Joint Pricing and Ordering Policy for an Advance Sales System of Perishable items with Partial Order Cancellations and Price Varying Linearly Decreasing Demand

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Abstract
The present paper deals with the study of joint pricing and ordering policy foran advance sales system of perishable items with partial order cancellations and price dependent linearly decreasing demand. A single period planning horizon is considered in which the cycle length is divided into two periods:advance sales period and spot sales period. In reality, the discount is offered to the customers for booking the product in advance along with the reservations to cancel their orders before receiving. The purpose of our study is to optimizing the total profit during a given cycle. A numerically example is also given to show the applicability of the developed model.
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Introduction

The analysis of pricing decision is an interesting topic in the field of inventory control for the competitive super market. The demand of the products cannot be constant. It depends upon price. The demand of the product decreases when its selling price increases. A very high selling price reduces both the demand and profit of any firm. Hence pricing of the product becomes the critical decision for any firm which affects its profitability.

A common strategy used by any firm to increase the revenue is to offer some discount to customers for advance booking of the product. This is the way for attracting the customers to purchase the product. Another way of attracting the customers is the influence of advertisement. Hence both the factors are necessary to increase the revenue/profit of any firm. In this area, some scholars have their worth mentioning. Abad [1] constructed an optimal price and lot size inventory model with temporary price reduction offer by the supplier over an interval.Abiodun [2] presented a case study on impact of advertising on sales volume of a product. Aggarwal et al. [3] studied an inventory model for coordinating ordering, pricing and advertisement policy for an advance sales system. Balcer [4] worked on inventory model with optimal advertisement and inventory control of perishable products. Chen and Simchi- Levi developed two inventory models [5, 6]. In the model [5] they were worked on coordinating inventory control and pricing strategies with random demand and constant ordering cost along with finite planning horizon. And in model [6] they were analyzed coordinating inventory control and pricing strategies with random demand and fixed ordering cost with infinite planning period. Cheng and Suresh [7] worked on a periodic continuous review inventory model with promotion decision dependent demand. Das et al. [8] analyzed a production inventory model with partial trade credit policy and reliability. Dye and Hsieh [9] presented a joint pricing and ordering policy for an inventory system with advance booking system and partial order cancellations. Huand and Susan [10] discussed manufacture retailer supply chain cooperative advertising models.Kunreuther and Jean [11] considered an inventory model with optimal pricing and inventory decisions for non-seasonal products. Li et al. [12] constructed a coordinating supply chain inventory model with advertisement and price varying stochastic demand. Mills [13] discussed the uncertainty and price theory for an inventory model. Kumar [14] proposed an optimal pricing policy for non-instantaneous deteriorating items with price and advertisement varying demand under infinite planning horizon. Seyed et al. [15] considered a manufacture supply chain inventory model with game theoretic approach to coordinate pricing and co-op advertising. Szmerekovsky and Jiang [16] analyzed one manufacture and one retailer type inventory model with pricing and two tier advertising. Tsao [17] worked on inventory model with retailers optimal ordering and discounting policies under advance sales discount and trade credits. Sharma and Sharma [18] constructed EOQ model of non-instantaneous perishable items with price and advertisement dependent demand and controllable deterioration rate with partial backlogging. Xie and Alexandre[19] developed a manufacture retailer supply chain inventory model with co-op advertising and pricing. Xie and Jerry [20] worked on manufacture retailer inventory system with coordinating advertising and pricing. You [21] studied the ordering and pricing of service products in an advance sales system consisting price dependent demand. You and Wu [22] discussed the optimal ordering and pricing policy for an inventory model with partial order cancellations. Yue et al. [23] analyzed coordination of cooperative advertising in two level supply chain inventory models with discounts.

Notations and Assumptions:

We consider the following notations and assumptions as follows

1. The selling price p_A is offered during the advance sales period $[0, T_1]$ and the selling price p is offered during the spot sales period $[T_1, T]$,where $p_A = \gamma p, \quad 0 < \gamma < 1$.
2. Price and advertisement dependent demand rate is $D(p,M) = d(p)g(M)$, where p is selling price and M is the advertisement cost per unit item.
3. Price dependent demand function is a linearly decreasing function of selling price, $d(p) = (a - bp), \qquad a,\ b > 0$.
4. Advertisement dependent demand function is exponentially decreasing function of advertisement cost per unit item, $d(M) = e^{-kM}, \quad k,\ M > 0$.

5. Constant deterioration rate is θ.
6. Ordering cost per order is c_1.
7. Holding cost per unit per cycle is h.
8. Purchasing cost per unit item is c.
9. The ratio of refund to advance sales price is g, $0 < g < 1$.
10. The maximum order quantity is Q.
11. T_1 is the length of advance sales period and $(T - T_1)$ is the length of spot sales period.
12. The total profit per cycle is $\Pi(T, T_1, p, M)$.
13. The single period inventory problem with planning horizon T.
14. The replenishment rate is finite.
15. The lead time is zero.

Mathematical Derivation of the Problem

Graphically, the inventory problem consists the zero inventory level in the beginning of the cycle. During the advance sales period $[0, T_1]$, the demand is generated at a rate of $-D(p_A, M)$ and it becomes maximum at time $t = T_1$. The customers also have reservations to cancel their order depending upon the length of waiting time. The customers will wait at a rate of $\frac{1}{1+\delta(T_1 - t)}$, where t is the waiting time δ is the backlogging parameter. After time $t = T_1$, the maximum replenishment order quantity Q is obtained and the advance sales booking is fulfilled. During the spot sales period $(T - T_1)$, the inventory level reduces at a rate of spot sales demand $D(p, M)$ and becomes zero at time $t = T$.

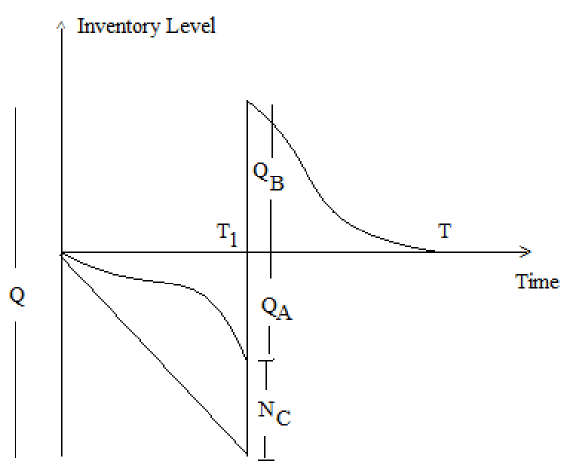

Figure 1, Graphical depiction of Inventory Model

In the cycle, the inventory level at any time t is given by the following differential equations,

$$\frac{dI}{dt} = -\frac{D(p_A, M)}{1+\delta(T_1 - t)}, \qquad 0 \le t \le T_1 \qquad (1)$$

With condition, $I(0) = 0$

$$\frac{dI}{dt} + \theta I = -D(p, M), \qquad T_1 \le t \le T \tag{2}$$

With condition, $I(T) = 0$

The solutions of the equations (1) and (2) are given by the equations (3) and (4) respectively

$$I = \frac{D(p_A, M)}{\delta} \log\left\{\frac{1+\delta(T_1 - T)}{1+\delta T_1}\right\} \tag{3}$$

$$I = D(p, M)\left[T - t - \theta T^2 + \theta t^2\right] \tag{4}$$

The maximum ordered quantity in the cycle is, $Q = Q_A + Q_B$, where Q_A and Q_B are the ordered quantities in the advance and spot sales period respectively.

The ordered quantity Q_A is obtained on putting $t = T_1$ in the equation (3), we have,

$$Q_A = \frac{D(p_A, M)}{\delta} \log\left(1+\delta T_1\right) \tag{5}$$

The ordered quantity Q_B is obtained on putting $t = T$ in equation (4), we have,

$$Q_B = D(p, M)\left[T - T_1 - \theta T^2 + \theta T_1^2\right] \tag{6}$$

Therefore,

$$Q = \frac{D(p_A, M)}{\delta} \log\left(1+\delta T_1\right) + D(p, M)\left[T - T_1 - \theta T^2 - \theta T_1^2\right] \tag{7}$$

The number of order cancellations during the advance sales period is,

$$N_C = \int_0^{T_1} D(p_A, M)\left[1 - \frac{1}{1+\delta(T_1 - t)}\right] dt$$

Or

$$N_C = \frac{D(p_A, M)}{2}(\delta T_1^2) \tag{8}$$

The refunded cost is partially refund to the customers, who cancelled their orders is given by,

$$C_R = g\, p_A . N_C$$

Or

$$C_R = \frac{(g\delta\, p_A . N_C) D(p_A, M)}{2} T_1^2 \tag{9}$$

The total sales in the advance sales period is,

$$N_A = D(p_A, M) T_1 \tag{10}$$

The total sales in the spot sales period is

$$N_S = D(p, M)(T - T_1) \tag{11}$$

The total sales revenue, during the cycle is

$$T_R = p_A N_A + p N_S$$

Or

$$T_R = p_A D(p_A,\ M)T_1 + pD(p,\ M)(T - T_1) \tag{12}$$

Per cycle, the ordering cost is,

$$C_O = c_1 \tag{13}$$

Per cycle, the holding cost is,

$$C_H = h\int_{T_1}^{T} D(p_A,\ M)\left[T - t - \theta T^2 + \theta t^2\right]dt$$

Or

$$C_H = \frac{hD(p_A,\ M)}{6}\left[3T^2 - 6TT_1 + 3T_1^2 - 4\theta T^3 + 6\theta T^2 T_1 - 2\theta T_1^3\right] \tag{14}$$

Per cycle, the purchasing cost is,

$$C_P = cQ$$

Or

$$C_P = \frac{c\,D(p_A,\ M)}{\delta}\log(1 + \delta T_1) + c\,D(p,\ M)\left[T - T_1 - \theta T^2 + \theta T_1^2\right] \tag{15}$$

The total profit, per cycle is

$$\Pi = \frac{1}{T}\left[T_R - (C_O + C_H + C_P + C_R)\right]$$

Or

$$\Pi(T, T_1, p, M) = \frac{1}{T}\Big[-c_1 + (1-kM)bp(1-\gamma)(2a - bp - b\gamma p)T_1 + (1-kM)(a-bp)(a-bp-c)T$$
$$+\frac{(1-kM)(a-bp)}{2}\{c(\delta + 2\theta) - h\}T_1^2 - \frac{g\delta(1-kM)(a-b\gamma p)^2}{2}T_1^2 + \frac{(1-kM)(a-bp)(2c\theta - h)}{2}T^2$$
$$+h(1-kM)(a-bp)TT_1 + \frac{2h\theta(1-kM)(a-bp)}{3}T^3 + \frac{h\theta(1-kM)(a-bp)}{3}T_1^3$$
$$-h\theta(1-kM)(a-bp)T_1T^2\Big] \tag{16}$$

(for the first order approximation of $e^{-kM} = (1-kM)$ and $D(p_A,\ M) = (a - b\gamma\, p)(1-kM)$)

After differentiating the equation (16), we obtain

$$\frac{\partial\Pi}{\partial T_1} = \frac{1}{T}\Big[(1-kM)bp(1-\gamma)(2a - bp - b\gamma p) + (1-kM)(a-bp)\{c(\delta + 2\theta) - h\}T_1 - g\delta(1-kM)(a-b\gamma p)^2 T_1$$
$$-h(1-kM)(a-bp)T - h\theta(1-kM)(a-bp)T^2 + h\theta(1-kM)(a-bp)T_1^2\Big] \tag{17}$$

$$\frac{\partial\Pi}{\partial T} = \frac{(1-kM)}{T}\Big[(a-bp)(a-bp-c) + (a-bp)(2c\theta - h)T + h(a-bp)T_1 + 2h\theta(a-bp)T^2$$
$$-2h\theta(a-bp)TT_1\Big] - \frac{1}{T^2}\Big[-c_1 + (1-kM)bp(1-\gamma)(2a - bp - b\gamma p)T_1 + (1-kM)(a-bp)(a-bp-c)T$$
$$+\frac{(1-kM)(a-bp)}{2}\{c(\delta + 2\theta) - h\}T_1^2 - \frac{g\delta(1-kM)(a-b\gamma p)^2}{2}T_1^2 + \frac{(1-kM)(a-bp)}{2}(2c\theta - h)T^2$$

$$+h(1-kM)(a-bp)TT_1+\frac{2h\theta(1-kM)(a-bp)}{3}T^3+\frac{h\theta(1-kM)(a-bp)}{3}T_1^3$$
$$-h\theta(1-kM)(a-bp)T_1T^2\Big] \quad (18)$$

$$\frac{\partial\Pi}{\partial p}=\frac{(1-kM)}{T}\big[b(1-\gamma)\{(2a-bp-b\gamma p)-bp(1+\gamma)\}T_1-b(2a-2bp-c)T-\frac{b}{2}\{c(\delta+2\theta)-h\}T_1^2$$
$$+bg\gamma\delta(a-b\gamma p)T_1^2-\frac{b(2c\theta-h)}{2}T^2-bhTT_1-\frac{2bh\theta}{3}T^3-\frac{bh\theta}{3}T_1^3+bh\theta T_1T^2\big] \quad (19)$$

$$\frac{\partial\Pi}{\partial M}=-\frac{k}{T}\Big[bp(1-\gamma)(2a-bp-b\gamma p)T_1+(a-bp)(a-bp-c)T+\frac{(a-bp)}{2}\{c(\delta+2\theta)-h\}T_1^2$$
$$-\frac{g\delta(a-b\gamma p)^2}{2}T_1^2+\frac{(a-bp)(2c\theta-h)}{2}T^2+h(a-bp)TT_1+\frac{2h\theta(a-bp)}{3}T^3$$
$$+\frac{h\theta(a-bp)}{3}T_1^3-h\theta(a-bp)T_1T^2\Big] \quad (20)$$

The equations $\frac{\partial\Pi}{\partial T_1}=0,\quad \frac{\partial\Pi}{\partial T}=0,\quad \frac{\partial\Pi}{\partial p}=0,\quad \frac{\partial\Pi}{\partial M}=0$, give the necessary conditions for total profit to be maximum. On solving these equations we find the optimum values of T, T_1, p and M for which the profit is optimum.

Table 1, variation in total profit with respect to δ

δ	T	T_1	p	M	$\Pi(T, T_1, p, M)$
0.1	106.75160	53.41630	30.24580	0.33334	5.5012×10^6
0.3	96.72455	21.70200	25.64512	0.33335	7.975×10^3
0.5	93.91062	13.99706	24.39965	0.33337	6.9396×10^3
0.7	92.34698	10.58976	23.77716	0.33340	6.4457×10^3
0.9	91.28064	8.61803	23.38257	0.33343	6.1281×10^3

The table 1, shows as we increase the parameter δ, the total profit is decreased. The values of T, T_1 and p are decreased and the values of M are increased. Therefore, to increase the profit, we have to decrease the number of orders.

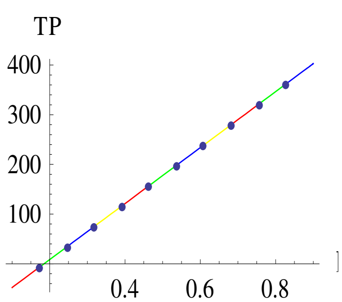

Figure 2, variation in total profit w.r.to δ

Table 2, variation in total profit with respect to γ

γ	T	T_1	p	M	$\Pi(T, T_1, p, M)$
0.4	106.75160	53.41630	30.24580	0.33334	5.5012×10^6
0.5	98.12850	41.74881	27.45825	0.33335	8.8229×10^3
0.6	90.95403	29.69294	24.97830	0.33341	6.2404×10^3
0.7	85.40662	16.53832	22.81022	0.33339	4.8594×10^3
0.8	81.84389	5.26679	21.23123	0.33375	4.0180×10^3

The table 2, shows as we increase the parameter γ, the total profit is decreased. The values of T, T_1 and p are decreased and the values of M are increased. Therefore, to increase the profit, we have to decrease the discount.

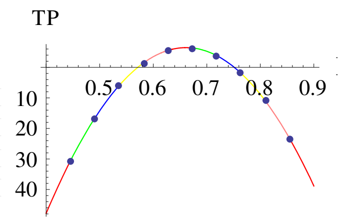

Figure 3, variation in total profit w.r.to γ

Table 3, variation in total profit with respect to k

k	T	T_1	p	M	$\Pi(T, T_1, p, M)$
3	106.75160	53.41630	30.24580	0.33334	5.5012×10^6
4	106.75161	53.41631	30.24581	0.25000	9.3907×104
5	106.75162	53.41623	30.24576	0.20009	1.2069×10^4
6	106.75164	53.41628	30.24577	0.16667	1.2051×10^4
7	106.75165	53.41629	30.24579	0.14286	1.0322×10^4

From the table 3, we observe that as we increase the parameter k, the total profit is decreased. The values of T, T_1 and p are decreased and the values of M are increased. Therefore, to increase the profit, we have to decrease the advertisement cost.

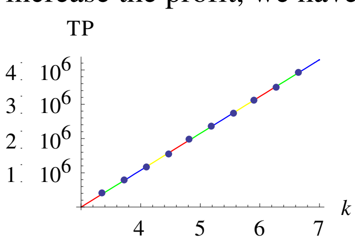

Figure 4, variation in total profit w.r.to k

Table 4, variation in total profit with respect to a

a	T	T_1	p	M	$\Pi(T, T_1, p, M)$
200	106.75160	53.41630	30.24580	0.33330	5.5012×10^{6}
300	124.78187	71.11901	46.75555	0.33334	2.4625×10^{4}
400	141.18337	86.12342	63.23998	0.33335	4.5096×10^{4}
500	156.21446	99.50699	79.61779	0.33335	7.5940×10^{5}
600	170.03357	111.64019	95.81387	0.33336	1.1255×10^{5}
700	182.83749	122.77707	111.79611	0.33337	1.5738×10^{5}
800	194.77321	133.07783	127.54231	0.33338	5.1624×10^{5}

From the table 4, we see that as we increase the parameter a, the total profit is decreased. The values of T ,T_1, M and p are increased. Therefore, to increase the profit, we have to decrease either advance sales period or the advertisement cost.

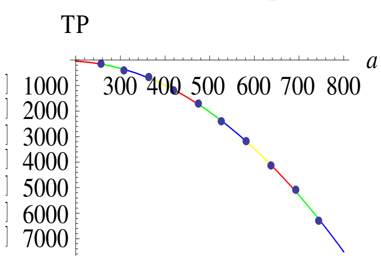

Figure 5, variation in total profit w.r.to a

Conclusion

In this paper, a profit optimization model for deteriorating items is discussed by considering the price and advertisement dependent demand with reservation to cancel their advance orders. From the observations based on the given numerical example, the total profit can be increased to decrease the advance sales period, advertisement cost, discount and number of orders. The reason is that in the long advance sales period there are more possibilities to the customers to cancel their booked orders. In future this model can be generalized by incorporating some realistic features on the demand, discount, deterioration, reliability, lead time, installment, backlogging, reservations etc..

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Appendix

The total profit $\Pi(T,\ T_1,\ p,\ M)$ will be maximum, if all principal minors of the Hessian matrix or H matrix are negative definite, where H matrix is defined as follows

$$H = \begin{bmatrix} \frac{\partial^2 \Pi}{\partial T_1^2} & \frac{\partial^2 \Pi}{\partial T_1 \partial T} & \frac{\partial^2 \Pi}{\partial T_1 \partial p} & \frac{\partial^2 \Pi}{\partial T_1 \partial M} \\ \frac{\partial^2 \Pi}{\partial T \partial T_1} & \frac{\partial^2 \Pi}{\partial T^2} & \frac{\partial^2 \Pi}{\partial T \partial p} & \frac{\partial^2 \Pi}{\partial T \partial M} \\ \frac{\partial^2 \Pi}{\partial p \partial T_1} & \frac{\partial^2 \Pi}{\partial p \partial T} & \frac{\partial^2 \Pi}{\partial p^2} & \frac{\partial^2 \Pi}{\partial p \partial M} \\ \frac{\partial^2 \Pi}{\partial M \partial T_1} & \frac{\partial^2 \Pi}{\partial M \partial T} & \frac{\partial^2 \Pi}{\partial M \partial p} & \frac{\partial^2 \Pi}{\partial M^2} \end{bmatrix}$$

Or

$$H = \begin{bmatrix} 0.00049 & -0.00110 & 0.00278 & 0.00240 \\ -0.00007 & 0.00131 & 0.00258 & 682.46139 \\ 0.00338 & 0.002144 & -0.02259 & 9.95650 \\ -99.78296 & 622.62997 & 9.95650 & 0 \end{bmatrix}$$