A Freezing Method for Solving Bottleneck-Cost Transportation Problem

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Abstract:

A new method namely 'Ghadle-Munot freezing method' is proposed for finding optimal solution to bottleneck Transportation Problem. Further Ghadle-Munot Algorithm [Congruence modulo method] is used to find all efficient solution of bottleneck-Cost Transportation Problem. The method structured in the form of an algorithm and coded in MATLAB which makes it user friendly. The method is illustrated through numerical example.

Keywords - *Bottleneck Transportation Problem, Freezing Method, Congruence modulo Method, Bottleneck-Cost Transportation Problem, MATLAB.*

Introduction:

The Bottleneck or time minimizing Transportation Problem (TP) is one of the special subclass of TP where the time associated with each shipping route need to be minimized. In real life applications, the time minimizing TPs are firmly rooted for determining better ways to deliver goods to consumer within time. Along with it, transportation of perishable food, groceries, emergency services like fire control equipment, ambulance, military armaments, help to the people caught in disaster etc. needs prompt delivery. Particularly, having a sight on COVID-19 pandemic around the globe; timely transportation of medical facilities plays a vital role, as delay in time may result in larger loss than any cost advantage. All such issues are tackled in BTP.

In standard TP, objective function is sum of shipping costs where as in BTP, objective function is maximum time of shipping a commodity. Because, time of transportation is independent of amount of commodity sentif all $x_{ij} \geq 0$. In this article, Author analyzes bi-criteria TP i.e. time and cost which is also called Bottleneck-Cost TP.

(Illija N. 2007) has developed an algorithm which finds minimum of total transportation time to time TP. BCTP has been proposed by **(Aneja& Nair 1979)**.**(Ahmed &Reshi 2014)**, **(Isserman 1984)** developed various ways to get the set of efficient solutions. **(Pandian&Natrajan 2011)** solved BCTP by blocking method to get set of time and then used zero point method to get all efficient solutions. (**Pandian&Anuradha 2011)** proposed dripping method to solve BCTP.

In this article, author propose a new method 'Ghadle – Munot Freezing method' for finding the set of all efficient solution to BCTP which is based on Congruence Modulo method (Ghadle – Munot algorithm). The proposed method will enable decision maker to handle time oriented logistic problem and from set of efficient solution select an appropriate transportation schedule depending on financial position and timely need. Also, proposed algorithm has been coded in MATLAB which makes it very easy to use and get the solution within fraction of seconds.

Further the article is organized as Mathematical formulation and Preliminaries in section 2, proposed algorithm in section 3, section 4 contains a numerical example illustrated with proposed algorithm, Result is discussed in section 5 and finally concluding remarks has been provided in section 6.

Mathematical Formulation:

Minimize $z_1 = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$

Minimize $z_2 = [\, Maximize t_{ij} | x_{ij} > 0]$

Subject to

$\sum_{j=1}^{n} x_{ij} = a_i$, i= 1,2,…,m

$\sum_{i=1}^{m} x_{ij} = b_j$, j= 1,2,…,n

$x_{ij} \geq 0, \quad \forall\, i, j integers$

Where a_i is the supply available atithsource ;b_j is the demand required at jth destination. $x_{ij}$is the number of units transported from ith source to jth destination; c_{ij} is the cost of transporting a unit from ith source to jth destination; t_{ij} is the time of transporting goods from ith source to jth destination.m is the number of sources and n is number of destinations.

Definition 1:

A point (P,T) is said to be feasible solution if P satisfies above three conditions where P=$\{x_{ij} |\, i = 1,2, \ldots, m \,\&\, j = 1,2, \ldots, n\}$ and T is a time.

Definition 2:

A feasible point (P0, T0) is said to be efficient for BTP if there doesn't exist any other feasible point (P,T) such that

$z_1(P) \leq z_1(P_0)$and$z_2(P) < z_2(P_0)$

or

$z_1(P) < z_1(P_0)$and$z_2(P) \leq z_2(P_0)$

Definition 3:

For a BCTP, a cost TP for any time Tm is active if it is the minimum time transportation corresponding to that cost TP

Definition 4:

Congruence Modulo:

If two numbers b & c have the property that their difference is integrally divisible by a number m i.e. m|(b-c) is an integer then we say that b is congruent to c mod m .

It means c is the remainder when b is divided by m.

In proposed algorithm, this concept of congruence modulo is used.

Ghadle-Munot Freezing Method:

Step 1: Construct the time transportation problem from the given BCTP.

Step 2: Solve the time transportation problem by the congruence modulo method (Ghadle-Munot Algorithm) . Let the optimal solution be T0.

Step 3. Construct the cost transportation problem from the given BCTP.

Step 4: Solve the cost transportation problem by the congruence modulo method (Ghadle-Munot Algorithm) and also, find the corresponding time transportation. Let it be Tc.

Step 5: For each time tm in [T0, Tc] Construct the active cost transportation problem by freezing time greater than tm and solve it by the congruence modulo method (Ghadle-Munot Algorithm).

Step 6: For each time tm, an optimal solution to the cost transportation problem , P is obtained from the Step 5. Then, the vector (P, tm) is an efficient solution to BCP.

Now, the Ghade-Munot Freezing method is illustrated by the Numerical example.

Example:

Let oxygen cylinders need to be transported from factory F1, F2, F3 to city C1, C2, C3, C4 with given transportation cost per unit in thousand and time of transportation on the corresponding route in hours. Consider the following 3 X 4 bottleneck-cost transportation problem representing given data where, upper left corner in each cell display cost of transportation and lower left corner displays time of transportation.

City **Factory**	**C_1**	**C_2**	**C_3**	**C_4**	**Supply**
F_1	5 10	6 68	10 73	11 52	8
F_2	6 66	7 95	12 30	14 21	19
F_3	14 97	11 63	9 19	7 23	17
Demand	11	3	14	16	

Firstwe will consider the time TP from given BCTP,

City **Factory**	**C_1**	**C_2**	**C_3**	**C_4**	**Supply**
F_1	10	68	73	52	8
F_2	66	95	30	21	19
F_3	97	63	19	23	17
Demand	11	3	14	16	

Using the congruence modulo method (Ghadle-Munot Algorithm), we get the optimal solution of time TP as 66. i.e. $T_0 = 66$

Now we will consider the cost TP from given BCTP,

City **Factory**	**C_1**	**C_2**	**C_3**	**C_4**	**Supply**
F_1	5	6	10	11	8
F_2	6	7	12	14	19

F_3	14	11	9	7	17
	11	3	14	16	

By congruence modulo method, the optimal solution is $x_{13} = 8; x_{21} = 11; x_{22} = 3; x_{23} = 5; x_{33} = 1$ &$x_{34} = 16$ with the minimum transportation cost is 348 and

The corresponding time is T_c= max {10, 66, 95, 30, 19, 23} = 95

Now time between $T_0$and $T_c$in given table are {66, 68, 73, 95 }.

Now we consider active cost transportation table corresponding to time 66 by freezing point having time greater than 66

City / Factory	**C_1**	**C_2**	**C_3**	**C_4**	**Supply**
F_1	5	-	-	11	8
F_2	6	-	12	14	19
F_3	-	11	9	7	17
	11	3	14	16	

Using congruence modulo method, the optimal solution is$x_{11} = 8; x_{21} = 3; x_{23} = 14; x_{24} = 2;$

$x_{32} = 3; x_{34} = 14$ With the transportation cost is 385.

Now we consider active cost transportation table corresponding to time 68 by freezing all the cost point having time greater than 68,

City / Factory	**C_1**	**C_2**	**C_3**	**C_4**	**Supply**
F_1	5	6	-	11	8
F_2	6	-	12	14	19
F_3	-	11	9	7	17
	11	3	14	16	

By congruence modulo method, the optimal solution is $x_{11} = 5; x_{12} = 3; x_{21} = 6; x_{23} = 13; x_{33} = 1; x_{34} = 16$ with the transportation cost is 356.

Now we consider active cost transportation table corresponding to time 73 by freezing all the cost point having time greater than 73,

City / Factory	C_1	C_2	C_3	C_4	Supply
F_1	5	6	10	11	8
F_2	6	-	12	14	19
F_3	-	11	9	7	17
	11	3	14	16	

By congruence modulo method, the optimal solution is $x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1; x_{34} = 16$ with the transportation cost is 351.

Result and Discussion:

The set of efficient solutions to given BCTP are given below:

Sr. No.	Efficient Solution of BCTP	Objective value of BCTP
1	$x_{11} = 8; x_{21} = 3; x_{23} = 14; x_{24} = 2;$ $x_{32} = 3; x_{34} = 14$ with time 66	(385,66)
2	$x_{11} = 5; x_{12} = 3; x_{21} = 6; x_{23} = 13; x_{33} = 1; x_{34} = 16$ with time 68	(356,68)
3	$x_{12} = 3; x_{13} = 5; x_{21} = 11; x_{23} = 8; x_{33} = 1; x_{34} = 16$ with time 73	(351,73)
4	$x_{13} = 8; x_{21} = 11; x_{22} = 3; x_{23} = 5; x_{33} = 1$ &$x_{34} = 16$ with time 95	(348,95)

From this set of efficient solutions, one can decide which route to select as per her/his timely need as well as financial conditions.

Conclusion:

In this article, the proposed 'Ghadle-Munot freezing method' gives the set of efficient solutions to the BCTP, which enables decision maker to take correct decision as per budgetary constraint and timely need. Proposed method requires less iterations so easy to apply and its coding in MATLAB makes it more user friendly.

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