

Fuzzy Quotient-3 Cordial Labeling of Generalized Jahangir Graph and its Subdivision Graph

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Abstract: Let the function $\sigma : V \rightarrow [0,1]$ defined by $\sigma(\vartheta) = \frac{\gamma}{10}, \gamma \in Z_4 - \{0\}$ is called the fuzzy quotient-3 cordial labeling if for each edges $\omega\vartheta \in E$, define $\mu : E \rightarrow [0,1]$ by $\mu(\omega\vartheta) = \frac{1}{10} \left\lfloor \frac{3\sigma(\omega)}{\sigma(\vartheta)} \right\rfloor$ where $\sigma(\omega) \leq \sigma(\vartheta)$ such that $v_\sigma(i)$ and $v_\sigma(j)$ differ by atmost 1 and $e_\mu(i)$ and $e_\mu(j)$ differ by atmost 1 where $v_\sigma(i)$ and $e_\mu(i)$ represents the number of vertices and edges assigned with $i \in \{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\}$. When the graph $G(V, E)$ admits fuzzy quotient-3 cordial labeling, then $G(V, E)$ is fuzzy quotient-3 cordial graph. Fuzzy quotient -3 cordial labeling on generalized Jahangir graph and its subdivision are investigated and the works are presented in this paper.

Keywords: Subdivision graph, Generalised Jahangir graph, Subdivision generalised Jahangir graph, Fuzzy quotient-3 cordial graph.

1. Introduction

Assigning of values under certain rules to the graph G is called graph labeling. Rosa (1967) or Graham & Sloane (1980) introduced the graph labeling methods. Among many researchers, graph labeling has been creating a lot of interest and motivation besides their practical applications, there are also various applications in other fields of mathematics. The excellent dynamic survey by Gallian on graph labeling gives an exhaustive survey on the results of graph labeling. Motivated by these labels, we introduced a cordial fuzzy quotient-3 label and proved to be a fuzzy quotient-3 cordial label for some graph families. In this paper the generalized Jahangir graph and its subdivisions are investigated and proved that the graphs are fuzzy quotient - 3 cordial.

2. Definitions

Definition 2.1. Subdivision graph

The subdivision graph is the graph obtained by inserting an additional vertex into each edge of G , denoted as $S(G)$.

Definition 2.2. Generalized Jahangir graph $J_{s,t}$

The generalized Jahangir graph $J_{s,t}$ is a graph with $st + 1$ vertices consisting of a cycle C_{st} such that one vertex is adjacent to t vertices at a distance s to each other on C_{st} .

Definition 2.3. $S^1(J_{s,t})$ graph

The graph denoted by $S^1(J_{s,t})$ is obtained by inserting an additional vertex into each edge of a cycle C_{st} in the graph $J_{s,t}$.

Definition 2.4. $S^2(J_{s,t})$ graph

The graph denoted by $S^2(J_{s,t})$ is obtained from the graph $J_{s,t}$ by inserting an additional vertex into each edge which are all incident with the centre vertex of $J_{s,t}$.

Definition 2.5. $S(J_{s,t})$ graph

The graph denoted by $S(J_{s,t})$ is obtained by inserting an additional vertex into each edge of the graph $J_{s,t}$.

Definition 2.6. Fuzzy quotient-3 cordial graph

A graph G with order p and size q . Let the function $\sigma : V \rightarrow [0,1]$ defined by $\sigma(\vartheta) = \frac{\gamma}{10}, \gamma \in Z_4 - \{0\}$ is called the fuzzy quotient-3 cordial labeling if for each edges $\omega\vartheta \in E$, define $\mu : E \rightarrow [0,1]$ by $\mu(\omega\vartheta) = \frac{1}{10} \left\lfloor \frac{3\sigma(\omega)}{\sigma(\vartheta)} \right\rfloor$ where $\sigma(\omega) \leq \sigma(\vartheta)$ such that $v_\sigma(i)$ and $v_\sigma(j)$ differ by atmost 1 and $e_\mu(i)$ and $e_\mu(j)$ differ by atmost 1 where $v_\sigma(i)$ and $e_\mu(i)$ represents the number of vertices and edges assigned with $i \in \{\frac{\gamma}{10}, \gamma \in Z_4 - \{0\}\}$. When the graph $G(V, E)$ admits fuzzy quotient-3 cordial labeling, then $G(V, E)$ is fuzzy quotient-3 cordial graph.

3. Main Result

Theorem 3.1. The generalized Jahangir graph $\mathcal{J}_{s,t}$ is fuzzy quotient - 3 cordial, for $s \geq 2$ and $t \geq 2$.

Proof: Let G be a generalized Jahangir graph $\mathcal{J}_{s,t}$. $V(G) = \{c\} \cup \{c_i: 1 \leq i \leq st\}$ and $E(G) = \{c_i c_{i+1}: 1 \leq i \leq st - 1\} \cup \{c_1 c_{st}\} \cup \{c c_{s(i-1)+1}: 1 \leq i \leq t\}$. $p = st + 1$ and $q = st + t$. To define $\sigma: V(G) \rightarrow [0,1]$ the following cases are to be considered.

Case 1: $s \equiv 0 \pmod{6}$

$$\sigma(c) = 0.1$$

For $k \equiv 1 \pmod{3}$ $1 \leq k \leq t$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 0,1 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 3,4 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 2,5 \pmod{6} \quad 1 \leq i \leq s$$

For $k \equiv 2 \pmod{3}$ $1 \leq k \leq t$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 4,5 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 1,2 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 0,3 \pmod{6} \quad 1 \leq i \leq s$$

For $k \equiv 0 \pmod{3}$ $1 \leq k \leq t$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 2,3 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 0,5 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 1,4 \pmod{6} \quad 1 \leq i \leq s$$

Case 2: $s \equiv 1 \pmod{6}$

Subcase 2.1: $t \equiv 0 \pmod{3}$

$$\sigma(c) = 0.1$$

For $k \equiv 1 \pmod{3}$ $1 \leq k \leq t$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 0,1 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 3,4 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 2,5 \pmod{6} \quad 1 \leq i \leq s$$

For $k \equiv 2 \pmod{3}$ $1 \leq k \leq t$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 2,3 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 0,5 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 1,4 \pmod{6} \quad 1 \leq i \leq s$$

For $k \equiv 0 \pmod{3}$ $1 \leq k \leq t$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 0,5 \pmod{6} \quad 1 \leq i \leq s - 1$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 2,3 \pmod{6} \quad 1 \leq i \leq s - 1$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 1,4 \pmod{6} \quad 1 \leq i \leq s - 1$$

$$\sigma(c_{s(k)}) = 0.2$$

Subcase 2.2: $t \equiv 1 \pmod{3}$

$$\sigma(c) = 0.1$$

For $k \equiv 1 \pmod{3}$ $1 \leq k \leq t - 1$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 0,1 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 3,4 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 2,5 \pmod{6} \quad 1 \leq i \leq s$$

For $k \equiv 2 \pmod{3}$ $1 \leq k \leq t - 1$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 2,3 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 0,5 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 1,4 \pmod{6} \quad 1 \leq i \leq s$$

For $k \equiv 0 \pmod{3}$ $1 \leq k \leq t - 1$

$$\sigma(c_{s(k-1)+i}) = 0.1 \quad \text{if } i \equiv 0,5 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.2 \quad \text{if } i \equiv 2,3 \pmod{6} \quad 1 \leq i \leq s$$

$$\sigma(c_{s(k-1)+i}) = 0.3 \quad \text{if } i \equiv 1,4 \pmod{6} \quad 1 \leq i \leq s$$

For $k = t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) = 0.1 & \quad ifi \equiv 3,4(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.2 & \quad ifi \equiv 0,1(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.3 & \quad ifi \equiv 2,5(mod 6) & \quad 1 \leq i \leq s \end{aligned}$$

Subcase 2.3: $t \equiv 2(mod 3)$

If t is even

$$\begin{aligned} \sigma(c_i) = 0.1 & \quad ifi \equiv 0,1(mod 6) & \quad 1 \leq i \leq st - 1 \\ \sigma(c_i) = 0.2 & \quad ifi \equiv 3,4(mod 6) & \quad 1 \leq i \leq st - 1 \\ \sigma(c_i) = 0.3 & \quad ifi \equiv 2,5(mod 6) & \quad 1 \leq i \leq st - 1 \\ & \quad \sigma(c_{st}) = 0.2 \end{aligned}$$

If t is odd

$$\begin{aligned} \sigma(c_i) = 0.1 & \quad ifi \equiv 1,2(mod 6) & \quad 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \quad ifi \equiv 4,5(mod 6) & \quad 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \quad ifi \equiv 0,3(mod 6) & \quad 1 \leq i \leq st \end{aligned}$$

Case 3: $s \equiv 2(mod 6)$

Subcase 3.1: $t \equiv 0(mod 3)$

$$\begin{aligned} \sigma(c_i) = 0.1 & \quad ifi \equiv 1,2(mod 6) & \quad 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \quad ifi \equiv 4,5(mod 6) & \quad 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \quad ifi \equiv 0,3(mod 6) & \quad 1 \leq i \leq st \end{aligned}$$

Subcase 3.2: $t \equiv 1(mod 3)$

$$\begin{aligned} \sigma(c) = 0.1 \\ \text{For } k \equiv 1(mod 3) 1 \leq k \leq t - 1 \\ \sigma(c_{s(k-1)+i}) = 0.1 & \quad ifi \equiv 0,1(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.2 & \quad ifi \equiv 3,4(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.3 & \quad ifi \equiv 2,5(mod 6) & \quad 1 \leq i \leq s \end{aligned}$$

For $k \equiv 2(mod 3) 1 \leq k \leq t - 1$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) = 0.1 & \quad ifi \equiv 2,3(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.2 & \quad ifi \equiv 0,5(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.3 & \quad ifi \equiv 1,4(mod 6) & \quad 1 \leq i \leq s \end{aligned}$$

For $k \equiv 0(mod 3) 1 \leq k \leq t - 1$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) = 0.1 & \quad ifi \equiv 4,5(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.2 & \quad ifi \equiv 1,2(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.3 & \quad ifi \equiv 0,3(mod 6) & \quad 1 \leq i \leq s \end{aligned}$$

For $k = t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) = 0.1 & \quad ifi \equiv 3,4(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.2 & \quad ifi \equiv 0,1(mod 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) = 0.3 & \quad ifi \equiv 2,5(mod 6) & \quad 1 \leq i \leq s \end{aligned}$$

Subcase 3.3: $t \equiv 2(mod 3)$

$$\begin{aligned} \sigma(c) = 0.3 \\ \sigma(c_i) = 0.1 & \quad ifi \equiv 1,2(mod 6) & \quad 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \quad ifi \equiv 4,5(mod 6) & \quad 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \quad ifi \equiv 0,3(mod 6) & \quad 1 \leq i \leq st \end{aligned}$$

Case 4: $s \equiv 3(mod 6)$

Subcase 4.1: $s = 3$ and $t \equiv 0,1(mod 3)$

$$\begin{aligned} \sigma(c_{is-2}) = 0.1 & \quad ifi \equiv 0,1(mod 3) & \quad 1 \leq i \leq t \\ \sigma(c_{is-2}) = 0.3 & \quad ifi \equiv 2(mod 3) & \quad 1 \leq i \leq t \\ \sigma(c_{is-1}) = 0.1 & \quad ifi \equiv 2(mod 3) & \quad 1 \leq i \leq t \\ \sigma(c_{is-1}) = 0.2 & \quad ifi \equiv 0,1(mod 3) & \quad 1 \leq i \leq t \\ \sigma(c_{is}) = 0.2 & \quad ifi \equiv 0(mod 3) & \quad 1 \leq i \leq t \\ \sigma(c_{is}) = 0.3 & \quad ifi \equiv 1,2(mod 3) & \quad 1 \leq i \leq t \end{aligned}$$

Subcase 4.2: $s = 3$ and $t \equiv 2(mod 3)$

$$\sigma(c) = 0.1$$

$$\begin{aligned} \sigma(c_{is-2}) &= 0.1 & ifi &\equiv 0,1(mod\ 3) & 1 \leq i \leq t-2 \\ \sigma(c_{is-2}) &= 0.3 & ifi &\equiv 2(mod\ 3) & 1 \leq i \leq t-2 \\ \sigma(c_{is-1}) &= 0.1 & ifi &\equiv 2(mod\ 3) & 1 \leq i \leq t-2 \\ \sigma(c_{is-1}) &= 0.2 & ifi &\equiv 0,1(mod\ 3) & 1 \leq i \leq t-2 \\ \sigma(c_{is}) &= 0.2 & ifi &\equiv 0(mod\ 3) & 1 \leq i \leq t-2 \\ \sigma(c_{is}) &= 0.3 & ifi &\equiv 1,2(mod\ 3) & 1 \leq i \leq t-2 \\ \sigma(c_{s(t-1)-2}) &= 0.2, \sigma(c_{s(t-1)-1}) &= 0.1, \sigma(c_{s(t-1)}) &= 0.3 \\ \sigma(c_{s(t)-2}) &= 0.3, \sigma(c_{s(t)-1}) &= 0.1, \sigma(c_{s(t)}) &= 0.2 \end{aligned}$$

Subcase 4.3: $n \geq 9$ and $t \equiv 0,1,2(mod\ 3)$

$$\sigma(c) = 0.2$$

For $1 \leq k \leq t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & ifi &\equiv 4,5(mod\ 6) & 1 \leq i \leq s-3 \\ \sigma(c_{s(k-1)+i}) &= 0.2 & ifi &\equiv 1,2(mod\ 6) & 1 \leq i \leq s-3 \\ \sigma(c_{s(k-1)+i}) &= 0.3 & ifi &\equiv 0,3(mod\ 6) & 1 \leq i \leq s-3 \\ \sigma(c_{is-2}) &= 0.1 & 1 \leq i \leq t & \\ \sigma(c_{is-1}) &= 0.2 & ifi &\equiv 0,1(mod\ 3) & 1 \leq i \leq t \\ \sigma(c_{is-1}) &= 0.3 & ifi &\equiv 2(mod\ 3) & 1 \leq i \leq t \\ \sigma(c_{is}) &= 0.2 & ifi &\equiv 2(mod\ 3) & 1 \leq i \leq t \\ \sigma(c_{is}) &= 0.3 & ifi &\equiv 0,1(mod\ 3) & 1 \leq i \leq t \end{aligned}$$

Case 5: $s \equiv 4(mod\ 6)$

Subcase 5.1: $t \equiv 0,1(mod\ 3)$

$$\sigma(c) = 0.3$$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi &\equiv 1,2(mod\ 6) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 & ifi &\equiv 4,5(mod\ 6) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 & ifi &\equiv 0,3(mod\ 6) & 1 \leq i \leq st \end{aligned}$$

Subcase 5.2: $t \equiv 2(mod\ 3)$

$$\sigma(c) = 0.1$$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi &\equiv 1,2(mod\ 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) &= 0.2 & ifi &\equiv 4,5(mod\ 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) &= 0.3 & ifi &\equiv 0,3(mod\ 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_{(st-1)+i}) &= 0.1 & ifi &\equiv 0,5(mod\ 6) & 1 \leq i \leq s \\ \sigma(c_{(st-1)+i}) &= 0.2 & ifi &\equiv 2,3(mod\ 6) & 1 \leq i \leq s \\ \sigma(c_{(st-1)+i}) &= 0.3 & ifi &\equiv 1,4(mod\ 6) & 1 \leq i \leq s \end{aligned}$$

Case 6: $s \equiv 5(mod\ 6)$

Subcase 6.1: $t \equiv 0(mod\ 3)$

$$\sigma(c) = 0.1$$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi &\equiv 0,1(mod\ 6) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 & ifi &\equiv 3,4(mod\ 6) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 & ifi &\equiv 2,5(mod\ 6) & 1 \leq i \leq st \end{aligned}$$

Subcase 6.2: $t \equiv 1(mod\ 3)$

$$\sigma(c) = 0.1$$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi &\equiv 0,1(mod\ 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) &= 0.2 & ifi &\equiv 3,4(mod\ 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) &= 0.3 & ifi &\equiv 2,5(mod\ 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_{s(t-1)+i}) &= 0.1 & ifi &\equiv 0,1(mod\ 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) &= 0.2 & ifi &\equiv 3,4(mod\ 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) &= 0.3 & ifi &\equiv 2,5(mod\ 6) & 1 \leq i \leq s \end{aligned}$$

Subcase 6.3: $t \equiv 2(mod\ 3)$

$$\sigma(c) = 0.1$$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi &\equiv 0,1(mod\ 6) & 1 \leq i \leq s(t-2) \\ \sigma(c_i) &= 0.2 & ifi &\equiv 3,4(mod\ 6) & 1 \leq i \leq s(t-2) \\ \sigma(c_i) &= 0.3 & ifi &\equiv 2,5(mod\ 6) & 1 \leq i \leq s(t-2) \\ \sigma(c_{s(t-2)+i}) &= 0.1 & ifi &\equiv 0,1(mod\ 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-2)+i}) &= 0.2 & ifi &\equiv 3,4(mod\ 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-2)+i}) &= 0.3 & ifi &\equiv 2,5(mod\ 6) & 1 \leq i \leq s \end{aligned}$$

$$\begin{aligned} \sigma(c_{s(t-1)+i}) &= 0.1 & ifi &\equiv 2,3(mod 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) &= 0.2 & ifi &\equiv 0,5(mod 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) &= 0.3 & ifi &\equiv 1,4(mod 6) & 1 \leq i \leq s \end{aligned}$$

Table.1. $v_\sigma(i)$ and $e_\mu(i)$ for the graph $J_{s,t}$, $i \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$.

Nature of s and t	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$s \equiv 0(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 0(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$s \equiv 0(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 1(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 1(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 1(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$
$s \equiv 2(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s = 3$ $t \equiv 0(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s = 3$ $t \equiv 1(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$
$s = 3$ $t \equiv 2(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 3(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 3(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$
$s \equiv 3(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$
$s \equiv 4(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 4(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 4(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$

$s \equiv 5(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 5(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 5(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$

From the table 1, we find that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ which satisfies the condition of fuzzy quotient - 3 cordial labeling. Hence we concluded that the generalized Jahangir graph $J_{s,t}$ is fuzzy quotient - 3 cordial.

Theorem 3.2. The graph $S^1(J_{s,t})$ is fuzzy quotient - 3 cordial, for $s \geq 2$ and $t \geq 2$.

Proof: Let G be a $S^1(J_{s,t})$. $V(S^1(J_{s,t})) = \{c\} \cup \{c_i: 1 \leq i \leq 2st\}$ and $E(J_{s,t}) = \{c_i c_{i+1}: 1 \leq i \leq 2st - 1\} \cup \{c_1 c_{2st}\} \cup \{c c_{2s(i-1)+1}: 1 \leq i \leq t\}$. $p = 2st + 1$ and $q = 2st + t$.

To define $\sigma: V(G) \rightarrow [0,1]$ the following cases are to be considered.

Case 1: $s \equiv 0,3(mod 6)$

$$\sigma(c) = 0.1$$

For $k \equiv 1(mod 3) 1 \leq k \leq t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & if i \equiv 1(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & if i \equiv 2(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & if i \equiv 0(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & if i \equiv 0(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & if i \equiv 2(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & if i \equiv 1(mod 3) & 1 \leq i \leq s \end{aligned}$$

For $k \equiv 2(mod 3) 1 \leq k \leq t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & if i \equiv 0(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & if i \equiv 1(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & if i \equiv 2(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & if i \equiv 2(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & if i \equiv 1(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & if i \equiv 0(mod 3) & 1 \leq i \leq s \end{aligned}$$

For $k \equiv 0(mod 3) 1 \leq k \leq t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & if i \equiv 2(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & if i \equiv 0(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & if i \equiv 1(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & if i \equiv 1(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & if i \equiv 0(mod 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & if i \equiv 2(mod 3) & 1 \leq i \leq s \end{aligned}$$

Case 2: $s \equiv 1,4(mod 6)$

Subcase 2.1: $t \equiv 0(mod 3)$

$$\sigma(c) = 0.1$$

$$\begin{aligned} \sigma(c_i) &= 0.1 & if i \equiv 1(mod 3) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 & if i \equiv 0(mod 3) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 & if i \equiv 2(mod 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.1 & if i \equiv 1(mod 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.2 & if i \equiv 2(mod 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.3 & if i \equiv 0(mod 3) & 1 \leq i \leq st \end{aligned}$$

Subcase 2.2: $t \equiv 1(mod 3)$

$$\sigma(c) = 0.1$$

For $k \equiv 1(mod 3) 1 \leq k \leq t - 1$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & if i \equiv 1(mod 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & if i \equiv 2(mod 3) & 1 \leq i \leq s \end{aligned}$$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.3 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \end{aligned}$$

For $k \equiv 2(mod\ 3) 1 \leq k \leq t - 1$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \end{aligned}$$

For $k \equiv 0(mod\ 3) 1 \leq k \leq t - 1$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \end{aligned}$$

For $k = t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.1 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.2 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{s(k-1)+i}) &= 0.3 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \end{aligned}$$

Subcase 2.3: $t \equiv 2(mod\ 3)$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.2 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.3 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq st \end{aligned}$$

Case 3: $s \equiv 2,5(mod\ 6)$

Subcase 3.1: $t \equiv 0,1(mod\ 3)$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.2 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq st \\ \sigma(e_i) &= 0.3 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq st \end{aligned}$$

Subcase 3.2: $t \equiv 2(mod\ 3)$

$$\begin{aligned} \sigma(c_i) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq st - 1 \\ \sigma(c_i) &= 0.2 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq st - 1 \\ \sigma(c_i) &= 0.3 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq st - 1 \\ \sigma(e_i) &= 0.1 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq st - 1 \\ \sigma(e_i) &= 0.2 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq st - 1 \\ \sigma(e_i) &= 0.3 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq st - 1 \\ \sigma(c_{(st-1)+i}) &= 0.1 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{(st-1)+i}) &= 0.2 & ifi \equiv 2(mod\ 3) & 1 \leq i \leq s \\ \sigma(c_{(st-1)+i}) &= 0.3 & ifi \equiv 1(mod\ 3) & 1 \leq i \leq s \\ \sigma(e_{(st-1)+i}) &= 0.1 & ifi \equiv 0(mod\ 3) & 1 \leq i \leq s \end{aligned}$$

$$\begin{aligned} \sigma(e_{(st-1)+i}) &= 0.2 & \text{if } i \equiv 1(\text{mod } 3) \quad 1 \leq i \leq s \\ \sigma(e_{(st-1)+i}) &= 0.3 & \text{if } i \equiv 2(\text{mod } 3) \quad 1 \leq i \leq s \end{aligned}$$

Table.2. $v_\sigma(i)$ and $e_\mu(i)$ for the graph $S^1(\mathcal{J}_{s,t})$, $i \in \{\frac{r}{10}, r \in \mathbb{Z}_4 - \{0\}\}$.

Nature of s and t	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$s \equiv 0,3(\text{mod } 6)$ $t \equiv 0(\text{mod } 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 0,3(\text{mod } 6)$ $t \equiv 1(\text{mod } 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$s \equiv 0,3(\text{mod } 6)$ $t \equiv 2(\text{mod } 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 1,4(\text{mod } 6)$ $t \equiv 0(\text{mod } 3)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 1,4(\text{mod } 6)$ $t \equiv 1(\text{mod } 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 1,4(\text{mod } 6)$ $t \equiv 2(\text{mod } 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2,5(\text{mod } 6)$ $t \equiv 0(\text{mod } 3)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2,5(\text{mod } 6)$ $t \equiv 1(\text{mod } 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 2,5(\text{mod } 6)$ $t \equiv 2(\text{mod } 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$

From the table 2, we find that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i, j \in \{\frac{r}{10}, r \in \mathbb{Z}_4 - \{0\}\}$ which satisfies the condition of fuzzy quotient - 3 cordial labeling. Hence we concluded that the graph $S^1(\mathcal{J}_{s,t})$ is fuzzy quotient - 3 cordial.

Theorem 3.3. The graph $S^2(\mathcal{J}_{s,t})$ is fuzzy quotient-3 cordial graph, for $s \geq 2$ and $t \geq 2$.

Proof: Let G be the $S^2(\mathcal{J}_{s,t})$ graph. $V(S(\mathcal{J}_{s,t})) = \{c_i: 1 \leq i \leq st\} \cup \{c\} \cup \{d_i: 1 \leq i \leq t\}$ and $E(S(\mathcal{J}_{s,t})) = \{c_i c_{i+1}: 1 \leq i \leq st - 1\} \cup \{c_1 c_{st}\} \cup \{c d_j: 1 \leq j \leq t\} \cup \{d_j c_{s(j-1)+1}: 1 \leq j \leq t\}$. $p = st + t + 1$ and $q = st + 2t$. To define $\sigma: V(S(\mathcal{J}_{s,t})) \rightarrow [0,1]$ the following cases are to be considered.

Case 1: $s \equiv 0(\text{mod } 6)$

$$\begin{aligned} \sigma(c) &= 0.2 \\ \sigma(d_j) &= 0.1 & \text{if } i \equiv 1(\text{mod } 3) & \quad 1 \leq j \leq t \\ \sigma(d_j) &= 0.2 & \text{if } i \equiv 0(\text{mod } 3) & \quad 1 \leq j \leq t \\ \sigma(d_j) &= 0.3 & \text{if } i \equiv 2(\text{mod } 3) & \quad 1 \leq j \leq t \end{aligned}$$

If $k \equiv 1,2(\text{mod } 3), 1 \leq k \leq t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & \text{if } i \equiv 0,1(\text{mod } 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & \text{if } i \equiv 3,4(\text{mod } 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & \text{if } i \equiv 2,5(\text{mod } 6) & \quad 1 \leq i \leq s \end{aligned}$$

If $k \equiv 0(\text{mod } 3), 1 \leq k \leq t$

$$\begin{aligned} \sigma(c_{s(k-1)+i}) &= 0.1 & \text{if } i \equiv 4,5(\text{mod } 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.2 & \text{if } i \equiv 1,2(\text{mod } 6) & \quad 1 \leq i \leq s \\ \sigma(c_{s(k-1)+i}) &= 0.3 & \text{if } i \equiv 0,3(\text{mod } 6) & \quad 1 \leq i \leq s \end{aligned}$$

Case 2: $s \equiv 1(\text{mod } 6)$

Subcase 2.1: $t \equiv 0(\text{mod } 6)$

$$\sigma(c) = 0.3$$

$$\begin{array}{lll} \sigma(d_j) = 0.1 & \text{if } i \equiv 0,4(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2,3(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

Subcase 2.2: $t \equiv 1(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.3 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 0,4(\text{mod } 6) & 1 \leq j \leq t - 1 \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq j \leq t - 1 \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq j \leq t - 1 \\ \sigma(d_t) = 0.1 & & \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2,3(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\ \sigma(c_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 4,5(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 0,3(\text{mod } 6) & 1 \leq i \leq s \end{array}$$

Subcase 2.3: $t \equiv 2(\text{mod } 6)$

If $t = 2$

$$\sigma(c) = 0.2$$

$$\sigma(d_1) = 0.1, \sigma(d_2) = 0.3$$

$$\begin{array}{lll} \sigma(c_i) = 0.1 & \text{if } i \equiv 0,1(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

If $t \geq 8$

$$\begin{array}{lll} \sigma(c) = 0.3 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 0,4(\text{mod } 6) & 1 \leq j \leq t - 2 \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq j \leq t - 2 \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq j \leq t - 2 \end{array}$$

$$\sigma(d_{t-1}) = 0.1, \sigma(d_t) = 0.2$$

$$\begin{array}{lll} \sigma(c_i) = 0.1 & \text{if } i \equiv 2,3(\text{mod } 6) & 1 \leq i \leq s(t - 2) \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq i \leq s(t - 2) \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq i \leq s(t - 2) \\ \sigma(c_{s(t-2)+i}) = 0.1 & \text{if } i \equiv 0,1(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-2)+i}) = 0.2 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-2)+i}) = 0.3 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 2,3(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq i \leq s \end{array}$$

Subcase 2.4: $t \equiv 3(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.3 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 1,5(\text{mod } 6) & 1 \leq j \leq t - 3 \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0,3(\text{mod } 6) & 1 \leq j \leq t - 3 \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 2,4(\text{mod } 6) & 1 \leq j \leq t - 3 \end{array}$$

$$\sigma(d_{t-2}) = 0.2, \sigma(d_{t-1}) = 0.1, \sigma(d_t) = 0.1$$

$$\begin{array}{lll} \sigma(c_i) = 0.1 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq i \leq st - 1 \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0,1(\text{mod } 6) & 1 \leq i \leq st - 1 \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq i \leq st - 1 \\ \sigma(c_{st}) = 0.3 & & \end{array}$$

Subcase 2.5: $t \equiv 4(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 1,5(\text{mod } 6) & 1 \leq j \leq t - 4 \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0,3(\text{mod } 6) & 1 \leq j \leq t - 4 \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 2,4(\text{mod } 6) & 1 \leq j \leq t - 4 \end{array}$$

$$\sigma(d_{t-3}) = 0.2, \sigma(d_{t-2}) = 0.3, \sigma(d_{t-1}) = 0.3, \sigma(d_t) = 0.1$$

$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq s(t-1)$
$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq s(t-1)$
$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq s(t-1)$
$\sigma(c_{s(t-1)+i}) = 0.1$	<i>if</i> $i \equiv 4,5 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(t-1)+i}) = 0.2$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(t-1)+i}) = 0.3$	<i>if</i> $i \equiv 0,3 \pmod{6}$	$1 \leq i \leq s$

Subcase 2.6: $t \equiv 5 \pmod{6}$

$\sigma(c) = 0.3$		
$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1,5 \pmod{6}$	$1 \leq j \leq t-5$
$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0,3 \pmod{6}$	$1 \leq j \leq t-5$
$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 2,4 \pmod{6}$	$1 \leq j \leq t-5$

$$\sigma(d_{t-4}) = 0.1, \sigma(d_{t-3}) = 0.1, \sigma(d_{t-2}) = 0.2, \sigma(d_{t-1}) = 0.3, \sigma(d_t) = 0.2$$

$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq st$
$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq st$
$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq st$

Case 3: $s \equiv 2 \pmod{6}$

Subcase 3.1: $t \equiv 0 \pmod{6}$

$\sigma(c) = 0.2$		
$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t$

If $k \equiv 1,2 \pmod{3}, 1 \leq k \leq t$

$\sigma(c_{s(k-1)+i}) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq s$

If $k \equiv 0 \pmod{3}, 1 \leq k \leq t$

$\sigma(c_{s(k-1)+i}) = 0.1$	<i>if</i> $i \equiv 4,5 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.2$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.3$	<i>if</i> $i \equiv 0,3 \pmod{6}$	$1 \leq i \leq s$

Subcase 3.2: $t \equiv 1 \pmod{6}$

$\sigma(c) = 0.2$		
$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t$

If $k \equiv 1,2 \pmod{3}, 1 \leq k \leq t$

$\sigma(c_{s(k-1)+i}) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq s$

If $k \equiv 0 \pmod{3}, 1 \leq k \leq t$

$\sigma(c_{s(k-1)+i}) = 0.1$	<i>if</i> $i \equiv 4,5 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.2$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.3$	<i>if</i> $i \equiv 0,3 \pmod{6}$	$1 \leq i \leq s$

Subcase 3.3: $t \equiv 2 \pmod{6}$

$\sigma(c) = 0.3$		
$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 4,5 \pmod{6}$	$1 \leq j \leq t$
$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq st$
$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq st$
$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq st$

Subcase 3.4: $t \equiv 3 \pmod{6}$

$\sigma(c) = 0.2$		
$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t$

$$\begin{array}{lll} \sigma(d_j) = 0.2 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 4,5(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 0,3(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

Subcase 3.5: $t \equiv 4(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.2 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 4,5(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 0,3(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

Subcase 3.6: $t \equiv 5(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 2,4(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0,1(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 3,5(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 4,5(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 0,3(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 0,1(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq i \leq s \end{array}$$

Case 4: $s \equiv 3(\text{mod } 6)$

Subcase 4.1: $n = 3$ and $t \equiv 0(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2,3(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

Subcase 4.2: $n = 3$ and $t \equiv 1(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2,3(\text{mod } 6) & 1 \leq i \leq st-1 \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq i \leq st-1 \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1,4(\text{mod } 6) & 1 \leq i \leq st-1 \\ \sigma(c_{st}) = 0.2 & & \end{array}$$

Subcase 4.3: $n = 3$ and $t \equiv 2(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2,5(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 0,1(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

Subcase 4.4: $n = 3$ and $t \equiv 3(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \end{array}$$

	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq st$
Subcase 4.5:	$n = 3$ and $t \equiv 4 \pmod{6}$		
	$\sigma(c) = 0.1$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq st$
Subcase 4.6:	$n = 3$ and $t \equiv 5 \pmod{6}$		
	$\sigma(c) = 0.1$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq j \leq t - 1$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t - 1$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq j \leq t - 1$
	$\sigma(d_{t-1}) = 0.2, \sigma(d_t) = 0.3$		
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq st$
Subcase 4.7:	$n \geq 9$ and $t \equiv 0,1,2,3,4,5 \pmod{6}$		
	$\sigma(c) = 0.1$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq j \leq t$
For $1 \leq k \leq t$			
	$\sigma(c_{s(k-1)+i}) = 0.1$	<i>if</i> $i \equiv 4,5 \pmod{6}$	$1 \leq i \leq s - 3$
	$\sigma(c_{s(k-1)+i}) = 0.2$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq i \leq s - 3$
	$\sigma(c_{s(k-1)+i}) = 0.3$	<i>if</i> $i \equiv 0,3 \pmod{6}$	$1 \leq i \leq s - 3$
	$\sigma(c_{is-2}) = 0.1$	<i>if</i> $i \equiv 0,2 \pmod{3}$	$1 \leq i \leq t$
	$\sigma(c_{is-2}) = 0.3$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq i \leq t$
	$\sigma(c_{is-1}) = 0.1$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq i \leq t$
	$\sigma(c_{is-1}) = 0.2$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq i \leq t$
	$\sigma(c_{is-1}) = 0.3$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq i \leq t$
	$\sigma(c_{is}) = 0.2$	<i>if</i> $i \equiv 0,1 \pmod{3}$	$1 \leq i \leq t$
	$\sigma(c_{is}) = 0.3$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq i \leq t$
Case 5:	$s \equiv 4 \pmod{6}$		
Subcase 5.1:	$t \equiv 0 \pmod{6}$		
	$\sigma(c) = 0.1$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq st$
Subcase 5.2:	$t \equiv 1 \pmod{6}$		
	$\sigma(c) = 0.3$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_{s(t-1)+i}) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq s$
	$\sigma(c_{s(t-1)+i}) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq s$

	$\sigma(c_{s(t-1)+i}) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq s$
Subcase 5.3:	$t \equiv 2 \pmod{6}$		
	$\sigma(c) = 0.1$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq j \leq t-2$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq j \leq t-2$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq j \leq t-2$
	$\sigma(d_{t-1}) = 0.2, \sigma(d_t) = 0.3$		
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_{s(t-1)+i}) = 0.1$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq s-1$
	$\sigma(c_{s(t-1)+i}) = 0.2$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq s-1$
	$\sigma(c_{s(t-1)+i}) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq s-1$
	$\sigma(c_{st}) = 0.1$		
Subcase 5.4:	$t \equiv 3 \pmod{6}$		
	$\sigma(c) = 0.2$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 2 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 1 \pmod{3}$	$1 \leq j \leq t$
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq st$
Subcase 5.5:	$t \equiv 4 \pmod{6}$		
	$\sigma(c) = 0.1$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq j \leq t-4$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq j \leq t-4$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq j \leq t-4$
	$\sigma(d_{t-3}) = 0.2, \sigma(d_{t-2}) = 0.3, \sigma(d_{t-1}) = 0.3, \sigma(d_t) = 0.2$		
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq s(t-1)$
	$\sigma(c_{s(t-1)+1}) = 0.2$		
	$\sigma(c_{s(t-1)+i}) = 0.1$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$2 \leq i \leq s$
	$\sigma(c_{s(t-1)+i}) = 0.2$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$2 \leq i \leq s$
	$\sigma(c_{s(t-1)+i}) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$2 \leq i \leq s$
Subcase 5.6:	$t \equiv 5 \pmod{6}$		
	$\sigma(c) = 0.3$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 1,2 \pmod{6}$	$1 \leq j \leq t-5$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq j \leq t-5$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq j \leq t-5$
	$\sigma(d_{t-4}) = 0.1, \sigma(d_{t-3}) = 0.2, \sigma(d_{t-2}) = 0.1, \sigma(d_{t-1}) = 0.2, \sigma(d_t) = 0.1$		
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 2,3 \pmod{6}$	$1 \leq i \leq st-1$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 0,5 \pmod{6}$	$1 \leq i \leq st-1$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 1,4 \pmod{6}$	$1 \leq i \leq st-1$
	$\sigma(c_{st}) = 0.3$		
Case 6:	$s \equiv 5 \pmod{6}$		
Subcase 6.1:	$t \equiv 0 \pmod{6}$		
	$\sigma(c) = 0.3$		
	$\sigma(d_j) = 0.1$	<i>if</i> $i \equiv 0,4 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.2$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(d_j) = 0.3$	<i>if</i> $i \equiv 1,3 \pmod{6}$	$1 \leq j \leq t$
	$\sigma(c_i) = 0.1$	<i>if</i> $i \equiv 0,1 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.2$	<i>if</i> $i \equiv 3,4 \pmod{6}$	$1 \leq i \leq st$
	$\sigma(c_i) = 0.3$	<i>if</i> $i \equiv 2,5 \pmod{6}$	$1 \leq i \leq st$

Subcase 6.2: $t \equiv 1(mod 6)$

$$\begin{aligned} \sigma(c) &= 0.3 \\ \sigma(d_j) &= 0.1 && \text{if } i \equiv 0,4(mod 6) && 1 \leq j \leq t-1 \\ \sigma(d_j) &= 0.2 && \text{if } i \equiv 2,5(mod 6) && 1 \leq j \leq t-1 \\ \sigma(d_j) &= 0.3 && \text{if } i \equiv 1,3(mod 6) && 1 \leq j \leq t-1 \\ \sigma(d_t) &= 0.1 \\ \sigma(c_i) &= 0.1 && \text{if } i \equiv 0,1(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 && \text{if } i \equiv 3,4(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 && \text{if } i \equiv 2,5(mod 6) && 1 \leq i \leq st \end{aligned}$$

Subcase 6.3: $t \equiv 2(mod 6)$

$$\begin{aligned} \sigma(c) &= 0.3 \\ \sigma(d_j) &= 0.1 && \text{if } i \equiv 0,4(mod 6) && 1 \leq j \leq t-1 \\ \sigma(d_j) &= 0.2 && \text{if } i \equiv 2,5(mod 6) && 1 \leq j \leq t-1 \\ \sigma(d_j) &= 0.3 && \text{if } i \equiv 1,3(mod 6) && 1 \leq j \leq t-1 \\ \sigma(d_t) &= 0.1 \\ \sigma(c_i) &= 0.1 && \text{if } i \equiv 0,1(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 && \text{if } i \equiv 3,4(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 && \text{if } i \equiv 2,5(mod 6) && 1 \leq i \leq st \end{aligned}$$

Subcase 6.4: $t \equiv 3(mod 6)$

$$\begin{aligned} \sigma(c) &= 0.3 \\ \sigma(d_j) &= 0.2 && \text{if } i \equiv 0,2(mod 3) && 1 \leq j \leq t-1 \\ \sigma(d_j) &= 0.3 && \text{if } i \equiv 1(mod 3) && 1 \leq j \leq t-1 \\ \sigma(c_i) &= 0.1 && \text{if } i \equiv 1,2(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 && \text{if } i \equiv 4,5(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 && \text{if } i \equiv 0,3(mod 6) && 1 \leq i \leq st \end{aligned}$$

Subcase 6.5: $t \equiv 4(mod 6)$

$$\begin{aligned} \sigma(c) &= 0.3 \\ \sigma(d_j) &= 0.2 && \text{if } i \equiv 0,2(mod 3) && 1 \leq j \leq t-1 \\ \sigma(d_j) &= 0.3 && \text{if } i \equiv 1(mod 3) && 1 \leq j \leq t-1 \\ \sigma(c_i) &= 0.1 && \text{if } i \equiv 1,2(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 && \text{if } i \equiv 4,5(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 && \text{if } i \equiv 0,3(mod 6) && 1 \leq i \leq st \end{aligned}$$

Subcase 6.6: $t \equiv 5(mod 6)$

$$\begin{aligned} \sigma(c) &= 0.1 \\ \sigma(d_j) &= 0.1 && \text{if } i \equiv 0,4(mod 6) && 1 \leq j \leq t \\ \sigma(d_j) &= 0.2 && \text{if } i \equiv 2,5(mod 6) && 1 \leq j \leq t \\ \sigma(d_j) &= 0.3 && \text{if } i \equiv 1,3(mod 6) && 1 \leq j \leq t \\ \sigma(c_i) &= 0.1 && \text{if } i \equiv 0,1(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.2 && \text{if } i \equiv 3,4(mod 6) && 1 \leq i \leq st \\ \sigma(c_i) &= 0.3 && \text{if } i \equiv 2,5(mod 6) && 1 \leq i \leq st \end{aligned}$$

Table.3. $v_\sigma(i)$ and $e_\mu(i)$ for the graph $S^2(J_{s,t})$, $i \in \{\frac{r}{10}, r \in \mathbb{Z}_4 - \{0\}\}$.

Nature of s and t	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$s \equiv 0(mod 6)$ $t \equiv 0,3(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 0(mod 6)$ $t \equiv 1,4(mod 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$s \equiv 0(mod 6)$ $t \equiv 2,5(mod 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$
$s \equiv 1(mod 6)$ $t \equiv 0,3(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$

$s \equiv 1(mod 6)$ $t \equiv 1,4(mod 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 1(mod 6)$ $t \equiv 2,5(mod 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2(mod 6)$ $t \equiv 0(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2(mod 6)$ $t \equiv 1(mod 6)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$	$\frac{q-1}{3}$
$s \equiv 2(mod 6)$ $t \equiv 2(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$s \equiv 2(mod 6)$ $t \equiv 3(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2(mod 6)$ $t \equiv 4(mod 6)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$s \equiv 2(mod 6)$ $t \equiv 5(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$
$s \equiv 3(mod 6)$ $t \equiv 0,3(mod 6)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 3(mod 6)$ $t \equiv 1,4(mod 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$
$s \equiv 3(mod 6)$ $t \equiv 2,5(mod 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$
$s \equiv 4(mod 6)$ $t \equiv 0(mod 6)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 4(mod 6)$ $t \equiv 1,4(mod 6)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 4(mod 6)$ $t \equiv 2,5(mod 6)$	$\frac{p+1}{3}$	$\frac{p+1}{3} - 1$	$\frac{p+1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 5(mod 6)$ $t \equiv 0,3(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 5(mod 6)$ $t \equiv 1(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$	$\frac{q-1}{3}$
$s \equiv 5(mod 6)$ $t \equiv 2(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$
$s \equiv 5(mod 6)$ $t \equiv 4(mod 6)$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{p-1}{3} + 1$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$s \equiv 5(mod 6)$ $t \equiv 5(mod 6)$	$\frac{p-1}{3} + 1$	$\frac{p-1}{3}$	$\frac{p-1}{3}$	$\frac{q+1}{3} - 1$	$\frac{q+1}{3}$	$\frac{q+1}{3}$

From the table 3, we find that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i, j \in \{\frac{r}{10}, r \in \mathbb{Z}_4 - \{0\}\}$ which satisfies the condition of fuzzy quotient - 3 cordial labeling. Hence we concluded that the graph $S^2(\mathcal{J}_{4,6})$ is fuzzy quotient - 3 cordial.

Theorem 3.4. The generalised subdivision Jahangir graph $S(\mathcal{J}_{s,t})$ is fuzzy quotient-3 cordial graph, for $s \geq 2$ and $t \geq 2$.

Proof: Let G be the $S(\mathcal{J}_{s,t})$ graph. $V(S(\mathcal{J}_{s,t})) = \{c_i: 1 \leq i \leq st\} \cup \{c\} \cup \{d_i: 1 \leq i \leq t\}$ and $E(S(\mathcal{J}_{s,t})) = \{c_i c_{i+1}: 1 \leq i \leq st - 1\} \cup \{c_1 c_{st}\} \cup \{c d_j: 1 \leq j \leq t\} \cup \{d_j c_{s(j-1)+1}: 1 \leq j \leq t\}$. $p = st + t + 1$ and $q = st + 2t$. To define $\sigma: V(S(\mathcal{J}_{s,t})) \rightarrow [0,1]$ the following cases are to be considered.

If $k \equiv 0(\text{mod } 3), 1 \leq k \leq t$

$\sigma(c_{s(k-1)+i}) = 0.1$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.2$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.3$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(e_{s(k-1)+i}) = 0.1$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(e_{s(k-1)+i}) = 0.2$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(e_{s(k-1)+i}) = 0.3$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq s$

Subcase 2.3: $t \equiv 2(\text{mod } 6)$

$\sigma(c) = 0.3$		
$\sigma(d_j) = 0.1$	if $i \equiv 0,1(\text{mod } 6)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	if $i \equiv 2,3(\text{mod } 6)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	if $i \equiv 4,5(\text{mod } 6)$	$1 \leq j \leq t$
$\sigma(c_{s(k-1)+i}) = 0.1$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.2$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(c_{s(k-1)+i}) = 0.3$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(e_{s(k-1)+i}) = 0.1$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(e_{s(k-1)+i}) = 0.2$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(e_{s(k-1)+i}) = 0.3$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s$

Subcase 2.4: $t \equiv 3(\text{mod } 6)$

$\sigma(c) = 0.2$		
$\sigma(d_j) = 0.1$	if $i \equiv 2(\text{mod } 3)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	if $i \equiv 0(\text{mod } 3)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	if $i \equiv 1(\text{mod } 3)$	$1 \leq j \leq t$
$\sigma(c_i) = 0.1$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(c_i) = 0.2$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(c_i) = 0.3$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(e_i) = 0.1$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(e_i) = 0.2$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(e_i) = 0.3$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq st$

Subcase 2.5: $t \equiv 4(\text{mod } 6)$

$\sigma(c) = 0.2$		
$\sigma(d_j) = 0.1$	if $i \equiv 2(\text{mod } 3)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	if $i \equiv 0(\text{mod } 3)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	if $i \equiv 1(\text{mod } 3)$	$1 \leq j \leq t$
$\sigma(c_i) = 0.1$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(c_i) = 0.2$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(c_i) = 0.3$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(e_i) = 0.1$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(e_i) = 0.2$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq st$
$\sigma(e_i) = 0.3$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq st$

Subcase 2.6: $t \equiv 5(\text{mod } 6)$

$\sigma(c) = 0.1$		
$\sigma(d_j) = 0.1$	if $i \equiv 2,4(\text{mod } 6)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.2$	if $i \equiv 0,1(\text{mod } 6)$	$1 \leq j \leq t$
$\sigma(d_j) = 0.3$	if $i \equiv 3,5(\text{mod } 6)$	$1 \leq j \leq t$
$\sigma(c_i) = 0.1$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq s(t-1)$
$\sigma(c_i) = 0.2$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s(t-1)$
$\sigma(c_i) = 0.3$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s(t-1)$
$\sigma(e_i) = 0.1$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s(t-1)$
$\sigma(e_i) = 0.2$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s(t-1)$
$\sigma(e_i) = 0.3$	if $i \equiv 0(\text{mod } 3)$	$1 \leq i \leq s(t-1)$
$\sigma(c_{s(t-1)+i}) = 0.1$	if $i \equiv 2(\text{mod } 3)$	$1 \leq i \leq s$
$\sigma(c_{s(t-1)+i}) = 0.2$	if $i \equiv 1(\text{mod } 3)$	$1 \leq i \leq s$

$$\begin{array}{lll} \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq i \leq s \\ \sigma(e_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq i \leq s \\ \sigma(e_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq i \leq s \\ \sigma(e_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq i \leq s \end{array}$$

Case 3: $s \equiv 2,5(\text{mod } 6)$

Subcase 3.1: $t \equiv 0(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(e_i) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(e_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq st \\ \sigma(e_i) = 0.3 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq st \end{array}$$

Subcase 3.2: $t \equiv 1(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.3 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq j \leq t \\ \sigma(c_i) = 0.1 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(e_i) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(e_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(e_i) = 0.3 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(e_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(e_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s \\ \sigma(e_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s \end{array}$$

Subcase 3.3: $t \equiv 2(\text{mod } 6)$

$$\begin{array}{lll} \sigma(c) = 0.1 & & \\ \sigma(d_j) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq j \leq t-2 \\ \sigma(d_j) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq j \leq t-2 \\ \sigma(d_j) = 0.3 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq j \leq t-2 \end{array}$$

$\sigma(d_{t-1}) = 0.2, \sigma(d_t) = 0.3$

$$\begin{array}{lll} \sigma(c_i) = 0.1 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_i) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(e_i) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(e_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(e_i) = 0.3 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s(t-1) \\ \sigma(c_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s-1 \\ \sigma(c_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s-1 \\ \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s-1 \\ \sigma(e_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s-1 \\ \sigma(e_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s-1 \\ \sigma(e_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s-1 \\ \sigma(e_{st}) = 0.1 & & \end{array}$$

Subcase 3.4: $t \equiv 3(\text{mod } 6)$

$$\sigma(c) = 0.2$$

$$\begin{array}{lll}
 \sigma(d_j) = 0.1 & \text{if } i \equiv 2(\text{mod } 3) & 1 \leq j \leq t \\
 \sigma(d_j) = 0.2 & \text{if } i \equiv 0(\text{mod } 3) & 1 \leq j \leq t \\
 \sigma(d_j) = 0.3 & \text{if } i \equiv 1(\text{mod } 3) & 1 \leq j \leq t \\
 \sigma(c_i) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq st \\
 \sigma(c_i) = 0.2 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq st \\
 \sigma(c_i) = 0.3 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq st \\
 \sigma(e_i) = 0.1 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq st \\
 \sigma(e_i) = 0.2 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq st \\
 \sigma(e_i) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq st
 \end{array}$$

Subcase 3.5: $t \equiv 4(\text{mod } 6)$

$$\begin{array}{lll}
 \sigma(c) = 0.1 & & \\
 \sigma(d_j) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq j \leq t - 4 \\
 \sigma(d_j) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq j \leq t - 4 \\
 \sigma(d_j) = 0.3 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq j \leq t - 4
 \end{array}$$

$\sigma(d_{t-3}) = 0.2, \sigma(d_{t-2}) = 0.3, \sigma(d_{t-1}) = 0.3, \sigma(d_t) = 0.2$

$$\begin{array}{lll}
 \sigma(c_i) = 0.1 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\
 \sigma(c_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\
 \sigma(c_i) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\
 \sigma(e_i) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\
 \sigma(e_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq s(t - 1) \\
 \sigma(e_i) = 0.3 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq s(t - 1)
 \end{array}$$

$\sigma(c_{s(t-1)+1}) = 0.2$

$$\begin{array}{lll}
 \sigma(c_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 2(\text{mod } 6) & 2 \leq i \leq s \\
 \sigma(c_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 2 \leq i \leq s \\
 \sigma(c_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 2 \leq i \leq s \\
 \sigma(e_{s(t-1)+i}) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 2 \leq i \leq s \\
 \sigma(e_{s(t-1)+i}) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 2 \leq i \leq s \\
 \sigma(e_{s(t-1)+i}) = 0.3 & \text{if } i \equiv 0(\text{mod } 6) & 2 \leq i \leq s
 \end{array}$$

Subcase 3.6: $t \equiv 5(\text{mod } 6)$

$$\begin{array}{lll}
 \sigma(c) = 0.3 & & \\
 \sigma(d_j) = 0.1 & \text{if } i \equiv 1,2(\text{mod } 6) & 1 \leq j \leq t - 5 \\
 \sigma(d_j) = 0.2 & \text{if } i \equiv 0,5(\text{mod } 6) & 1 \leq j \leq t - 5 \\
 \sigma(d_j) = 0.3 & \text{if } i \equiv 3,4(\text{mod } 6) & 1 \leq j \leq t - 5
 \end{array}$$

$\sigma(d_{t-4}) = 0.1, \sigma(d_{t-3}) = 0.2, \sigma(d_{t-2}) = 0.1, \sigma(d_{t-1}) = 0.2, \sigma(d_t) = 0.1$

$$\begin{array}{lll}
 \sigma(c_i) = 0.1 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq st - 1 \\
 \sigma(c_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq st - 1 \\
 \sigma(c_i) = 0.3 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq st - 1 \\
 \sigma(e_i) = 0.1 & \text{if } i \equiv 1(\text{mod } 6) & 1 \leq i \leq st - 1 \\
 \sigma(e_i) = 0.2 & \text{if } i \equiv 0(\text{mod } 6) & 1 \leq i \leq st - 1 \\
 \sigma(e_i) = 0.3 & \text{if } i \equiv 2(\text{mod } 6) & 1 \leq i \leq st - 1 \\
 \sigma(e_{st}) = 0.3 & &
 \end{array}$$

Table.4. $v_\sigma(i)$ and $e_\mu(i)$ for the graph $S(\mathcal{J}_{s,t})$, $i \in \{\frac{r}{10}, r \in \mathbb{Z}_4 - \{0\}\}$.

Nature of s and t	$v_\sigma(0.1)$	$v_\sigma(0.2)$	$v_\sigma(0.3)$	$e_\mu(0.1)$	$e_\mu(0.2)$	$e_\mu(0.3)$
$s \equiv 0,3(\text{mod } 6)$ $t \equiv 0(\text{mod } 3)$	$\frac{p}{3} + 1$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 0,3(\text{mod } 6)$ $t \equiv 1(\text{mod } 3)$	$\frac{p}{3} + 1$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$
$s \equiv 0,3(\text{mod } 6)$ $t \equiv 2(\text{mod } 3)$	$\frac{p}{3} + 1$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 2,5(\text{mod } 6)$ $t \equiv 0(\text{mod } 3)$	$\frac{p}{3} + 1$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$

$s \equiv 2,5(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 2,5(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p+2}{3}$	$\frac{p+2}{3} - 1$	$\frac{p+2}{3}$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 1,4(mod 6)$ $t \equiv 0(mod 3)$	$\frac{p}{3}$	$\frac{p}{3}$	$\frac{p}{3} + 1$	$\frac{q}{3}$	$\frac{q}{3}$	$\frac{q}{3}$
$s \equiv 1,4(mod 6)$ $t \equiv 1(mod 3)$	$\frac{p+2}{3}$	$\frac{p+2}{3} - 1$	$\frac{p+2}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3}$	$\frac{q+1}{3} - 1$
$s \equiv 1,4(mod 6)$ $t \equiv 2(mod 3)$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{p+1}{3}$	$\frac{q-1}{3}$	$\frac{q-1}{3} + 1$	$\frac{q-1}{3}$

From the table 4, we find that $|v_\sigma(i) - v_\sigma(j)| \leq 1$ and $|e_\mu(i) - e_\mu(j)| \leq 1$ for $i, j \in \{\frac{r}{10}, r \in Z_4 - \{0\}\}$ which satisfies the condition of fuzzy quotient - 3 cordial labeling. Hence we concluded that the graph $S(\mathcal{J}_{s,t})$ is fuzzy quotient - 3 cordial.

4. Conclusion

In this work we have discussed and established the existence of fuzzy quotient-3 cordial labeling on the generalized Jahangir graph $\mathcal{J}_{s,t}$ and its subdivision graphs $S^1(\mathcal{J}_{s,t})$, $S^2(\mathcal{J}_{s,t})$ and $S(\mathcal{J}_{s,t})$ for $s \geq 2$ and $t \geq 2$. Investigating the existence of fuzzy quotient-3 cordial labeling concept on other families of graphs and finding the application of this labeling will be our future.

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