AN INNOVATIVE APPROACH OF INTUIONISTIC FUZZY NUMBERS IN CALCULATION OF STRESS AND DEFLECTION OF A COMPOSITE BAR

J.Akila Padmasree
Department of Mathematics, Vellalar college for women, Erode.
E-mail: akilaa6666@gmail.com
R.Parvathi
Department of Mathematics, Vellalar college for women, Erode.
paarvathis@rediffmail.com

Abstract: In the analysis and design of structures, an important aspect is the deformation caused by the loads applied at them. Analyzing the deformations helps in calculating the stress in the structures. Usually the distribution of stress cannot be clearly determined along the structure as it is uncertain. This paper proposes an innovative method to find the stress and deflection of a composite bar using Horizontal relative trapezoidal intuitionistic fuzzy number (HRTrIFN) as Young's modulus values. The bending point at any location along the beam can be calculated since the stress and deflection values are found as range of values, the actual bending point, peak and the breaking point of the beam can be calculated efficiently which is far more better than the traditional method where it is a single crisp value. This helps the analyst to determine very accurately the strength of the structure.
Keywords: Horizontal relative trapezoidal intuitionistic fuzzy number (HRTrIFN), bending point of the beam

1Introduction

A classical or crisp set is a collection of objects or elements out of some universal set X characterised by some well-defined common property. If an element shows this property, it belongs to the set, if the set is A then it is written as $x \in A$, if it is excluded then $x \notin A$. But classical set theory reaches its limit when there is no clear distinction of membership or exclusion is possible and there comes into existence the fuzzy sets. Fuzzy sets allow elements to belong to the set to a certain degree while the intuitionistic fuzzy sets, the extension of fuzzy sets defines not only the belonging of an element but also the non-belongingness of an element to a specific set. Fuzzy set theory was developed by Zadeh and intuitionistic fuzzy sets by Atanassov describing a membership μ, non-membership ν and hesitancy degree π such that the sum of membership and non-membership degree must not exceed one. It plays an important role in decision making under an uncertain environment. For example, if an employer is suggesting an idea to his head, the head can accept the idea (membership μ) or he can flatly reject the idea (non-membership ν) or he can just put the idea on hold (undeterministicπ). Among the various types of intuitionistic fuzzy sets, the sets defined on the set of all real numbers R are of particular importance. They are called intuitionistic fuzzy numbers with certain conditions.

In real life computations, uncertainity is natural because the processed values are from human measurements which cannot be really accurate. Interval analysis defined by Moore in

(Moore R, E 1996) produces a closed interval inside which the result lies. Since intuitionistic fuzzy numbers is in the interval form, it will be a very clear mathematical model of real-life problems. Of all the intuitionistic fuzzy numbers, trapezoidal intuitionistic fuzzy number has wide applications in mathematical modelling. For example, in **(Jayagowri.P, et al.,2014),** an algorithm is developed to find intuitionistic fuzzy optimized path and optimized distance for a network with its arc length as trapezoidal intuitionistic fuzzy number. In real life networks, intuitionistic fuzzy optimized path length and distance are vital information for decision makers in the field of logistics. In (**Parvathi.R et al., 2018**), complex trapezoidal intuitionistic fuzzy numbers were introduced. In **(Moore.R.E 1996),** the interval arithmetic was introduced which involves only the extremities of intervals in calculations. The multi-dimensional relative distance measure interval arithmetic (RDM - IA) was developed in **(Piegat.A., et al., 2017c)** which includes a multiplier $\alpha \in [0,1]$ that converts the membership and non-membership function from linear/curvilinear into planar form. The horizontal membership functions were introduced by Prof. A.Piegat in **(Piegat.A., et al., 2015)** which defines a fuzzy number not in the form of $\mu = f(x)$ but as $x = f(\mu)$. This form enables using fuzzy values in mathematical formulas of type $y = f(x_1,.......,x_n)$ together with crisp values. Constructing horizontal membership functions requires using multi-dimensional relative distance measure interval arithmetic. Horizontal relative trapezoidal intuitionistic fuzzy number (HRTrIFN) is defined and its advantage over other intuitionistic fuzzy numbers is that it enables relatively easy aggregation of crisp and intuitionistic fuzzy values together in arithmetic operations.

In this paper, the stress and deflection of a composite bar using HRTrIFN as Young's modulus values are calculated. In reality, the initiation of damage cannot be defined exactly. The young's modulus assumptions are based on the interpretation of material data which accounts for some amount of uncertainity and expressing them as a constant value may not produce accurate initial value of deflection. So, representing the young's modulus as intuitionistic fuzzy number takes into consideration of all the uncertainties and provides a better result for deflections in real life applications. Usage of horizontal membership function provides a multi-dimensional solution with a single expression and also rules out the usage of Zadeh's extension principle which is time consuming. The traditional method provides a single crisp value for deflections and stress with a single formula but usage of horizontal functions provides a multidimensional solution with a single expression. With the varying values of μ and α, a multidimensional accurate result may be found whereas in traditional method numerical calculations provides only a single crisp result. Since the stress and deflection values are found as range of values , the actual bending point, peak and the breaking point of the beam can be calculated efficiently. Simplicity of calculation and multi-dimensional solution is an added advantage in using the horizontal functions. It also provides a better and more accurate result compared to traditional method.

2 Prerequisites

Definition 2.1. (Shaw.A.K. et al.,2013) An *intuitionistic fuzzy number (IFN) A* is defined as follows:

- It is an intuitionistic fuzzy subset of the real line
- It is normal, that is, there exists a $x \in R$ such that $\mu_A(x) = 1$, (So, $\nu_A(x) = \pi_{A(x)} = 0$)

- It is a convex set for the membership function $\mu_A(x)$
  That is, $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_A(x_1),\mu_A(x_2))$ for all $x_1,x_2 \in R$ and $\lambda \in [0,1]$
- It is a concave set for the non-membership function $\nu_A(x)$
  That is, $\nu_A(\lambda x_1 + (1-\lambda)x_2) \geq max(\nu_A(x_1),\nu_A(x_2))$ for all $x_1,x_2 \in R$ and $\lambda \in [0,1]$

Definition 2.2. . (Shaw.A.K. et al.,2013) A *Trapezoidal intuitionistic fuzzy number* (TrIFN) A is an intuitionistic fuzzy set in R with the membership and non-membership functions as given below:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & x \in [a,b] \\ 1 & x \in [b,c] \\ \frac{d-x}{d-c} & x \in [c,d] \\ 0 & \text{otherwise} \end{cases} \qquad \nu_A(x) = \begin{cases} \frac{b-x}{b-a'} & x \in [a',b] \\ 0 & x \in [b,c] \\ \frac{x-c}{d'-c} & x \in [c,d'] \\ 1 & \text{otherwise} \end{cases}$$

where $b \leq c$ and $(b-a) \geq 0$ which gives $b \geq a$. $(d-c) \geq 0$ which gives $d \geq c$. Therefore, $a \leq b \leq c \leq d$. Also, $(b-a) \leq b - a'$. Therefore, $a' \leq a$ and similarly, $d \leq d'$. Hence, $a' \leq a \leq b \leq c \leq d \leq d'$. Thus$A_{TrIFN}= [a,b,c,d; a',b,c,d']$.

Definition 2.3: A *horizontal relative trapezoidal intuitionistic fuzzy number* (HRTrIFN) H of A, is defined as

$$H = \{\langle x, x(\mu,\alpha_x), x(\nu,\beta_x)\rangle : x \in X, \mu, \nu, \alpha_x, \beta_x \in [0,1]\}$$

where $x(\mu,\alpha_x)$ denotes the horizontal relative membership function of H given by $x(\mu,\alpha_x) = [a + (b-a)\mu] + [(d-a) - (d-c+b-a)\mu]\alpha_x$, $\mu,\alpha_x \in [0,1], x \in X$ and $x(\nu,\beta_x)$ denotes the horizontal relative non-membership function of H given by $x(\nu,\beta_x) = [a' + (b-a')\nu] + [(d' - a') - (d' - c + b - a')\nu]\beta_x$, $\nu,\beta_x \in [0,1]$,$x \in X$ and α_x and β_x are the relative distance measure of membership and non-membership values of H respectively.

For example, consider the trapezoidal intuitionistic fuzzy number$\langle[3,4,5,6; 2,4,5,7]\rangle$. The horizontal relative membership function $x(\mu,\alpha_x) = [3+\mu]+[3-2\mu]\alpha_x$ and horizontal relative non-membership function $x(\nu,\beta_x) = [2 + 2\nu] + [5 - 4\nu]\beta_x$are represented in the Figure 1.

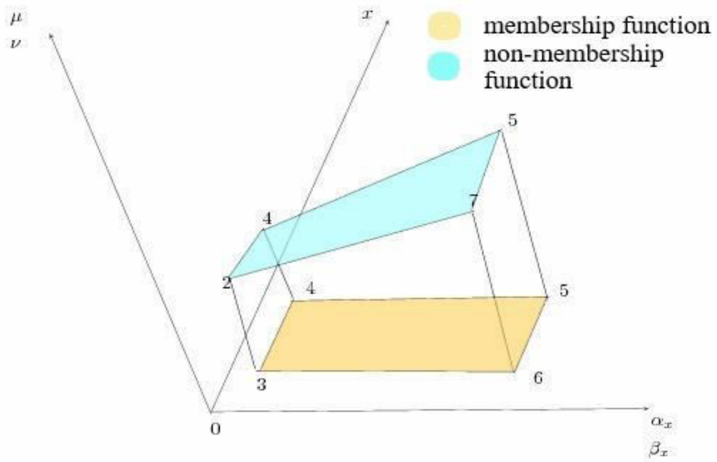

Figure 1: Horizontal membership and non-membership function

3 An application in structural engineering

3.1 Uncertainity in Young's modulus

Uncertainity has come a long way from being ignored to the idea that it should be actively controlled. It is the result of some information deficiency. The various information deficiencies associated with different types of uncertainity can be measured and processed by different theories. Traditional approach assumes that all data are known with mathematical precision. But, in practice the problems are imprecise in nature because of the inherent fuzziness in them. So, the mathematical models are represented with some levels of imprecision. There are two major categories of uncertainity: unintentional uncertainity and intentional uncertainity. The former arises due to the partial or complete lack of information while the latter is due to the consequence of simplification. Unintentional uncertainities include scatter or variability of the model parameters such as material properties or geometry parameters arising due to irregularities with material or defects of fabrication**. (Michael Hanns).** There are several ways to represent this imprecise parameters in engineering calculations and the simplest approach is to assign single value for each parameter and obtain a single valued output and repeat the process in order to arrive at a conclusion called the deterministic method, clearly ignoring all possible uncertainities. The method is very simple but time consuming and expensive. Historically, the civil engineering community introduced probability as a framework for quantifying uncertainity. However the uneasiness in its usage has found entrance of interval arithmetic, fuzzy sets, random sets. Using probability theory, requires a vast knowledge of probability distribution, its types and measures which is a clear problem. Interval analysis represents the uncertain parameters as [a,b] with signifying bounds but no detailed information on its distribution over the interval. Fuzzy sets use fuzzy numbers to model the uncertainty and its output is interpreted as the degree of possibility that the parameter takes the value but it doesn't speak of non-belongingness and neutral case. Modelling of uncertainty as intuitionistic fuzzy numbers produces the output as range of values with information over the distribution of uncertainty in the interval representing the degree of possibility that the parameter takes the value, degree of non-possibility that the parameter takes the value. And the usage of horizontal intuitionistic fuzzy numbers approaches the problem with crisp information and intuitionistic fuzzy parametric values to arrive at a range of output value. Irrespective of the vagueness of input data and uncertainties with respect to the model, the engineer comes up with a crisp value, but the intuitionistic fuzzy set theory provides a insight to reflect the lack of information and uncertainties in a more clear way. For example, consider the problem of composite massless rod under a tensile load with l^1 and l^2 as length parameters, A^1 *and* A^2 as cross-sectional areas and Young's moduli E^1 *and* E^2. The rod is clamped at one end and subjected to a tensile force F at another end. To determine the displacement $u(x)$ at any position x within the rod using finite element method, the rod is discretized into two elements as illustrated in the figure. The first component is assumed to be steel and the second is aluminium with material and geometry parameters as $A^1 = 1^0 0mm^2$, $A^2 = 75mm^2$, $l^1 = 500mm^2$, $l^2 = 500mm^2$, external load $F = 1000N$, $E^1 = 2 \times 10^5 N/mm^2$, $E^2 = 6.9 \times 10^4 N/mm^2$. The problem isfirst solved using the crisp values by the finite element method to find the displacements as u^2 and u^3. The finite element equations are derived from the principle of virtual work. The virtual work equation is given by

$E^{(i)}A^{(i)}\int_{l(i)}\epsilon^{(i)}\delta\epsilon^{(i)}dx = F_i^{(i)}\delta u_i^{(i)} + F_j^{(i)}\delta u_j^{(i)}$ where the subscript indicates the number of the node and the

superscript denotes the number of the element. Employing fundamental lemma of variational principles, the linear system of equations $\frac{E^{(i)}A^{(i)}}{l^{(i)}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}u_i^{(i)}\\ u_j^{(i)}\end{bmatrix} = \begin{bmatrix}F_i\\ F_j\end{bmatrix}$ is obtained. Furthermore, the continuity of displacements at the node 2 requires $u_2^{(1)} = u_2^{(1)}$ and applying boundary conditions $u_1 = 0$ and $F_3 = F$, the system finally reduces to the following expressions for $u_2 = \frac{F}{c^{(1)}}$ and $u_3 = F\left(\frac{1}{c^{(1)}} + \frac{1}{c^{(2)}}\right)$. The displacement at node 2 and node 3 is calculated as $u^2 = 0.025$ and $u^3 = 0.1216$. The same problem when solved for fuzzy valued material parameters where the young's moduli E^1 and E^2 are considered as trapezoidal fuzzy numbers $E^1 = <1.8,1.9,2.0,2.1>\times 10^5 N/mm^2$ and $E^2 = <6.555,6.9,7.245,7.3>\times 10^4 N/mm^2$. After applying horizontal membership functions for E^1 and E^2, the expressions for the displacements u^2 and u^2 are obtained as $u^2(\mu) = \frac{1}{36+2\mu+20\alpha_1(0.3-0.2\mu)}$ and $u^3(\mu) = \frac{1}{36+2\mu+20\alpha_1(0.3-0.2\mu)} + \frac{1}{96+1.5\mu+15\alpha_2(0.8-0.4\mu)}$. Here, when giving values for μ and α, the displacement at various points throughout the rod can be found as a range of trapezoidal fuzzy numbers whereas in case of crisp solution displacement was a single crisp value. Using horizontal membership functions facilitates easy calculation since it is a mix of both fuzzy and crisp values in a single expression and delivers a multi-dimensional solution. When $\mu = 1$ and $\alpha = 1$, the displacements values are $u^2(\mu) = 0.025$ and $u^3(\mu) = 0.12$. When the material parameters are chosen as intuitionistic fuzzy numbers, the Young's moduli is written as trapezoidal intuitionistic fuzzy numbers as $E^1 = \langle[1.8,1.9,2.0,2.1; 1.7,1.9,2,2.2]\rangle\times 10^5 N/mm^2$ and $E^2 = \langle[6.555,6.9,7.245,7.3; 6.4,6.9,7.245,7.4]\rangle \times 10^4 N/mm^2$ .The displacements of membership values for the nodes are same as the fuzzy problem. Applying horizontal non-membership functions for $E^1(\nu)$ and $E^2(\nu)$, the expressions for the displacements $u^2(\nu)$ and $u^3(\nu)$ are obtained as $u^2(\nu) = \frac{1}{34+4\nu+20\beta_1(0.5-0.4\mu)}$ and $u^3(\mu) = \frac{1}{34+4\nu+20\beta_1(0.5-0.4\nu)} + \frac{1}{94.5+3\nu+15\beta_2(1.3-0.6\nu)}$. Giving values for ν and β, the displacement at various points throughout the rod can be found as a range of trapezoidal intuitionistic fuzzy numbers. When $\nu = 0$ and $\beta = 0$, $u^2 = 0.0294$ and $u^3 = 0.03998$ which shows that the displacement values of the rod will not go beyond these values.

So, the usage of trapezoidal intuitionistic fuzzy numbers and the horizontal membership and non-membership functions enables to aggregate the crisp and uncertain values to provide a better output. In case of steel, the tensile tests have consistently shown that the modulus of elasticity varies with grade and thickness of steel and addition of impurities may increase or decrease the elasticity of steel. Tensile tests of steel of different grades and thickness shows different elasticity values. So, assuming 200*Gpa* for all grades of steel may be inaccurate **(Mahen Mahendran (1996))**. Hence assuming a range of possible values for the young's modulus value reduces the inaccuracy and provides a better result in real life applications.

3.2 Application of HRTrIFN

Consider the composite bar, 2 vertical bars in parallel. *S* is the steel bar considered as bar 1 and *C*, the copper bar as bar 2. Each bar has different properties *A*,*E* and *L* as shown. The horizontal bar is rigid and remains horizontal when load is applied. Determine the force, stress, deflection in each bar. Assume that the axial force in bars 1 and 2 are P_1 and P_2 (tension) respectively. Equilibrium of the free body in terms of forces requires that $P = P_1 +$

P_2. Elongation of each bar is δ_1(elongation of bar 1) = $\frac{P_1L_1}{E_1A_1}$ and δ_2(elongation of bar 2) =$\frac{P_2L_2}{E_2A_2}$. Assume that the horizontal bar moves down a distance δ, when the load P is applied. Since the horizontal bar is required to remain horizontal, the elongation of each of the vertical bars must be equal. So, the displacement boundary condition is $\delta_1 = \delta_2 = \delta$ = (movement of horizontal bar)= $\frac{P_1L_1}{E_1A_1} = \frac{P_2L_2}{E_2A_2}$ leads to

$$\begin{bmatrix} 1 & 1 \\ \frac{L_1}{A_1E_1} & \frac{-L_2}{A_2E_2} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix}$$

Solving for $P_1$and P_2 , from above gives,

$P_1 = \frac{P}{1+\frac{A_2E_2L_1}{A_1E_1L_2}}$ $P_2 = \frac{P}{1+\frac{A_1E_1L_2}{A_2E_2L_1}}$

Stress in each bar is $\sigma_1 = \frac{P_1}{A_1} = \frac{\frac{P}{A_1}}{1+\frac{A_2E_2L_1}{A_1E_1L_2}}$ $\sigma_2 = \frac{P_2}{A_2} = \frac{\frac{P}{A_2}}{1+\frac{A_1E_1L_2}{A_2E_2L_1}}$

and the deflection of each bar is equal$\delta_1 = \frac{PL_1}{A_1E_1} = \delta_2 = \frac{PL_1L_2}{A_1E_1L_2+A_2E_2L_1}$.

Numerical example:

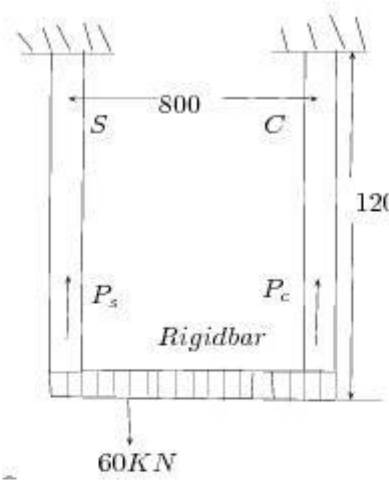

Figure 2: Composite bar

A composite bar consisting of two vertical bars of one steel and the other copper are 12*mm* long and 20*mm* × 20*mm* size of area of cross section are considered as shown in the Figure 2 . They are rigidly fixed at 800*mm* apart and are connected by a rigid horizontal bar at their bottom. A vertical load of 60*KN* is positioned on the rigid bar and it has to be horizontal when carrying the load. Determine the stresses in the bars and their elongations(deflections) if $E_s = 2 \times 10^5 N/mm^2$ and $E_c = 1.17 \times 10^5 N/mm^2$.

Traditional method with crisp values:

Area of cross section of steel and copper bars, A_s and A_c = 400mm^2. Total load (P) = 60*KN*. Let the loads carried by the steel and copper bars be P_s and P_c respectively. If the rigid bar is to be horizontal when carrying the load the deformation of both the bars are to

be equal, that is,$\frac{P_sL_s}{A_sE_s} = \frac{P_cL_c}{A_cE_c}$. Length and area of the bars are equal, so $L_s = L_c$ = 1200*mm*, $A_s = A_c$ = 400mm^2 therefore,$P_s = \frac{E_sP_c}{E_c} = 1.795P_c$. But, P_s+P_c= 60. So, P_s = 38.533*KN* and P_c = 21.467*KN*. Stress

is force per unit area and is denoted by σ. The stress and deflection values are calculated as Stress in copper bar, σ_c= 53.668N/mm^2. Sress in steel bar , σ_s= 96.333N/mm^2. Elongation or deflection of the copper bar,δ_c= 0.55*mm* and Elongation or deflection of the steel bar, δ_s= 0.55*mm*.

Intuitionistic fuzzy values for the problem:

In relation to the modulus of elasticity, it is assumed to be 200 GPa for all steel grades. However the tensile tests have consistently shown that the modulus of elasticity varies with the grade of steel and thickness.It was found that it increases to values as high as 240 GPa for smaller thicknessess and higher grades of steel. The Table 1 (**Mahen Mahendran (1996)**). presents the report of determining whether the modulus of elasticity (E) is constsnt for all grades of thickness of steel using laboratory experiments of tensile steel specimens. It shows that the variation in E values are within the range 190 to 230 GPa.

Minimum Yield Stress (MPa)	Measured Yield Stress (MPa)	Nominal Thickness (mm)	Modulus of Elasticity E (GPa)			Reference
			bmt	tct	bmt or tct	
550	653	0.6	230	220		Bernard et al. (1992)
550	670	0.75			218	Mahendran (1995)
350	392	6.0			208	Sully and Hancock (1994)
-	324	2.5			206.7	Chen et al. (1994)
-	420	1.3			190	Davies et al. (1994)

Table 1: E values for steel

With the increasing use of thin steels in the building and construction industry, a good knowledge of the basic material properties namely, tensile strength, modulus of elasticity and ductility parameters is needed. The tensile tests of steels with different grade and thickness revealed that the current practice of assuming the modulus of elasticity value as $200GPa$ may be inaccurate. Also, due to the irregularities in the material or defects of fabrication the material and geometry parameters has some natural unintentional uncertainities which can be modelled as intuitionistic fuzzy parameters. There are several ways to represent this imprecise parameters, representing as horizontal intuitionistic fuzzy numbers models the problem with crisp information and intuitionistic fuzzy parametric values to arrive at a range of output value, irrespective of the vagueness of input data. So, in this problem Young's modulus value is considered as trapezoidal intuitionistic fuzzy number. Consider the elasticity parameters of the rod as trapezoidal intuitionistic fuzzy numbers. Let us considerE_s and E_c as

$E_s = <1.5,1.9,2,2.1;1.3,1.9,2,2.3> \times 10^5$ $E_c = <1.12,1.15,1.17,1.18;1.10,1.15,1.17,1.19> \times 10^5$

Membership and non-membership functions for E_s and E_c are shown in Figure 3 and Figure 4.

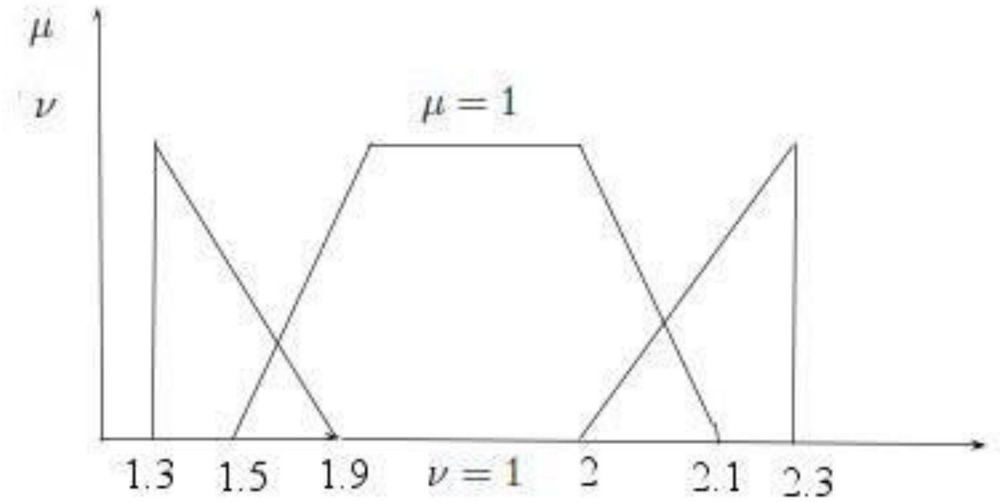

Figure 3: Membership and non-membership function of Es

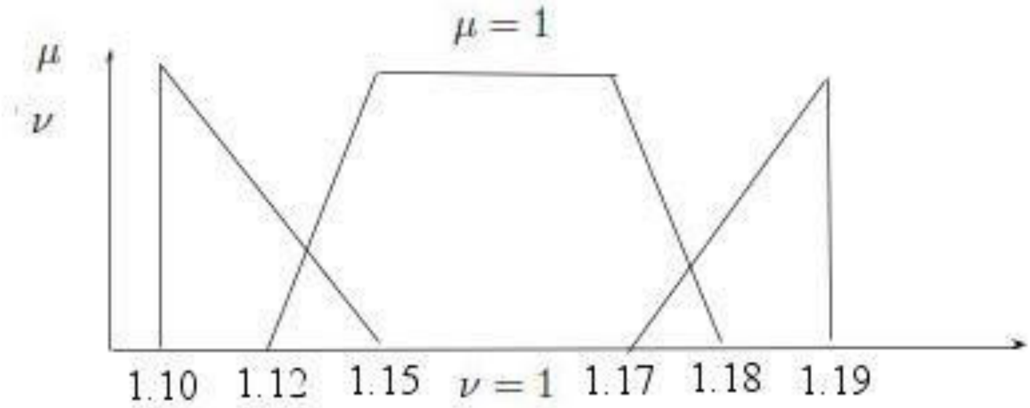

Figure 4: Membership and non-membership function of Ec

Now applying horizontal membership functions,we have $E_s = 1.5 + 0.4\mu + (0.6 - 0.5\mu)\alpha_1 \times 10^5 N/mm^2$ and $E_c = 1.12+0.03\mu+(0.06-0.04\mu)\alpha_2 \times 10^5 N/mm^2$. Calculating P_s and P_c as $P_s = \frac{60\times((1.5+0.4\mu+(0.6-0.5\mu))\alpha_1}{2.62+0.43\mu+(0.6-0.5\mu)\alpha_1+(0.06-0.04\mu}\alpha_2$

and $P_c = \frac{60\times(1.12+0.03\mu+(0.06-0.04\mu))\alpha_2}{2.62+0.43\mu+(0.6-0.5\mu}\alpha_1 + (0.06 - 0.4\mu)\alpha_2$.

The membership values for stress of steel bar is $\sigma_s(\mu) = \frac{3(1.5+o.4\mu+(0.6-0.5\mu)\alpha_1)}{20(2.62+0.43\mu+(0.6-0.5\mu)\alpha_1)+(0.06-0.04\mu)\alpha_2}$ and the non-membership values are $\sigma_s(\nu) = \frac{3(1.3+0.6\nu+(1-0.9\nu)\beta_1)}{20(2.4+0.65\nu+(1-0.9\nu)\beta\beta_1+(0.09-0.07\nu)\beta_2)}$. The membership values for stress of copper bar is $\sigma_c(\mu) = \frac{3(1.12+0.03\mu+(0.6-0.04\mu)\alpha_2)}{20(2.62+0.43\mu+(0.6-0.5\mu)\alpha_1)+(0.06-0.04\mu)\alpha_2}$ and the non-membership values are

$\sigma_s(\nu) = \frac{3(1.10+0.05\nu+(0.09-0.07\nu)\beta_2)}{20(2.4+0.65\nu+(1-0.9\nu)\beta\beta_1+(0.09-0.07\nu)\beta_2)}$. The deflections of both the bars are equal, so the membership values for deflection is $\delta_c(\mu) = \delta_s(\mu) = \frac{180}{2.62+0.43\mu+(0.6-0.5\mu)\alpha_1+(0.06-0.04\mu)\alpha_2\times 10^2}$ and for non-membership values, $P_s = \frac{60\times(1.3+0.6\nu+(1-0.9\nu))\beta_1}{2.4+0.65\nu+(1-0.9\nu)\beta_1+(0.09-0.07\nu)\beta_2}$

$P_c = \frac{60\times(1.10+0.05\mu+(0.09-0.07\mu))\beta_2}{2.4+0.65\nu+(1-0.9\nu)\beta_1+(0.09-0.07\nu)\beta_2}$

$\delta_c(\nu) = \delta_s(\nu) = \frac{180}{2.4+0.65\nu+(1-0.9\nu)\beta_1+(0.09-0.07\nu)\beta_2\times 10^2}$

Giving values for μ and α, the horizontal membership and non-membership functions of stress for steel and copper bar is given in Figure 5 and Figure 6.

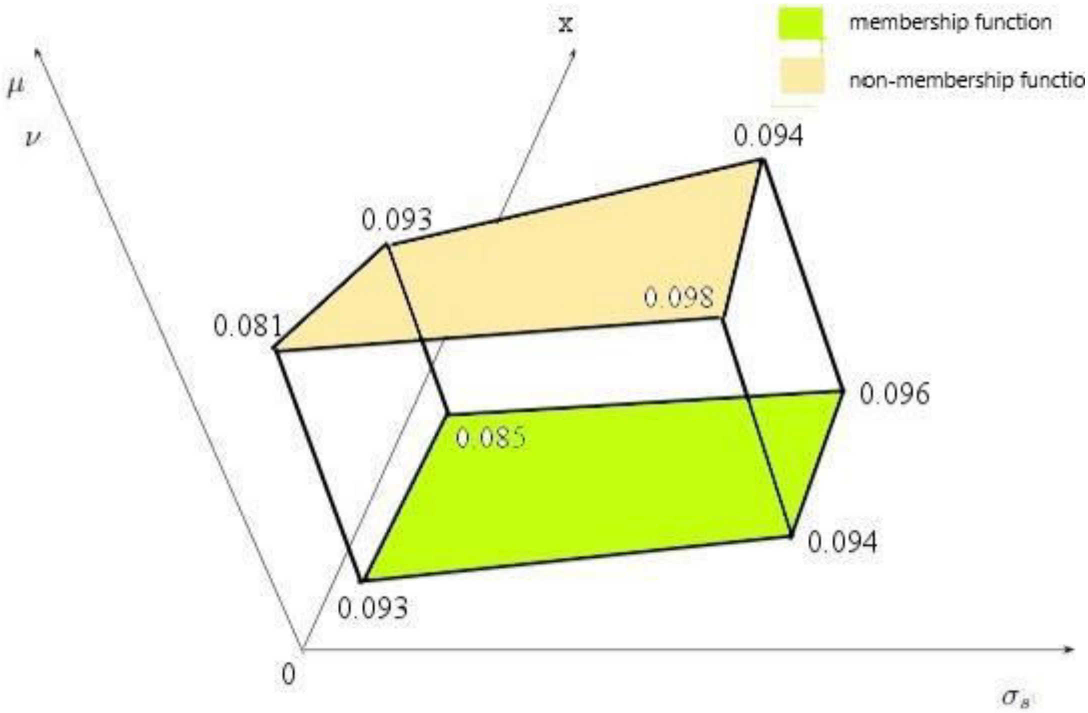

Figure 5: Horizontal membership and non-membership for stress of steel bar

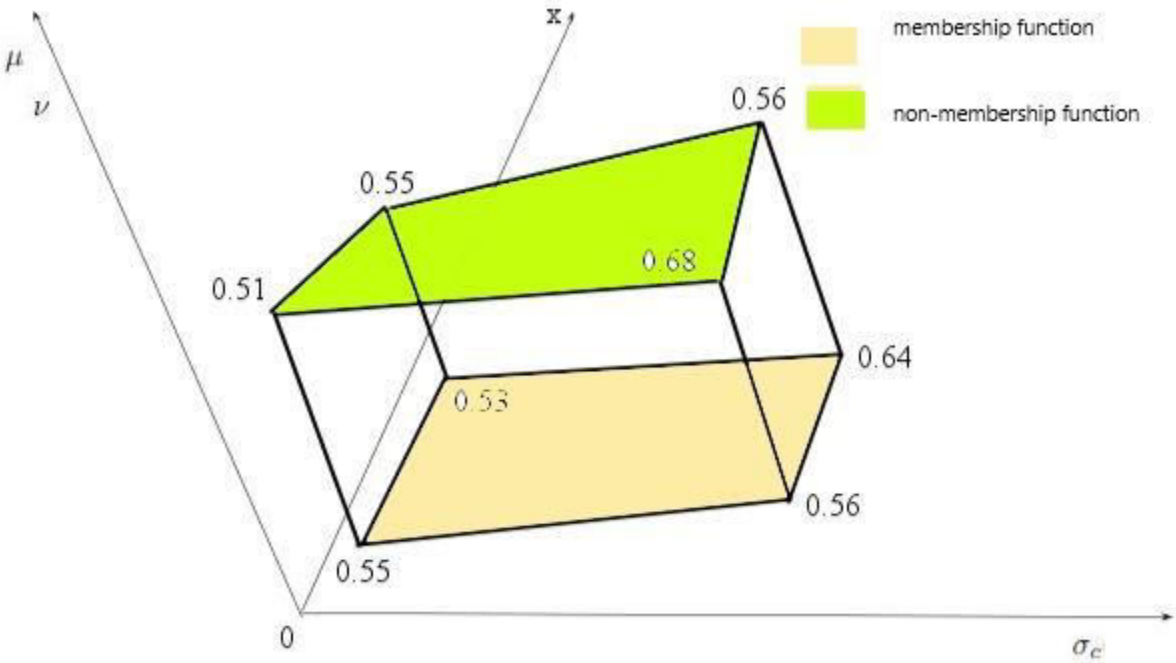

Figure 6: Horizontal membership and non-membership function for stress of copper bar

The horizontal membership and non-membership values for the deflections of both the bars are equal and is given in the figure 7.

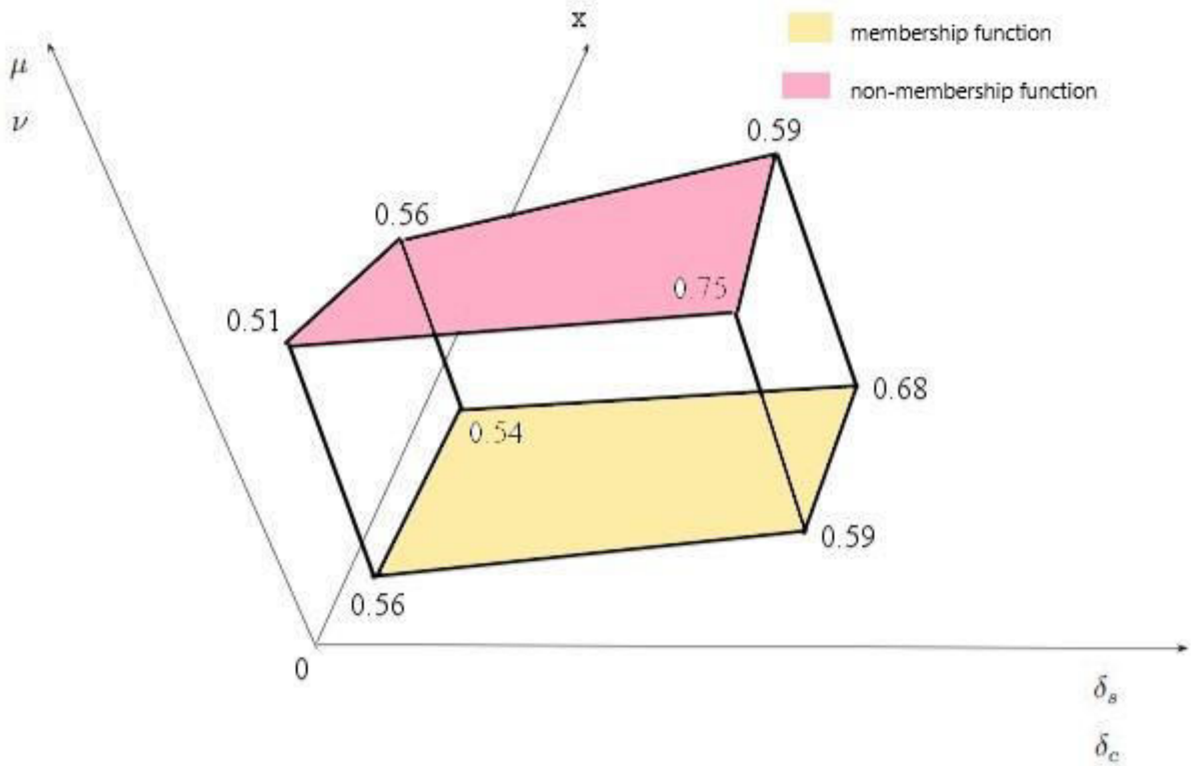

Figure 7: Horizontal membership and non-membership function of deflection of steel and copper bar

Results and discussion

Inference:

In the traditional method, a single crisp value of 0.55*mm* is the output for the deflection in the bars whereas in the intuitionistic fuzzy values it is arange of numbers. The beam starts bending at 0.51mm and reaches its peak at 0.56 to 0.59 mm and breaks at 0.75mm. Using intuitionistic fuzzy values, the initiation of deflection is found as a range of values, so the engineers can be vigilant and use this technique for effective construction but with the traditional method as seen above, the deflection is a single crisp value 0.55*mm*. Also, the initial point of beam bending is found. So, HRTrIFNs are effective in finding out the accurate deflection points in beams, thus useful for civil engineers in selecting the rods for construction. The traditional method provides a single crisp value for deflections and stress with a single formula but usage of horizontal functions provides a multidimensional solution with a single expression. With the varying values of μ and α, a multidimensional accurate result may be found without the need for experiment whereas in traditional method numerical calculations provides only a single crisp result arising the need for conducting laboratory experiment for accurate results.

Advantages:

The exact values for the parameters of the finite element model such as geometrical dimensions or material properties should be available but in reality, the initiation of damage cannot be defined exactly. The young's modulus assumptions are based on the interpretation of material data which accounts for some amount of uncertainty. The horizontal functions help in finding the stress and deflection at any single point throughout the rod with a single expression providing a multi-dimensional solution. So, representing the young's modulus as intuitionistic fuzzy number takes into consideration of all the uncertainties and provides a better result for deflections in real life applications. Simplicity of calculation and multi-dimensional solution is an added advantage in using the horizontal functions. It also provides a better and more accurate result compared to traditional method.

Conclusion

The finite element method is a well-known and time-tested technique for developing approximate solutions to static or dynamic problems in stress analysis. The application of intuitionistic fuzzy arithmetic to solve the finite element problems with uncertainties in the parameters is a powerful tool. An example is provided which is solved with crisp, fuzzy and intuitionistic fuzzy parameters to compare the results. To provide a clear idea of the method of using horizontal functions, problem of composite bar is chosen and solved for both crisp and intuitionistic fuzzy valued parameters. The results are compared showing the effectiveness of the range output of HrTrIFN and a single valued output.

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