected return. An appropriate portfolio can be utilized both for investment companies and stock exchange and lead to the absorbed excess liquidity from the financial market and thus lead to the allocative efficiency in capital market.

This study utilizes Saaty and Roger's model (1980) for portfolio selection and ranks the companies listed on stock exchange in terms of risk, liquidity rating and expected return according to the importance of their criteria by the Fuzzy AHP. The risk is defined as the potential changes or the standard deviation of portfolio expected return. Since the investors tend to achieve maximum return by minimizing the risk, this variable is selected as one of the variables of optimal portfolio selection. The information of this variable is prepared by Rahavard Novin software during 2014 to 2019. The liquidity of a sheet of stock refers to the ability to sell it quickly. The more the share is sold faster and with less cost, the more its liquidity is increased. The securities, which are traded daily and frequently, have higher liquidity and thus low risk than the in securities traded in limited numbers and low frequency. In general, it is not recommended to buy the shares with liquidity rate above 100. The more the liquidity rating of a share is decreased, the more the liquidity status of that share is improved. In other words, the lower value indicates the high liquidity potential of a share and

its higher value shows the low potential of liquidity for a share. Principally, the rate 1 has the best liquidity. Therefore, this variable is selected as the second variable. It is noteworthy that this study utilizes the liquidity rating of companies listed on Tehran Stock Exchange from Rahavard Novin software as liquidity criterion. Since the investors are always seeking to maximize their return on portfolio, it is reasonable to consider this variable as the third variable. According to the obtained results, it can be concluded that the investors in Tehran Stock Exchange pays attention to expected return and level of liquidity and liquidity rating of shares in making economic decisions including the optimal portfolio selection. According to the obtained result by Eslami Bidgoli and Sarang (2008) in this study, the rate of stock liquidity can affect the optimal portfolio selection from companies listed on Tehran Stock Exchange. Therefore, the better portfolio can be achieved during the portfolio selection by taking into account the level of stock liquidity. Furthermore, the investors need to consider the level of liquidity in their decisions during the portfolio selection. According to the advantages of hierarchical analysis, several variables can be easily considered in decision making. Furthermore, according to another result of study, the use of hierarchical analysis is so useful in optimal portfolio selection, because the investor should consider several factors (such as risk, return, etc.) in making decision about creating the portfolio and should compare these factors while selecting the shares of various companies for investment and thus select the stock, which is better than the other shares in terms of target factors, and then invest on it. Therefore, according to the obtained results, the results are as follows:

Prioritization of main factors in optimal portfolio selection using the FAHP

1. Expected return
2. Liquidity rating
3. Risk criterion

Prioritization of companies according to the risk criterion using the FAHP

1. Pipe and Machine Manufacturing Co.
2. Sadid Industrial Group
3. Tous Wool Weaving Co.
4. Esfahan's Mobarakeh Steel Co.
5. IRAN Merinos Co.
6. Rolling Mill & Steel Production Co.
7. Borujerd textile Co.

Prioritization of companies according to the liquidity rating using the FAHP

1. Esfahan's Mobarakeh Steel Co
2. Pipe and Machine Manufacturing Co.
3. Rolling Mill & Steel Production Co.
4. IRAN Merinos Co.
5. Sadid Industrial Group
6. Borujerd textile Co.
7. Tous Wool Weaving Co.

Prioritization of companies according to the expected return criterion using the FAHP

1. Borujerd textile Co.
2. Esfahan's Mobarakeh Steel Co.
3. IRAN Merinos Co.

4. Rolling Mill & Steel Production Co.
5. Pipe and Machine Manufacturing Co.
6. Tous Wool Weaving Co.
7. Sadid Industrial Group

4-1- Recommendations for future research

- Optimal portfolio selection using the data mining analysis method;
- Dynamic analysis of optimal portfolio selection using the TOPSIS network analysis;
- Providing an optimization model for evaluation and selection of risk responses in fuzzy environment;
- Analysis of optimal portfolio selection risks by combined fuzzy approach of VIKOR and grey relational analysis

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