Application of Set theory: with Specific Reference to Uncertain and Vague Data

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Abstract
Decision making has always remained a problem for the professionals, finding solution in the cases where the set of data is not random and fulfills all the parameters of the universe is easier, But there are certain cases where the data is not available in precise form i.e. it is either vague or uncertain, this makes the decision making more problematic. To deal with such issues the researcher evaluates the theory of Soft Set and proves the authenticity of the same on the basis of a real life problem.

Keywords: Soft Set, Decision Making, Algorithm.

Introduction

This is a well known fact that the application versatility of set theory is very abstract and general in nature, the number of applications of set theory can be examined in different branches of mathematics. Some of the prominent application are given as under:

- ***In Calculus***

In case of calculus set theory is applicable in both integral and differential calculus, as a matter of fact it is being used to evaluate and understand the limit points. On the other hand under the head of differential and integral calculus explanation for continuity of a function is also based on set theory.

- ***In Boolean Algebra***

After the development of soft set by Molodtsov in 1999, a number of generic application of same were introduced throughout the world, in the field of academics, research, management and even in scientific domains. Provided with the facility to deal with the uncertainty of data it became applicable for Boolean matrices as well. as Boolean is a algebraic structure and input data has a number of certainties in the same hence soft set have solved the problems in relation to the same.

Apart from this the basic operation of set theory i.e. intersection, union and even difference of set are being used in the same with high level of priority like the logical operation of ‘and’, ‘or’, ‘not’, etc.

- ***Topological Applications***

Collection of subsets and application of the same in defining topology has always remained a benchmark application of set theory, apart from the digital electronic circuitry and related variation sets are being used in extensive manner. Here the components are designed on the basis of subsets and even collection of the same.

- ***Importance in Computing***

Cantor’s diagnostics proof that every set has more subsets than it has elements is the central idea underlying the work of Godel, Turing, and others in showing that certain problems are not algorithmically solvable (recursion theory), and similarly in computational complexity theory.

Apart from the above given specific applications of set theory, this phenomenon is used in many of the other related fields as well, like the field of engineering, business, medical/health sciences and even in natural sciences. As far as the business operations are concerned, the application of the same exist in almost all the domains i.e. decision making on the basis of intersecting and non-intersecting sets of different decision variables. Then in the field of inventory management i.e. variation of holding/carrying cost, EOQ variations, logistics, etc.

A number of tools are being used for the purpose of computing and modeling, etc. are somehow deterministic in nature and have certain level of precision in their operations, but for any of the researcher or expert, the probability of getting precise and crisp data every time is less and such legitimate issue can be

arise in different field of study like economics, engineering, environment, social sciences and even in the field of the medical sciences. For a professional it is not always correct to use the classical methods of assessment on these data i.e. either the uncertainties of data is avoided or may be different model is used for the same.
As a matter of fact there are a number of theories available to deal with the above said uncertainties of data and can be listed as follows:

a. Probability Theory
b. Fuzzy Sets
c. Vague Sets
d. Interval Mathematics
e. Rough Sets, and others.

But all these theories and models are having certain limitation and these limitations are being mentioned in the article by **Molodtsov (1999)**. These limitations possess different reasons like inadequacy of parameters, etc. Molodtsov elaborated the use of '*Soft Set theory* ' as a tool to deal with the uncertain and vague nature of data, on the other hand this tool was suppose the handle the aforesaid limitation of data in a definite manner. The author has presented various examples from different field of studies to deal with the limitations like, economics, Engineering, etc.
As far as soft set are concerned they are basic and elementary in nature, and know as the neighborhood systems. Then on the other hand they are also considered as a special case fo fuzzy sets (dependent), this stature was given to them by **Thielle (1999)**. After the results drawn by Molodtsov and Thielle in the same year, various other experts also presented their views in this regard and gradually the application of soft set theory started to tighten its grip on other fields of study to handle the real life problems. Till the start of 21st century, people started to consider this system as an authentic one and by the year 2002, first practical application of soft sets was started in decision making problems in different fields. Like **Pawlak (1998)** have already provided a ready reference of algorithm to be used in case of soft sets, although it was based on the choice of optimal objects, but later it was found relevant in the formulation of soft sets. As a matter of fact the decision making was made easy using these soft sets and many of the real life problems were solved. For example Pawlak considered the marketing of real estate based on these soft sets and it was a success.

Basic of Soft Sets

- If '*U'* is an initial universe set and *E* is a set of all possible parameters parameters with aspect of *U*. here the term of parameter is used in relation of relevant characteristics or features of the objects present in the universe set *U*.

Definitions

a. According to **Molodtsov (1999)** any pair of objects like (*F,E*) is called a soft set of (*U*) in a condition where *F* is a mapping of *E* into the set of all subsets present in *U*. This can be stated as,

$F : E \rightarrow P(U)$
Where,
P (*U*) is the power set of *U*.
It can also be stated that the soft set is a parametric family of the subset of the universe set (*U*). Any random set i.e. *F(e) where* $e \in E$, can be stated as set of e-elements of the soft set (F, E) or approximately the elements of soft set.

b. This definition was given by **Pawlak (1991)** and was related to the presentation of rough sets. As per the definition, any system can be formulated as a pair i.e. *S=(U,A)* where U = a (non-empty) and finite set is called universe, also A = a (non-empty), finite set of primitive attributes.

Where,
$a \in A$ is a total function $a : U \rightarrow Va$, where Va is the set of the values of α called the domain of α.

c. Maji (2002);(2012) stated the use of soft sets in decision making science and fuzzy mathematics. He stated that if there are two soft sets i.e. (*F, A*) and (*G, B*) and the common universe of both the sets is *U* then it can be stated in the form:
- (*F, A*) is a soft subset of (*G, B*) if the given condition is, and *F*(e) ⊆*G* (e), for all e ∈ A. We write (F; A) $\tilde{\subseteq}$ (G; B).
- (*F, A*) is a soft subset of (*G, B*), then if (*G, B*) is soft subset of (F, A), it can be stated as (F, A) $\tilde{\supseteq}$ (G;B).

d. Then again Maji (2002);(2012), stated that a soft set (F, A) over X is said to be:
- Null soft set denoted by ϕ, if F (e) = ϕ, ∀ e ∈A.
- Absolute sor sect denoted by $\tilde{X}$, if F (e) = X, ∀ e ∈A.

Insertion in issues of Decision Making

Molodstov (2012) and the other experts of his time line have presented the insertion of this theory of soft set in different set of studies, like evaluation of functions, game theory, integration, probabilistic assessments, etc. then by 2012 some other experts presented the use of soft set in various decision making problems like Maji used the same in marketing of real estate services but the approach of the same was similar to that of the rough set.

Example

There are six types of houses, presented as a universe set i.e. *U* = {h_1, h_2, h_3, h_4, h_5, h_6}and the identifiers of the houses are, costly, good looking, made of wood, not so costly i.e. cheap, with green surrounding, modern style, etc. and these can also be called as parameters.

In the above given case the soft set (F, E) that is related to the 'attractiveness of the houses' and the same is presented by:

{F,E}=
Costly house = ϕ
Good Looking = {h_1, h_2, h_3, h_4, h_5, h_6}
Made of Wood = {h_1, h_2, h_6}
Not so costly = {h_1, h_2, h_6}
With Green surrounding = {h_2, h_4, h_5}
Modern Style = {h_1, h_3, h_6}

Now if Mr. Mohan is willing to buy a house according to the choice based parameters i.e. 'good looking, Made of wood, with green surrounding, Modern Style' and these frame the subset A = (e_1=good looking, e_2 = Made of Wood, e_3= With good surrounding, e_4= not so costly, e_5 = Modern Style) of the set *E*. These conditions state that a person who is willing to buy a house is required to select the same which qualifies the maximum number of parameters of the soft set A.

Then on the other hand Mr. Sohan, Mr. pritam also want to buy the house under the above given parameters, for Sohan the conditions are B⊂ E and for pritam the conditions are C ⊂ E. Here the problem is to identify a good house for Mr. Mohan and the suitability of that house may not be equal to the choices of other interested customers.

The problem is explained as:

U \ A	e_1	e_2	e_3	e_4	e_5
c_1	1	1	1	1	1
c_2	1	1	1	1	0
c_3	1	0	1	1	1
c_4	1	0	1	1	0
c_5	1	0	1	0	0

c_6	1	1	1	1	1

System of Selecting a House

System for Mr. X:

- Consider the soft set (*F, E*)
- Input the set A of choice parameters of Mr. X that is subset of *E*,
- Consider all soft sets to (*F, A*)
- Choose one soft set, Like (*F, B*) of (*F, A*)
- Arrange against constant k, where vk = Max vi

As per the last condition given above vkcan be considered as optimal choice. Now if the value of constt. k is more than one value Mr. X can chose any one house on the basis of options available to him.

Putting the weights of the available options in $vi = \sum_j cij$, the respective options are,

$v_1 = 4$
$v_2 = 3$
$v_3 = 3$
$v_4 = 2$
$v_5 = 1$
$v_6 = 4$
(viranges from 1 to 6)

In any given case of the above said values the parameters of A are not so relevant for Mr. Mohan or these are having equal importance for him, then he may be assigning weights to the choice based parameters and that too for each element.

These weights can be in the form of:

$a_i \in A$ are the elements, and

assigned weight can be of the form, $w_i \in (0, 1)$

After assigning the weights, the clear selection of the choice, for houses can be c_1 or c_6 for purchasing the house as per the parameters given in A.

Conclusion

As stated in the above given example, the inclusion of soft set theory in the decision making and optimization problems have played a significant role in finding solutions and that too in the cases where the data is not so perfect i.e. it is vague or uncertain. Many of such applications of the above said models are given by Molodtsov considering soft set theory as the basis of the same. On the other hand Maji also elaborated the results of above said model and tried to find the neutral values for each decision variable. Another application was presented by Sayed and he also explained the same in accordance of decision choices. Then the rough technique of Pawlak was some kind of exception to this model and can be used as an alternative of the same.

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