Results on Random Differential Inclusions

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Abstract: Here, we investigate the boundary value problem for fourth order of random differential inclusions and proved the existence result of random solution through Krasnosel'skii fixed point result.

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1.Description of the Problem

Consider the fourth order random differential inclusions as

$$p''''(t,\omega)\in\beta(t,\omega)\alpha(p(t,\omega),p'(t,\omega),\omega),\qquad 0\le t\le 1\ ,\omega\in\Omega \tag{1.1}$$

with respect to boundary conditions

$$p(0,\omega)=p'(0,\omega)=p''(1,\omega)=u'''(1,\omega)=0 \tag{1.2}$$

Suppose that

(A_1). $\alpha:[0,\infty)\times[0,\infty)\times\Omega\to[0,\infty)$ is continuous.

(A_2). $\beta:[0,1]\times\Omega\to[0,\infty)$ is a continuous function such that $\int_0^1\beta(t,\omega)\,dt>0$.

In classical problem (1.1) has been studied by Ma [11],Bai, Wang [2], Davis, Henderson [4], Elgindi,Guan [5], Also, in random problem (1.1) has been studied by D.S. Palimkar[12,13,14].S. B. Biradar, D. S. Palimkar[2].

In here, we have proved existence result of problem (1.1)-(1.2) applying Krasnosel'skii fixed point theorem.

The Green's function $G:[0,1]\times[0,1]\to[0,\infty)$ for(1.1)-(1.2) is

$$G(t,s)=\begin{cases}\frac{1}{6}t^2(3s-t), & \text{if}\quad 0\le t\le s\le 1,\\ \frac{1}{6}s^2(3t-s), & \text{if}\quad 0\le s\le t\le 1.\end{cases}$$

and (1.1)-(1.2) is equivalent to the integral equation

$$p(t,\omega)=\int_0^1 G(t,s)\beta(s,\omega)\alpha(p(s,\omega),p'(s,\omega),\omega)\,ds,\qquad 0\le t\le 1,\omega\in\Omega. \qquad (1.3)$$

The following Krasnosel'skii fixed point theorem is apply to prove key results .

Theorem1.1[9]**.**Let $(B,\|.\|)$ be a Banach space and let $K\subset B$ be a cone. Let B_1 and B_2 be real numbers such that $B_2>B_1>0$, and let

$U_i=\{q\in B|\ \|q\|<B_i\},\qquad i=1,2.$

If $M{:}K\bigcap(\overline{U}_2-U_1)\to K$ is a completely continuous operator such that, either

(C_1). $\|Mq\|\le\|q\|$ if $q\in K\bigcap\partial U_1$, and $\|Mq\|\ge\|q\|$ if $q\in\bigcap\partial U_2$,

Or

$$(C_2).\|Mq\|\ge\|q\|\ \ if\ \ q\in K\cap\partial U_1,\ and\ \ \|Mq\|\le\|q\|\ \ if\ \ q\in K\cap\partial U_2.$$

Then M has a fixed point in $K\cap(\overline{U}_2-U_1)$.

2 .Existence Results

We have listed some results about positive solutions of problem (1.1)-(1.2).

Define the functions $a:[0,1]\times\Omega\to[0,+\infty)$ and $b:[0,1]\times\Omega\to[0,+\infty)$ by

$$a(t,\omega)=\frac{3}{2}t^2-\frac{1}{2}t^3,\qquad 0\le t\le 1,$$

$$b(t,\omega)=2t^2-\frac{4t^3}{3}+\frac{t^4}{3},\qquad 0\le t\le 1.$$

Lemma 2.1. If $p\in C^4[0,1]$ satisfies the boundary conditions (1.2), and such that

$$p''''(t)\ge 0\quad for\quad 0\le t\le 1, \qquad (2.1)$$

$$p'''(t)\le 0,\ \ p''(t)\ge 0\ ,\ p'(t)\ge 0\ ,\ p(t)\ge 0\quad \text{for}\quad t\in[0,1] \qquad (2.2)$$

Lemma 2.2. If $p\in C^4[0,1]$ satisfies (1.2) and (2.1), then

$$a(t)p(1)\le p(t)\le t\,p(1)\qquad \text{for}\quad t\in[0,1] \qquad (2.3)$$

Lemma 2.3. If $p\in C^4[0,1]$ satisfies (1.2) and (2.1), and $p''''(t)$ is non decreasing on $[0,1]$, then

$$a(t)p(1)\le p(t)\le b(t)p(1)\qquad for\qquad t\in[0,1] \qquad (2.4)$$

Lemma 2.4. Suppose that (A_1) and (A_2) holds. If $p(t)$ is a nonnegative solution to the problem (1.1)-(1.2), then $p(t)$ satisfies (2.2) and (2.3).Define the operator $T:P\times\Omega\to B$ by

$$(T\,p)(t,\omega)=\int_0^1 G(t,s)\beta(s,\omega)\alpha(p(s,\omega),p'(s,\omega),\omega)\,ds,\qquad 0\le t\le 1,\omega\in\Omega$$

Lemma 2.5. Suppose that (A_1), (A_2), and the following condition hold.

(A_3). Both α and β are non decreasing functions.

If $p(t)$ is a nonnegative solution to the problem (1.1)-(1.2), then $p(t)$ satisfies (2.2) and (2.4).

3. Main Results

Here, we quote some constants terms

$$D=\int_0^1 G(1,\omega,s)\beta(s,\omega)a(s,\omega)\,ds,\ \mathrm{E}=\int_0^1 G(1,\omega,s)\beta(s,\omega)s\,ds,$$

$$F_0=\limsup_{w\to 0^+}\frac{\alpha(w)}{w},\quad f_0=\liminf_{w\to 0^+}\frac{\alpha(w)}{w},$$

$$F_\infty=\limsup_{w\to +\infty}\frac{\alpha(w)}{w},\quad f_\infty=\liminf_{w\to +\infty}\frac{\alpha(w)}{w},$$

let $B=C[0,1]$ be with norm $\|q\|=\max_{t\in[0,1]}|q(t,\omega)|,\quad \mathrm{q}\in B,$

and

let $K^+=\{q\in B\,|\,q(1,\omega)\ge 0,\ a(t,\omega)\,q(1,\omega)\le q(t,\omega)\le t\,q(1,\omega)\ \ for\ \ t\in[0,1],\omega\in\Omega\}.$

Where K^+ is a positive cone in B.

Result-I

Suppose that (A_1) and (A_2) holds. If $E\,F_0<1<D\,f_\infty$, then the problem (1.1)-(1.2) has at least one positive random solution.

Proof. Firstly, choose $\varepsilon>0$ such that $(F_0+\varepsilon)E\le 1$. From F_0, there exists $H_1>0$ such that

$$f(w)\le(F_0+\varepsilon)w\ \ for\ \ \ 0<w\le H_1.$$

For each $p\in K$ with $\|p\|=H_1$, we have

$$\begin{aligned}(T\,p)(1,\omega)&=\int_0^1 G(1,\omega,s)\beta(s,\omega)\alpha(p(s,\omega),p'(s,\omega),\omega)\,ds\\ &\le\int_0^1 G(1,\omega,s)\beta(s,\omega)(F_0+\varepsilon)p(s,\omega)\,ds\\ &\le(F_0+\varepsilon)\|p\|\int_0^1 G(1,\omega,s)\beta(s,\omega)\,ds\\ &\le(F_0+\varepsilon)\|p\|B\\ &\le\|p\|,\end{aligned}$$

which means $\|T\,p\|\le\|p\|$. Thus, if we let $U_1=\{p\in B\,|\ \|p\|<H_1\}$ then

$$\|T\,p\|\le\|p\|\quad for\quad \mathrm{p}\in k\cap\partial U_1.$$

For U_2, choose $\delta>0$ and $c\in(0,1/4)$ such that

$$\int_0^1 G(1,\omega,s)\beta(s,\omega)a(s,\omega)\,ds.(f_\infty-\partial)\ge 1.$$

There exists $H_3>0$ such that

$$f(w) \geq (f_\infty - \delta)w \ \ for \ \ \mathrm{w} \geq H_3.$$

Let $H_2 = \max\left\{H_3 c^{-2}, 2H_1\right\}$. If $p \in K$ such that $\|p\| = H_2$, then for each $t \in [c,1]$,

We have , $p(t,\omega) \geq H_2 a(t,\omega) \geq H_2 t^2 \geq H_2 c^2 \geq H_3.$

Therefore, for each $p \in K$ with $\|p\| = H_2$, we have

$$\begin{aligned}(T\,p)(1,\omega) &= \int_0^1 G(1,\omega,s)\beta(s,\omega)\alpha(p(s,\omega), p'(s,\omega),\omega)\,ds \\ &\geq \int_c^1 G(1,\omega,s)\beta(s,\omega)\alpha(p(s,\omega), p'(s,\omega),\omega)\,ds \\ &\geq \int_c^1 G(1,\omega,s)\beta(s,\omega)(f_\infty - \delta)p(s,\omega)\,ds \\ &\geq \int_0^1 G(1,\omega,s)\beta(s,\omega)a(s,\omega)\,ds\ .\ (f_\infty - \delta)\|p\| \\ &\geq \|p\|,\end{aligned}$$

which means $\|Tp\| \geq \|p\|$. Thus, if we let $U_2 = \left\{p \in B \mid \|p\| < H_2\right\}$, then $\overline{U_1} \subset U_2$, and

$$\|T\,p\| \geq \|p\| \ \ for \ \ \mathrm{p} \in K \cap \partial U_2.$$

Now, condition first of Theorem 1.1 is satisfied, there exists a random fixed point of T in $K \cap (\overline{U_2} \subset U_1)$.

Result-II

Assume that (A_1) and (A_2) holds. If $E\,F_\infty < 1 < D\,f_0$, then the problem (1.1)-(1.2) has at least one random positive solution.

Proof. Firstly **,** choose $\varepsilon > 0$ such that $(f_0 - \varepsilon)D \geq 1$. There exists $H_1 > 0$ such that

$$f(w) \geq (f_0 - \varepsilon)w \quad for \quad 0 < w < H_1.$$

For each $p \in K$ with $\|p\| = H_1$, we have

$$\begin{aligned}(T\,p)(1,\omega) &= \int_0^1 G(1,\omega,s)\beta(s,\omega)\alpha(p(s,\omega), p'(s,\omega),\omega)\,ds \\ &\geq \int_0^1 G(1,\omega,s)\beta(s,\omega)p(s,\omega)\,ds\ .\ (f_0 - \varepsilon) \\ &\geq \int_0^1 G(1,\omega,s)\beta(s,\omega)a(s,\omega)\,ds\ .\ (f_0 - \varepsilon)\|p\| \\ &= D(f_0 - \varepsilon)\|p\| \\ &\geq \|p\|,\end{aligned}$$

which means $\|Tp\| \geq \|p\|$. Thus, if we let $U_1 = \left\{p \in B \mid \|p\| < H_1\right\}$, , then

$$\|Tp\| \geq \|p\| \ \ for \ \ \mathrm{p} \in K \cap \partial U_1.$$

For U_2, choose $\delta \in (0,1)$ such that $(F_\infty + \delta)D < 1$. There exists $H_3 > 0$ such that

$$\alpha(w) \le (F_\infty + \delta)w \quad for \quad w \ge H_3.$$

If we let $M = \max_{0 \le w \le H_3} \alpha(w),$ then $\alpha(w) \le M + (F_\infty + \delta)w \quad for \quad w \ge 0.$

Let $N = M\int_0^1 G(1,\omega,s)\beta(s,\omega)\, ds,$ and let $H_2 = \max\left\{2H_1, N(1-(F_\infty+\delta)B)^{-1}\right\}.$

from (3.2) implies that $N + (F_\infty + \delta)EH_2 \le H_2.$

For each $p \in K$ with $\|p\| = H_2$, we have

$$(Tp)(1,\omega) = \int_0^1 G(1,\omega,s)\beta(s,\omega)\alpha(p(s,\omega), p'(s,\omega),\omega)\, ds$$
$$\le \int_0^1 G(1,\omega,s)\beta(s,\omega)(M + (F_\infty+\delta)\, p(s,\omega))\, ds$$
$$\le N + (F_\infty+\delta)\int_0^1 G(1,\omega,s)\beta(s,\omega)\, p(s,\omega)\, ds$$
$$\le N + (F_\infty+\delta)H_2\int_0^1 G(1,\omega,s)\beta(s,\omega)s\, ds$$
$$= N + (F_\infty+\delta)H_2E,$$
$$\le H_2,$$

which means $\|Tp\| \le \|p\|$ Thus, if we let $U_2 = \left\{p \in B \mid \|p\| < H_2\right\},$ then $\overline{U_1} \subset U_2$, and

$$\|Tp\| \le \|p\| \quad for \quad p \in K \cap \partial U_2.$$

Therefore, condition (C_2) of Theorem 1.1 is verified. Hence ,T has at least one random fixed point. Obviously , the problem (1.1)-(1.2) has at least one random positive solution.

4. Conclusion

We have proved the criteria for existence of random solution of the problem. This type of results are obtained in classical differential inclusions but not in random differential inclusions. Hence, this result is basic and new results in random differential inclusions.

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