An Algorithm for Solving Integer Goal Programming Problems with Fuzzy Parameters

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ABSTRACT
In this paper, we describe an algorithm for modeling and solving integer linear goal programming problems with fuzzy parameters. The proposed algorithm helps the decision maker and the analyst in determination the aspiration levels of the goals by a method based on the feasible domain, the differential weights of the sub-goals and the decision maker's preferences. The solution, which is obtained by the present algorithm, is most preferred by the decision maker. An illustrative example is presented.
Keywords: Multi – criteria decision making, Goal programming, Integer programming, Fuzzy parameters.

1. Introduction

Many decision making problems that arise in the real world need to be formulated as integer multi – criteria mathematical programming problems. In most practical problems, there are various factors should be reflected in the formulation of the objective functions and the constraints. Criteria of some problems involve many parameters whose possible values may be assigned by the decision maker (DM) or the experts, [4, 6, 7].

Goal programming (GP) is one of several approaches that have been proposed for soling multi – objective mathematical programming (MOMP) problems. It is a powerful technique for modeling, solving and analysis multi- criteria decision making (MCDM) problems. In general, a GP model consists of system constraints and goal constraints which are marked according to priority structure. It finds a solution to a (MCDM) problem by performance the following:

(i) Determination the aspiration levels of the objectives.
(ii) Calculation the degree of attainment for the goals.
(iii) Obtaining the optimal solution under the priority structure.

GP has been applied to a wide range of decision making problems but has not widely accepted by the DM because determination the aspiration levels of the goals is difficult and needs numerous calculations and information about the problem.

Several approaches are proposed for solving GP problems, [1, 2, 3, 5, and 8]. These methods deal with several types of GP problems but do not discuss determination the aspiration levels of the goals. In [7], an interactive algorithm is described for solving multi – objective nonlinear programming problems with fuzzy parameters. The concept of α- Pareto optimality is introduced in which the ordinary Pareto optimality is extended based on the α-level set of fuzzy numbers.

The aim of this research is to present an algorithm for solving integer linear goal programming problems with fuzzy parameters. The proposed algorithm helps the DM and the analyst in determination the aspiration levels of the goals by a method based on the information of the problem and the simplex method.

2. Problem Formulation and Algorithm

Consider the mathematical formulation of an integer linear goal programming problem with fuzzy parameters is:

$$FIGP: \ Goal1: \ f_1(x) \geq a_1 + z_1$$
$$Goal2: \ f_2(x) \geq a_2 + z_2$$

$$GoalK: \ f_k(x) \geq a_k + z_k$$

Subject to

$$Ax \leq b$$
$$x \geq 0$$
$$x_j, j \in j \subseteq \{1,2,\ldots\ldots,n\} are \ \text{int} \ egers$$

Where, x $\in R^n$
Is the vector of the decision variables, A is an mxn matrix, b is an mxl matrix, each goal
$f_i(x) \geq a_i + z_i$ consists of r_i linear sub-goals with differential weights $W_{il}, \ldots\ldots\ldots, W_{iri}, Z_i$
Is a vector of r_i fuzzy parameters $Z_{il}, \ldots\ldots\ldots\ldots, Z_{iri}$ with real coefficients?

$P_{iI}, \ldots\ldots\ldots\ldots, P_{iri}$ And the functions $f_i(x), i = 1,2,\ldots\ldots\ldots, k,$ are bounded from above on the feasible domain of the decision variables.

Assume that the aspiration levels a_i of the goals are unknown and the DM wishes to maximize the attainment degrees of the goals $f_j(x), j \in Q \subseteq \{1,2,\ldots\ldots\ldots\ldots, k\}$ under the same priority structure. Then, we determine these aspiration levels with respect to the function $f_j(x)$ which has a higher priority in Q by solving the following linear programming problems:

$$P_j : \max imize \sum_{S=1}^{t_j} W_{js} f_{js}(x)$$
$$s.t. \ \ Ax \leq b, x \geq 0$$

Let Y be the set of all optimal solutions of this problem.

$$P_i : \max imize \sum_{s=1}^{t_i} W_{is} f_{is}(x)$$
$$s.t.\ \ x \in Y$$
$$Where\ i - 1,2,......., k, i \neq j$$

Let b_i be the optimal value of the problem $P_i, i = 1,2,......, k. Then\ put$

$$a_i = b_i \Big/ \sum_{s=1}^{t_i} W_{is}, i = 1,2,........k.$$

Assume that th fuzzy parameters in our problem are real fuzzy numbers, [7], such that each real fuzzy number Z is a convex continuous fuzzy subset of the real line R whose membership function $g_z(z)$ is defined by:

(1) A continuous mapping from R to the closed interval [0,1].
(2) $g_z(z) = 0$ for all $Z \in (-\infty, z_1)$
(3) Strictly increasing on $[z_1, z_2]$
(4) $g_z(z) = 1$ $for\ all\ z \in [z_2, z_3]$
(5) Strictly decreasing on $[z_3, z_4]$
(6) $g_z(z) = 0$ $for\ all\ z \in [z_4, \infty)$

According to information and data of the DM about the fuzzy parameters of his problem, it is easy to derive membership functions for these fuzzy parameters which satisfy the above properties.

Definition: The α -level set of the fuzzy numbers $Z_{ij}, i = 1,2,........., k,\ \ j = 1,2,........., r_i$, is defined as the ordinary set:

$$L_\alpha(z) = \{z_{ij} : g_{z_{ij}}(z_{ij}) \geq \alpha\}$$

Remark: $L_{\alpha_1} \subset L_{\alpha_2}$ if and only if $\alpha_1 \geq \alpha_2$

For certain $\alpha, 0 \leq \alpha \leq 1,$ the fuzzy integer linear goal programming (FIGP) problem can be understood as the following non fuzzy mixed integer $\alpha - goal$ programming problem ($\alpha - MIGP$):

$\alpha - MIGP$: Goal 1: $h_1(x, z) \geq a_1$

Goal 2: $h_2(x, z_2) \geq a_2$

Goal k: $h_k(x, z_k) \geq a_k$

Subject to

$$Ax \le b, x \ge 0$$
$$z_i \in L_{ij}, i = 1,........,k$$
$$x_j j \in j \subseteq \{1,2,........,n\}$$

Where, each goal $h_i(x, z_i) \ge a_i$ consists of I_i sub-goals of the form:

$$f_{ij}(x) - p_{ij} z_{ij} \ge a_i$$

With the same differential weights $w_{il},......., w_{iri}, i = 1,2,......., k.$

$j = 1,......., r_i$

The proposed algorithm can be described in step form as follows:

Step 1: Determine the aspiration levels of the goals according to the DM's preferences.

Step 2: Derive the membership functions of the fuzzy parameters.

Step3: Elicit α from the DM

Step4: Find the α - level set and constitute the α - MIGP problem.

Step5: Solve the α - MIGP problem

Step6: Form the optimal solution of the FIGP problem.

Step7: Stop

3. Illustrative Example

Let us have the following problem:

FIGP: Goal 1: $\begin{bmatrix} 2x_1 + x_2 \\ x_1 + 3x_2 \end{bmatrix} \ge \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} + \begin{bmatrix} \tilde{z}_{11} \\ -\frac{1}{2}\tilde{z}_{12} \end{bmatrix}$

Goal 2: $-x_1 + x_2 \ge a_2 - \tilde{z}_2$

Subject to

$$x_1 + x_2 \leq 7$$
$$x_1 \leq 5$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0, \text{int } egers$$

Where the sub-goals of the first goal have differential weights 2 and 1 respectively.

Assume that the membership functions of the fuzzy parameters

$z_{11}, z_{12}, and\ z_2 are:$

$$g_{z_{11}} - (Z_{11}) = \begin{bmatrix} \frac{1}{2}(Z_{11} - 1), & 1 \leq Z_{11} \leq 3 \\ 1\ , & 3 \leq Z_{11} \leq 6 \\ \frac{-1}{2}(Z_{11} - 8), & 6 \leq Z_{11} \leq 8 \\ 0 & otherwise \end{bmatrix}$$

$$g_{z_{12}} - (Z_{12}) = \begin{bmatrix} \frac{1}{4}Z_{12}, & 0 \leq Z_{12} \leq 4 \\ 1\ , & 4 \leq Z_{12} \leq 5 \\ \frac{-1}{2}(Z_{12} - 7), & 5 \leq Z_{12} \leq 7 \\ 0 & otherwise \end{bmatrix}$$

$$g_{z_2} - (Z_2) = \begin{bmatrix} Z_2 - \frac{1}{2}, & \frac{1}{2} \leq Z_2 \leq \frac{3}{2} \\ 1\ , & \frac{3}{2} \leq Z_2 \leq 2 \\ \frac{-1}{2}(Z_2 - 4), & 2 \leq Z_2 \leq 4 \\ 0 & otherwise \end{bmatrix}$$

If $\alpha = \frac{1}{2}$, then the $\frac{1}{2}$ level set is:

$$L\frac{1}{2} = \{Z_{11}, Z_{12}, Z_2,: 2 \le Z_{11} \le 7, 2 \le Z_{12} \le 6, 1 \le Z_2 \le 3\}$$

(i) The aspiration levels with respect to the first goal are $a_{11} = a_{12} = 35/3, a_2 = 1$

And the optimal solution of the $\frac{1}{2}$

MIGP problem is:

$$(x_1^*, x_2^*) = (5,2), Z_{11} = 2, Z_{12} = 2, Z_2 = 1$$

(ii) The aspiration levels with respect to the second goal are $a_{11} = a_{12} = 20/3, a_2 = 4$

And the optimal solution of the $\frac{1}{2}$

MIGP problem is:

$$(x_1^*, x_2^*) = (3,4), Z_{11} = 2, Z_{12} = 2, Z_2 = 3$$

4. Conclusion and limitations

In the proposed algorithm, there is interaction between the analyst and the DM for determination the aspiration levels of the goals by a simple method based on the feasible domain of the decision variables, the differential weight of the sub-goals and the DM's preferences. This method determines the aspiration levels of the goals for some GP problems in which some of the sub-goals are not bounded on the feasible domain of the decision variables. The present algorithm solves mixed integer and pure integer linear goal programming problems with fuzzy parameters in the aspiration levels of the goals. The branch and bound method of integer programming, [5.9], is the appropriate technique because the α MIGP problem is always mixed integer.

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