A Novel Approach for Solving Non-Linear Equation

Noorzaman Bawari
Assistant Professor Science Faculty, Math Department, Nangarhar University, Nangarhar, Afghanistan
Email: noorzamanstd@uop.edu.pk

Abstract
The primary objective of this research is to discover the ideal way to solve the non-linear equation using iterative methods. This article examines and highlights the four iterative methods to dealing with non-linear equations. As a consequence of this approach, the pace of convergence between first-degree iterative procedures is explained. The graphical development is then built up utilizing the four iterative methods and the outcomes are evaluated using different functionalities. An example of the algebraic equation is presented to compare the approximation inaccuracy among the methods. In addition, two examples of algebraic and transcendental equations are utilized to verify both the optimum method and the extent of the visibly presented errors.
Keywords:Newton-Raphson Method, Secant Method, Non-Linear Equations, False Position Method, Bisection Method, Rate of Convergence.

Introduction

The most prevalent problem in numerical testing is that the root of an equation is quickly identified with f(x) = 0, in which f(x) = 0 kinds are referred to as algebraic or transcendental.

$f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x^1 + a_n, (a\neq 0)$

When f(x) is an algebraic, transcendental, or transcendental mix, if f(x) = 0 at x= the root of the equation is only known when f(x) =0. The equation f(x)=0 is an algebraic equation of k grade, such as x3-3x2+12=0, 2x3-3x-6=0, and so on; where f(x) is one of a number of functions; and where f(x) is one of a number of functions, such as trigonometric, logarithmic, exponential and so on, f(x)=0 is a transcendent equation, such as x sin x + cos x=0, logo In addition, numerous layers of algebraic functions are resolved by means of familiar techniques. The problems, on the other hand, are that we are trying to deal with the more severe or transcendental functions which do not need quick techniques to achieve the perfect arrangement. The use of imprecise methods offers an easy solution to these problems. Numerical methods are utilized to obtain nonlinear equation solutions. Many techniques are available to determine the root of a nonlinear equation. Many examples include graphics, Regula-falsi methods, bisection methods, Secant methods, Bairstow methods, Newton-Raphson methods, Graeffe methods for root squaring and iterative methods of Newton. R.L. In his works, he provides a few formulas for numerical methods. A.R., A.R., A.R., A.R., A.R., A.R., A.R., S.S. Vasishtha[2], Nonlinear Equations Enhancement by modified adomian decomposition process [4], Newton's Mid-point modified approach [10], a novel three-stage nonlinear equations settlement iterative process [7], Other researchers [13] found and demonstrated that Newton-fragmented Raphson's process has been accelerated Furthermore, some researchers[14] have examined the convergence of five numerical techniques. Matlab uses an interface to derive the roots of fundamental equations and unexpected articulations[15]. Moreover, some academics propose two additional compulsory methods for approximation solutions for non-linear algebraic problems[16]. In order to solve the nonlinear issue f(x)=0, an iterative connection was also suggested and explored,[17]. Furthermore, some other academics[18] suggest and study fully a prediction correction method based on a weighted combination of medium, squared and trapezoidal formules.

We have compared and contrasted in our working plan with current iterative approaches like the wrong positioning method, the seant method, the Newton method and the bisection method to find the optimal strategy. This research looks at the most effective way to determine the optimal concept for non-linear equation management. The roots of non-linear equations may readily be found using the information on this page. We have also shown numerous features to achieve convincing effects with less repeatability than the above-mentioned methods.

Objectives

The main goal of the research is to identify the best technique used by numerical methods to solve nonlinear equations. The following are the goals:

- ¨Finding the convergence rate, the optimum solution, as well as the degree of technical error.
- · Comparison of the techniques available to get the optimum strategy for tackling nonlinear problems.

2. Materials and methods

The Newton-Raphson technique, the Secant method, the False Position Method, and the Bisection Method are all names for procedures that are used to calculate distances.

2.1. Newton-Raphson Method

The Newton-Raphson technique is a well-known calculating approach that is often used to determine the root of an equation, such as f(x) = 0, where the f(x) derivative is thought to equal f. The Newton-Raphson method is also known as the Newton-Raphson formula. (x). The Newton-Raphson method is based on a basic concept.

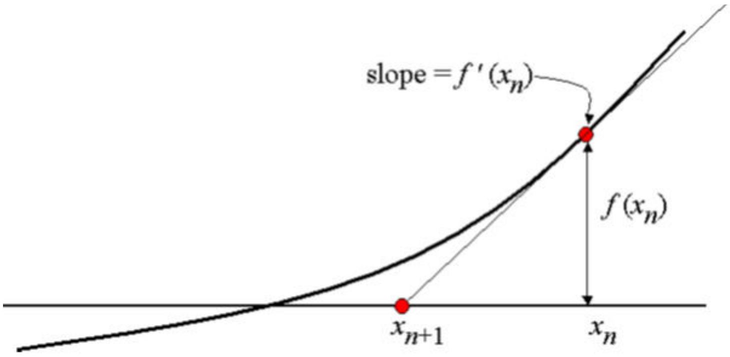

Figure 1. Newton-Raphson Method

The Newton method may also be accomplished by expanding the Taylor f(x) function series at point x0, as illustrated below:

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + \frac{(x - x_0)^2}{2!}f''(x_0) + \cdots$$

We may ignore the second, third, and higher words in (xx0), since it's little and we benefit from that.

$$(x)=f(x_0)+(x-x_0)f'(x_0) \text{ ……..(1)}$$

We get f(x) = 0 in (1),

$$f(x_0) + (x - x_0)f'(x_0) = 0$$

$$\text{Or. } (x - x_0) = -\frac{f(x_0)}{f'(x_0)}; f'(x_0) \neq 0$$

Hence,

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

In general, we obtain the overall formula;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \; for\; n = 0,1,2, \ldots$$

This is known as the formulation of Newton-Raphson.
Example: Solve the equation f(x)=x3+2x2+10x−20=0 using the technique of Newton.
Since f(x)=x3+2x2+10x−20=0 f′(x)=3x2+4x+10 Correct to five decimal points using the Newton technique for the equation.
We can now observe f(1) = - 7 < 0 and f(2) = 16 > 0.
The root is therefore somewhere in the range of 1 and 2.
We know Newton's formula as follows;

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Or,
$$x_{n+1} = x_n - [\frac{{x_n}^3+2{x_n}^2+10x_n-20}{3{x_n}^2+4x_n-10}]$$

After the right side yields are solved;

$$x_{n+1} = \frac{2({x_n}^3+2{x_n}^2+10)}{3{x_n}^2+4x_n-10}$$

Now, the Newton-Raphson technique convergence range: the Newton-Raphson method has provided below,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

It's a technique of iteration and here;

$$x_{n+1} = \emptyset(x_n);\ \emptyset(x_n) = x_n - \frac{f(x_n)}{f'(x_n)}$$

In general, x= $\emptyset(x)$, where $\emptyset(x) = x - \frac{f(x_n)}{f'(x_n)}$.
We know that the iteration method converges if

$|\ \emptyset'(x)\ | < 1$, i.e $|\ 1- \frac{[f'(x)]^2-f(x)f''(x)}{[f'(x)]^2}\ | < 1$

Or, $|\frac{f(x)f''(x)}{[f'(x)]^2}| < 1$

i.e $|$ f(x) f''(x)$| < [f'(x)]^2$

Therefore, Newton-Raphson formula is convergent if $|$ f(x) f''(x)$| < [f'(x)]^2$

2.2. False Position Method

False positioning technique, an extraordinarily ancient methodology for addressing a fundamental issue. The test and error method is regarded that uses test estimates for the variable and after this change an incentive to dismiss the outcome. This formula is sometimes termed "conjecture and control." The following image shows the fundamental concept of this approach,

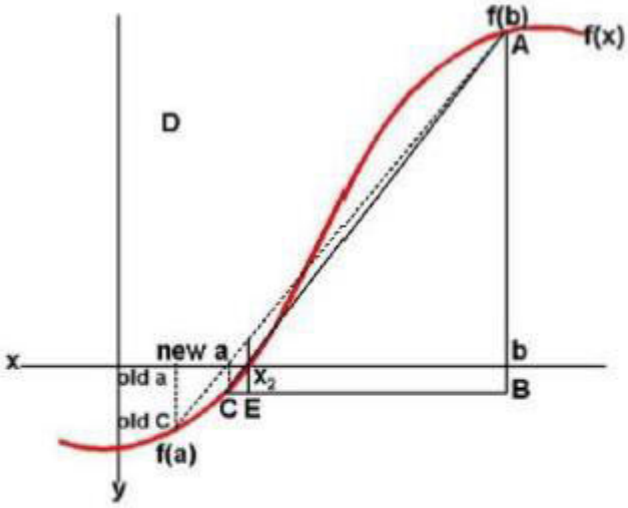

Figure 2: False Position Method

Consider the expression f(x) = 0, where f(x) is a variable that is not temporary. In this technique, we select two points a and b that have the opposite signs for f(a) and f(b) and are next to each other (a b). As a result, the y curve = f(x) will intersect the x-axis between A(a, f(a)) and B(b, f(b)), touching the x-axis. As a result, there is a root located halfway between these two centers. The chord equations connecting A(a, f(a)) and B(b, f(b)) are now provided in the next section; nevertheless,

$$\frac{x-a}{a-b} = \frac{y-f(a)}{f(a)-f(b)} \ldots\ldots\ldots\ldots\ldots(1)$$

By replacement of y = 0 in (1) for the x-axis chord equation crossing point we get the main x0 approximation for the root of f(x) = 0;

$$\frac{x_0-a}{a-b} = \frac{-f(a)}{f(a)-f(b)}$$

$$\text{Or, } \quad x_0 = a - \frac{f(a)(a-b)}{f(a)-f(b)}$$

$$\therefore \quad x_0 = \frac{af(a)-bf(a)}{f(b)-f(a)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The root is between a and x0 when the signs of f(a) and f(x0) are opposite. We then obtain approximation x1 by substituting b with x0 in (2). If f(a) and f(x0) have the same sign, f(x0) and f(b) have opposite sign, thus their root is between x0 and b. In (2), we get x1 by substituting an x0. This procedure continues until the root achieves the necessary accuracy. This is the generic formula:

$$x_{n+1} = \frac{af(a) - bf(a)}{f(b) - f(a)}$$

Example: Fix the equation f(x) = x3−3x−5=0 using the False Position process..
Solution:

Let f(x) = $x^3-3x-5=0$

Since f(2) = - 3<0

F(3) = 13>0

We know that,

$$x_0 = \frac{af(a) - bf(a)}{f(b) - f(a)}$$

When a=2, b=3 then

$$x_0 = \frac{af(a)-bf(a)}{f(b)-f(a)} = \frac{2\times13-3\times-3}{13+3} = 2.1875$$

Now f(x_0) = f(2.1875) = -1.095 < 0.

The root is in the range of 2.1875 and 3 with the ultimate aim a = 2.1875 and b = 3 etc.
False position convergence method: we know that,

$$x_{n+1} = x_n - [\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}]f(x_n)$$

It is necessary for one of the foci, x0, or x1, to remain constant, while the remainder of the point varies from n in order for the function f(x) in the span that takes root in the equation f(x) = 0 to be convex (x0-x1). The function f(x) is represented as a straight line passing through the foci (x0,f0), where n = 1, 2,... and x0 is a fixed point.
The error equation (2) resulting from the convergence of the secant method is now known as the secant error equation:

$$\varepsilon_{n+1} = C\varepsilon_n\varepsilon_{n-1}\ldots\ldots(1)$$

By equation (1), we obtain:

$$\varepsilon_{n+1} = C\varepsilon_n\varepsilon_0$$

$$\varepsilon_{n+1} = C\varepsilon_n\varepsilon_0$$

Where $C = \frac{1}{2}\frac{f''(\zeta)}{f'(\zeta)}$ and $\varepsilon_0 = x_0 - \zeta$ is independent of k. Therefore we can write

$$\varepsilon_{n+1} = C'\varepsilon_n$$

The constant asymptotic error is C = C0. As a consequence, the rate of convergence of the fake position technique is linear.

2.3. Secant Method

The secant approach is just a variation of Newton's method. The fundamental idea is illustrated here.

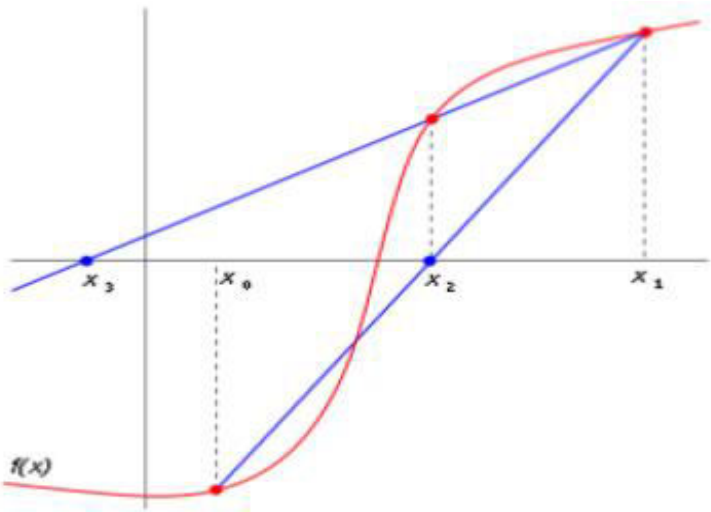

Figure 3: Secant Method

The Newton method of tackling a nonlinear equation f(x) = 0 is given by the iterative formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \ldots\ldots\ldots\ldots (1)$$

It is one of the disadvantages of Newton's method to have to evaluate the derivative of a function as one of the steps in the process. In order to address these shortcomings, the function f(xderivative)'s is approximated as

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}} \ldots\ldots\ldots (2)$$

By Putting equation (2) into (1), we obtain,

$$x_{n+1} = x_n - \frac{x_n f(x_n) - x_{n-1} f(x_n)}{f(x_n) - f(x_{n-1})}$$

By factoring out f(x_n)

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

This can also be written as

$$x_{n+1} = x_n - [\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}] f(x_n)$$

The solution to the preceding problem is provided by secant technology. This technique is similar to Newton-Raphson, with the exception of the fact that two previous assumptions are required.
As an example, consider the equation f(x) = x32x5=0, which may be solved using the secant technique.
It is recommended to utilize two initial estimates: x1=2 and x0=3.
The root is somewhere between 2 and 3 in number.

By secant method, first approximation is when i = 1; $x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$

when i = 2: $x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$

when i = 3: $x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)}$

when i = 4: $x_5 = x_4 - \frac{f(x_4)(x_4 - x_3)}{f(x_4) - f(x_3)}$

Secant Method Convergence: since we know that,

$$x_{n+1} = x_n - [\frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}] f(x_n) \ldots\ldots\ldots (1)$$

We assume that ζ is a simple root of f(x) = 0. Putting $x_n = \zeta + \epsilon_k$ in (1) we obtain

$$\epsilon_{n+1} = \epsilon_n - [\frac{(\epsilon_n - \epsilon_{n-1})}{f(\zeta + \epsilon_n) - f(\zeta + \epsilon_{n-1})}] f(\zeta + \epsilon_n)$$

Expanding f($\zeta + \epsilon_k$) and f($\zeta + \epsilon_{k-1}$) in taylor's series about the point ζ and noting that f(ζ) = 0 we get,

$$\varepsilon_{n+1} = \epsilon_n - \frac{(\epsilon_n - \epsilon_{n-1})[\epsilon_n f'(\zeta) + \frac{1}{2}\epsilon_n^2 f''(\zeta) + \cdots]}{(\epsilon_n - \epsilon_{n-1})f'(\zeta) + \frac{1}{2}(\epsilon_n^2 - \epsilon_{n-1}^2)f''(\zeta) + \cdots}$$

Or, $\varepsilon_{n+1} = \epsilon_n - [\epsilon_n f'(\zeta) + \frac{1}{2}\epsilon_n^2 f''(\zeta) + \cdots][1 + \frac{1}{2}(\epsilon_{k-1} + \varepsilon_k)f''(\zeta)/f'(\zeta) + \cdots]^{-1}$

Or, $\varepsilon_{n+1} = \frac{1}{2}\varepsilon_n\varepsilon_{n-1}f''(\zeta)/f'(\zeta) + 0(\varepsilon_n^2\varepsilon_{n-1} + \varepsilon_n^2\varepsilon_{n-1})$

Or, $\varepsilon_{n+1} = C\varepsilon_n\varepsilon_{n-1}$.........(2)

Where C = ½ $f''(\zeta)/f'(\zeta)$ and the higher forces of εn are ignored.

Where C = ½ $f''(\zeta)/f'(\zeta)$ and the higher powers of ε_n are neglected.

The relation of the form (2) is called the error equation. Keeping in mind the definition of convergence we want a relation of the form , $\varepsilon_{n+1} = A\varepsilon_k^p$.........(3)

Where A and p are to be determined.

From (3) we have $\varepsilon_n = A\varepsilon_{n-1}^p$ or $\varepsilon_{n+1} = A^{-1/p}\varepsilon_n^{1/p}$

Putting the value of ε_{n+1} and ε_{n-1} in (1)

$$\varepsilon_n^p = CA^{-(1+\frac{1}{p})}\varepsilon_n^{1+1/p}$$

Comparing the power of ε_n on both sides we get

p = 1+1/p which gives p = 1/2(1$\pm\sqrt{5}$).

Without respect to the negative sign, the convergence rate for the Secant technique is p = 1.618

2.4. Bisection Method

The method of bisection is very fundamental and is often employed in numerical approaches to find the root by solving nonlinear equations, which is why it is included in this list. Using the intermediate value theorem to look through all of the possible roots of a function takes a lengthy time (IVT). This method is often used since it is less complicated to monitor than other approaches. The underlying concept of this technique is explained in more detail below:

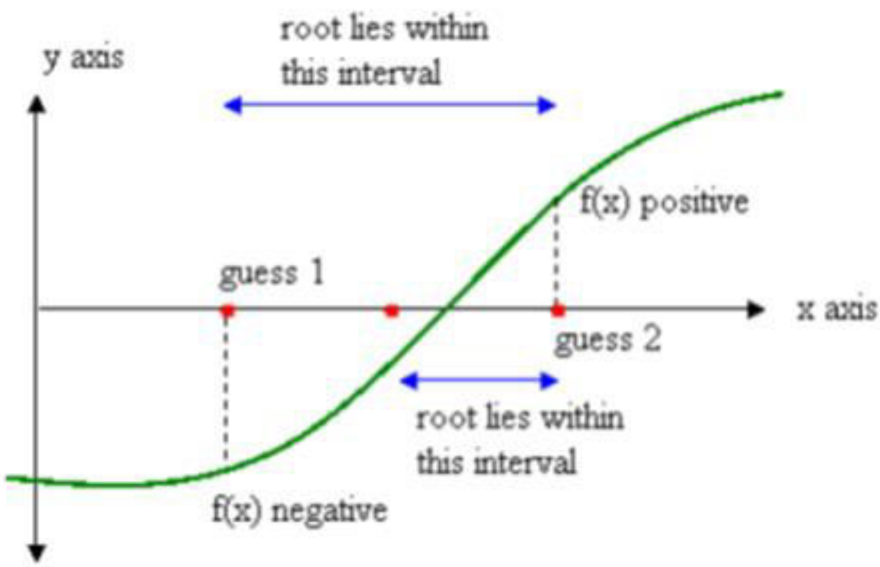

Figure 4: Bisection Method

If f(x) is persistent and the connection f(a)f(b) 0 between f(a) and f(b) is defined by the interval [a, b] as having opposing signs, then at least the root or null root of f(x) is described by f(a)f(b) 0, according to the Theorem for Intermediate Value (IVT) (a, b). To solve this problem, we must first identify two real numbers, called the root and the interval[a and b], that are both included inside the interval[a, b], and whose length at each step is approximately half of that of the starting interval. It is necessary to repeat this procedure until the interval is correct as a result of the required root. Because the interval contains several roots (a, b), the root should be distinct from the others. Additionally, the root converges in a linear and progressive manner. We may, however, choose to cease iterating when f(x0) is close to zero or very tiny. As a result, the full formula is provided:

$$x_n = \frac{a+b}{2}$$

Example: Solving the equation $f(x) = x^3 - x - 1 = 0$ by bisection method correct up to two decimal places.

Solution: Let $f(x) = x^3 - x - 1$

$\therefore f(1) = -1 < 0$

And $f(2) = 5 > 0$

Thus, at least one root of this equation lies between 1 and 2.

Let $x_0 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5.$

$f(x_0) = 0.875 > 0$

The root lies between 1 and 1.5 so forth.

Convergence of Bisection method:

Let us conventionally define the "approximation" at x_n after the n-th iteration as the midpoint

$$x_n = \frac{a_n + b_n}{2}$$

of I_n. Since the actual solution $f(c) = 0$ satisfies $c \in I_n$, we have

$$|x_n - c| \le 1/2|I_n|$$

where $|I_n|$ symbolizes the length of the interval I_n. Since the length of the current search interval gets divided in half in each iteration, we have,

$$|e_n| = |x_n - c| \le (\tfrac{1}{2})^n |I_0|$$

We interpret this behavior as linear convergence; although we cannot strictly guarantee that $|e_{n+1}| \le L\,|e_n|$ $(L < 1)$ at each iteration, then the convergence is called the first order and L is called the rate of convergence.

2.5. Comparison Of Convergence Rate Among These Four Methods

In order to determine the approximate speed with which issues may be resolved using any numerical technique, the arrangement of algebraic and transcendental equations is required.

The convergence rate is shown in the table below as a comparison.

Table 1: Comparison of convergence Rate

Method	Based on Equation	Convergence Rate
Newton-Raphson method	1^{st} degree	2
False Position method	1^{st} degree	1
Secant method	1^{st} degree	1.618
Bisection method	1^{st} degree	0.5

In this table, we see that Newton-Raphson has a preferable rate of convergence over different methods i.e., 2.

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3. Comparison and analysis

3.1. Graphical Development Of The Approximation Root Of These Methods To Achieve The Best Method

The graphical development of these four methods is shown bellow:

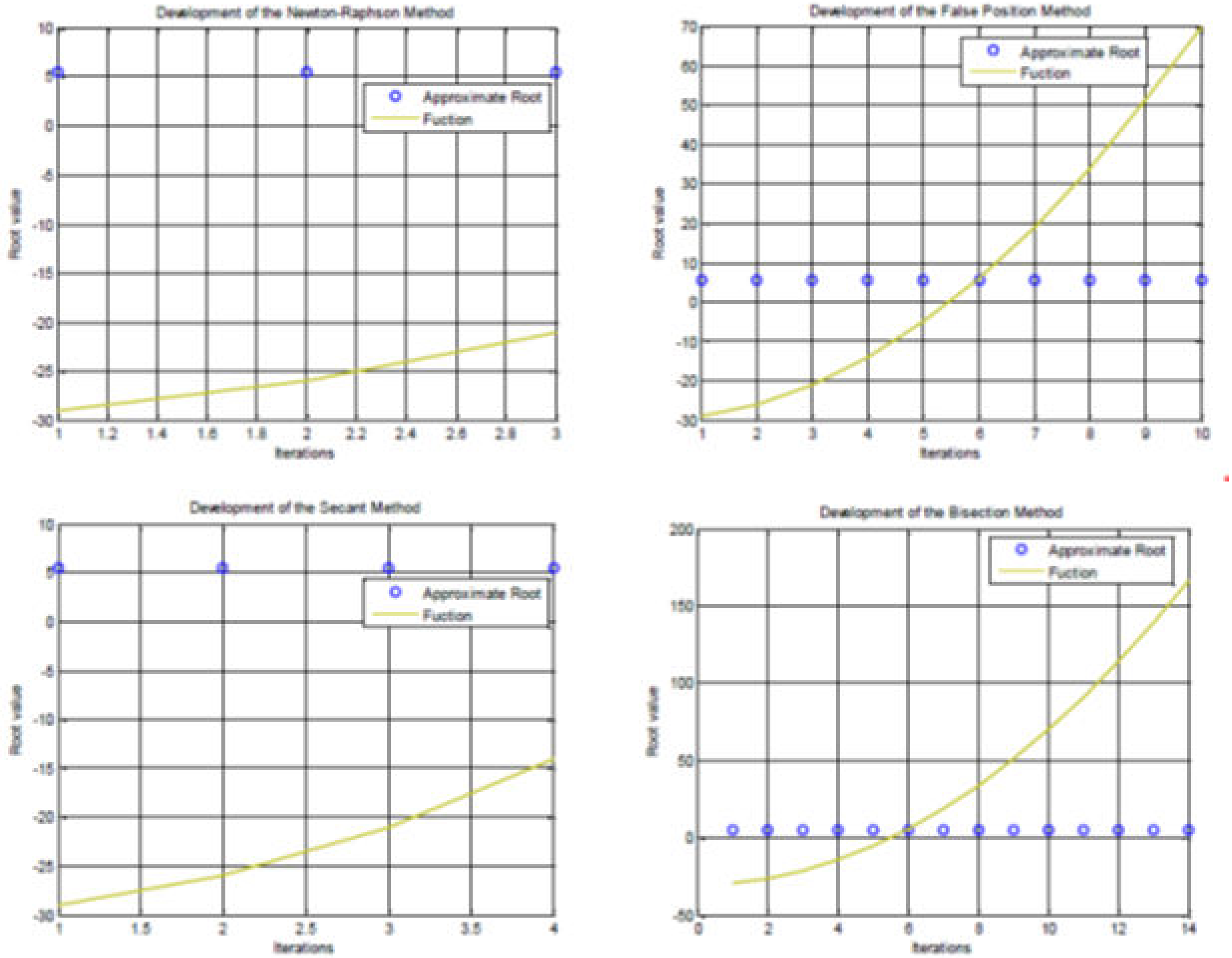

Figure 5: A graphical comparison of the four approaches.

The function employed is x2-30, which has a range of [5 6] and a slope error of 0.0001. Its range is [5 6] and its slope error is 0.0001. As seen in the chart above, the Newton-Raphson method shows the approximate root with just three iterations each iteration, since it converges more quickly and more correctly than other approaches, while other strategies postpone the equation's inclined roots for longer periods of time. Because of this, it takes a long time for the Bisection technique to gradually arrive at the optimum root as convergent. The Secant technique, as opposed to the Newton methodology, provides a higher degree of convergence, while the Newton methodology does not. False Position approaches converge more slowly than Newton and Secant approaches because at least ten iterations are needed before convergence can be achieved.

In order to compare the two methods, we will use just a few functions, including the 0.0001 error and the ideal data correctness. The findings obtained for the approximation root, errors, and iteration of the Newton-Raphson technique are summarized in Table 2, the false position method is described in Table 3, the Secant method is summarized in Table 4, and the Bisection approach is summarized in Table 5. The false position method is described in Table 3, the Secant method is summarized in Table 4, and the Bisection approach is summarized in Table 5. While the underlying numbers contain the required root, the Newton-Raphson algorithm only provides a limited amount of repetition.

Table 2: Data obtained by the Newton-Raphson Method

Type of Function	Approximate Root	Error value	Iterations
Linear: 3x+5	-1.6667	0.0000000	2
Quadratic: x^2– 12	3.46410	4.8917e-09	4
Cubic: x^3 —-48	3.6342	7.9622e-06	4
Trigonometric: 3x - cos (x) -1	0.60710	1.7597e-08	3
Exponential: $xe^x - 2$	0.85261	1.2179e-07	3
Logarithmic: $x^2 + lnx - 2$	1.31410	7.7547e-05	3
Combination: cos(x)-xe^x	0.51776	1.9482e-08	5
Combination:3x -$\sqrt{1 + sin\ (x)}$	0.39185	3.7502e-06	3

In the False position method, it needs a lot of iteration numbers to get the required root.

Table 3: Data obtained by the False Position Method

Type of Function	Approximate Root	Error value	Iterations
Linear: 3x+5	-1.66666	0.000000000000000	3
Quadratic: x^2– 12	3.464101	0.000000000001810	10
Cubic: x^3 —-48	3.634241	0.000000000000003	14
Trigonometric: 3x - cos (x) -1	0.60 101	0.000000000000156	9
Exponential: $xe^x - 2$	0.85260	0.000000000002899	12
Logarithmic: $x^2 + lnx - 2$	1.314096	0.000000000000001	17
Combination: cos(x)-xe^x	0.517757	0.000000000000061	15
Combination:3x -$\sqrt{1 + sin\ (x)}$	0.391846	0.000000000000877	8

We apply the secant method to get the outcome with a couple of quantities of iterations.

Table 4: Data obtained by the Secant Method.

Type of Function	Approximate Root	Error value	Iterations
Linear: 3x+5	-1.66666	0.0000000	2
Quadratic: x^2– 12	3.464101	1.3222e-05	4
Cubic: x^3 —-48	3.634241	2.0674e-07	5
Trigonometric: 3x - cos (x) -1	0.6071016	3.8551e-06	4
Exponential: $xe^x - 2$	0.8526054	1.2398e-05	5
Logarithmic: $x^2 + lnx - 2$	1.3140967	5.4035e-05	4
Combination: cos(x)-xe^x	0.51776	9.9055e-06	6
Combination:3x -$\sqrt{1 + sin\ (x)}$	0.39185	3.5903e-07	4

For the most part, the Bisection method gradually reveals the roots so long as the underlying values completely encircle the target root and the number of iterations is large enough.

Table 5: Data obtained by the Bisection Method.

Type of Function	Approximate Root	Error value	Iterations
Linear: 3x+5	-1.66664	0.000076293	16

Quadratic: $x^2 - 12$	3.464050	0.000355567	14
Cubic: x^3 —-48	3.634241	0.000223494	15
Trigonometric: 3x - cos (x) -1	0.607102	0.000164184	13
Exponential: $xe^x - 2$	0.852606	0.000109137	15
Logarithmic: $x^2 + lnx - 2$	1.314097	0.001207722	9
Combination: cos(x)-xe^x	0.517761	0.001956944	10
Combination:3x -$\sqrt{1 + sin\ (x)}$	0.391847	0.000076418	14

3.2. Comparison Of The Approximate Error

The following picture shows a comparison of four approaches with a 0.0001 accuracy in the interval [1 2] for the estimated error in finding the root of the function x4 - x - 10 with a 0.0001 accuracy in the interval [1 2].

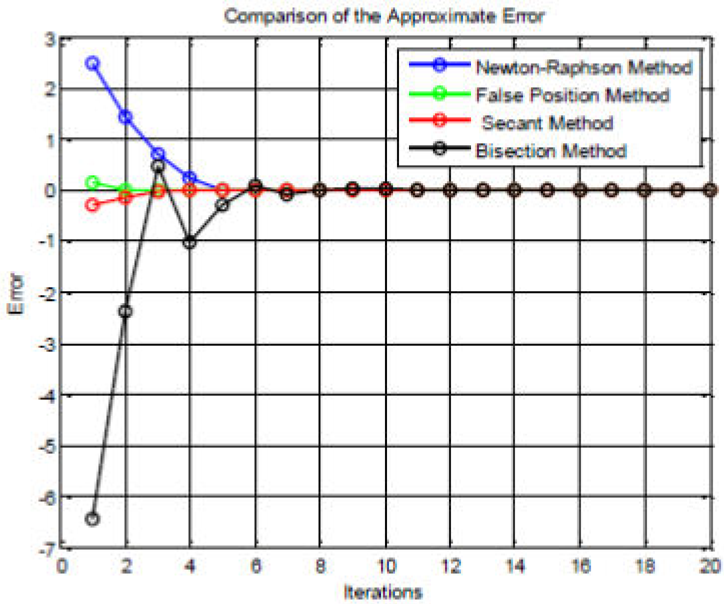

Figure 6: The estimated error is shown versus the number of iterations in this graph (1-20).

The following image illustrates a comparison of the estimated error in finding the root for function x4 - x - 10 using four interval[12] methods with a precision of 0.0001.

4. Test and results for varification of the best method

In the following cases, decipher the findings of the four nonlinear equation resolution methods, such as algebraic and transcendental equations.

Problem 1. Problem 1. Consider the equation [2 3], f(x) = x3-2x-5 = 0 Now start x0 = 2

Table 6: Iteration Numbers and the results obtained by these four methods.

N	Newton-Raphson Method	Error	False Position Method	Error	Secant Method	Error	Bisection Method	Error
1	2.12000	0.00544851	2.058823	0.03572795	2.05882	0.9411764	2.50000	5.6249937
2	2.09513	0.00001663	2.081263	0.01328782	2.08126	0.0224401	2.25000	1.8906187
3	2.09455	1.5 x 10^{-10}	2.089639	0.00491227	2.09482	0.0135604	2.12500	0.3456968
4	2.09455	0.00000000	2.092739	0.00181190	2.09454	0.0002747	2.06250	-0.351324

5	2.09455	0.00000000	2.093883	0.00066777	2.09455	0.0000020	2.09375	-0.008947
6	2.09455	0.00000000	2.094305	0.00024603	2.09455	0.0000000	2.10937	0.1668329
7	2.09455	0.00000000	2.094460	0.00009063	2.09455	0.0000000	2.10156	0.0785560
8	2.09455	0.00000000	2.094518	0.00003338	2.09455	0.0000000	2.09765	0.0347080
9	2.09455	0.00000000	2.094539	0.00001229	2.09455	0.0000000	2.09570	0.0128560
10	2.09455	0.00000000	2.094546	0.00000453	2.09455	0.0000000	2.09472	0.0019481
11	2.09455	0.00000000	2.094546	0.00000166	2.09455	0.0000000	2.09423	-0.003501
12	2.09455	0.00000000	2.094546	0.00000061	2.09455	0.0000000	2.09448	-0.000777
13	2.09455	0.00000000	2.094546	0.00000022	2.09455	0.0000000	2.09460	0.0005854
14	2.09455	0.00000000	2.094546	0.00000008	2.09455	0.0000000	2.09454	-0.000095
15	2.09455	0.00000000	2.094546	0.00000003	2.09455	0.0000000	2.09455	0.0002448
16	2.09455	0.00000000	2.094546	0.00000001	2.09455	0.0000000	2.09455	0.0000745
17	2.09455	0.00000000	2.094546	0.00000000	2.09455	0.0000000	2.09455	-0.000010
18	2.09455	0.00000000	2.094546	0.00000000	2.09455	0.0000000	2.09455	0.0000319
19	2.09455	0.00000000	2.094546	0.00000000	2.09455	0.0000000	2.09455	0.0000106
20	2.09455	0.00000000	2.094546	0.00000000	2.09455	0.0000000	2.09455	0.0000000

In addition, we can take a gander at the accompanying approximate error graph acquired from these four methods which illustrate the difference in errors starting with one method then onto the next method.

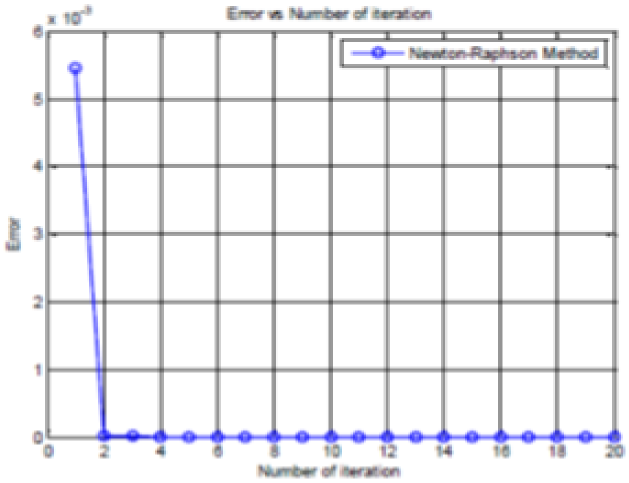

Figure 7. The graph of NRM

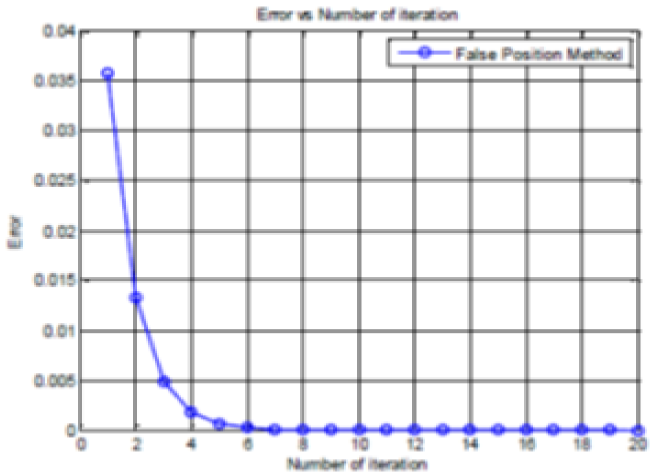

Figure 8. The graph of FPM

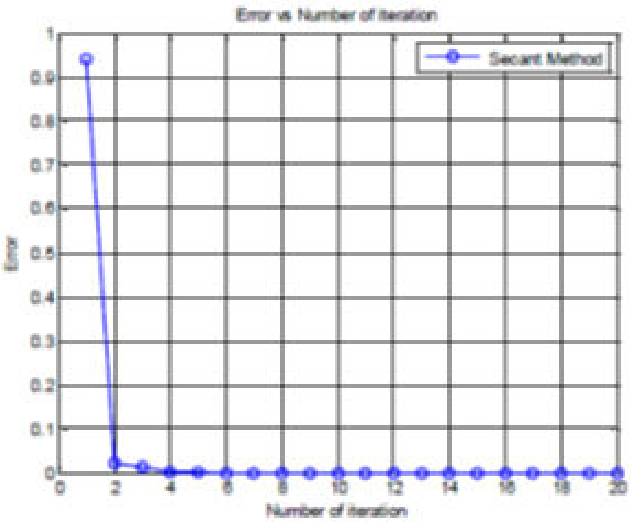

Figure 9. The graph of SM

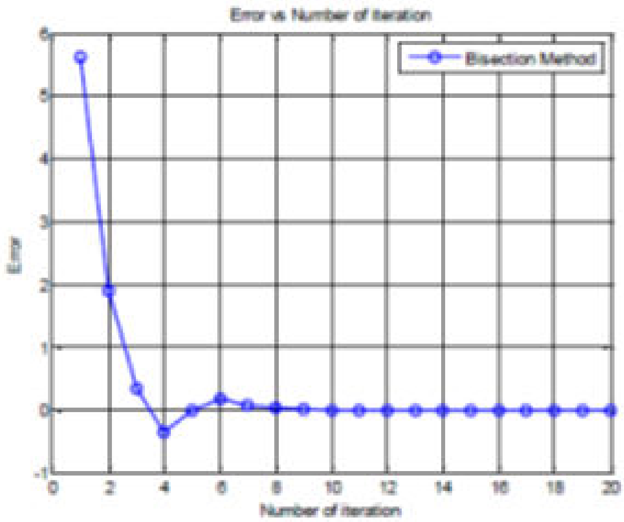

Figure 10. The graph of BM

When we compare the two graphs, the Newton-Raphson method is better than the others.
Problem 2 is also an issue. Consider [—1 - 2], f(x) = sin x – 1- x3 = 0
This x0 = -1

Table 7: Iteration Numbers and the results obtained by these four methods.

N	Newton-Raphson Method	Error	False Position Method	Error	Secant Method	Fimr	Bisection Method	Error
1	-1.27636	-0.09305126	-1.12138	-0.1276658	-1.12138	0.8786136	-1.50000	1.377501
2	-1.24974	-0.00738636	-1.18687	-0.0621759	-1.18687	0.0654899	-1.2500	0.004136
3	-1.24905	-0.00005222	-1.21958	-0.0294663	-1.25793	0.0710594	-1.1250	-0.47844
4	-1.24905	-2.6×10^{-9}	-1.23527	-0.0137771	-1.24849	0.0094389	-1.18750	-0.25288
5	-1.24905	0.000000000	-1.24265	-0.0064000	-1.24904	0.0005506	-1.21875	-0.12840
6	-1.24905	0.000000000	-1.24608	-0.0029641	-1.24905	0.0000047	-1.23437	-0.06315
7	-1.24905	0.000000000	-1.24768	-0.0013708	-1.24905	0.0000000	-1.24218	-0.02976
8	-1.24905	0.000000000	-1.24841	-0.0006335	-1.24905	0.0000000	-1.24609	-0.01287
9	-1.24905	0.000000000	-1.24875	-0.0002926	-1.24905	0.0000000	-1.24804	-0.00438
10	-1.24905	0.000000000	-1.24891	-0.0001351	-1.24905	0.0000000	-1.24902	-0.00012
11	-1.24905	0.000000000	-1.24891	-0.0000624	-1.24905	0.0000000	-1.24951	0.002002
12	-1.24905	0.000000000	-1.24891	-0.0000287	-1.24905	0.0000000	-1.24926	0.000936
13	-1.24905	0.000000000	-1.24891	-0.0000132	-1.24905	0.0000000	-1.24914	0.000403
14	-1.24905	0.000000000	-1.2489	-0.000006	-1.2490	0.0000000	-1.24908	0.000137

			1	0	5			
15	-1.24905	0.0000000 00	- 1.2489 1	- 0.000002 7	- 1.2490 5	0.000000 0	-1.24907	0.000004
16	-1.24905	0.0000000 00	- 1.2489 1	0.000000 0	- 1.2490 5	0.000000 0	-1.24907	-0.00006
17	-1.24905	0.0000000 00	- 1.2489 1	0.000000 0	- 1.2490 5	0.000000 0	-1.24907	-0.00002
18	-1.24905	0.0000000 00	- 1.2489 1	0.000000 0	- 1.2490 5	0.000000 0	-1.24907	-0.00001
19	-1.24905	0.0000000 00	- 1.2489 1	0.000000 0	- 1.2490 5	0.000000 0	-1.24907	0.000000
20	-1.24905	0.0000000 00	- 1.2489 1	0.000000 0	- 1.2490 5	0.000000 0	-1.24907	0.000000

We may also take a diagram of the following approximate error chart from the four techniques, which illustrates the error difference from one method to the next.

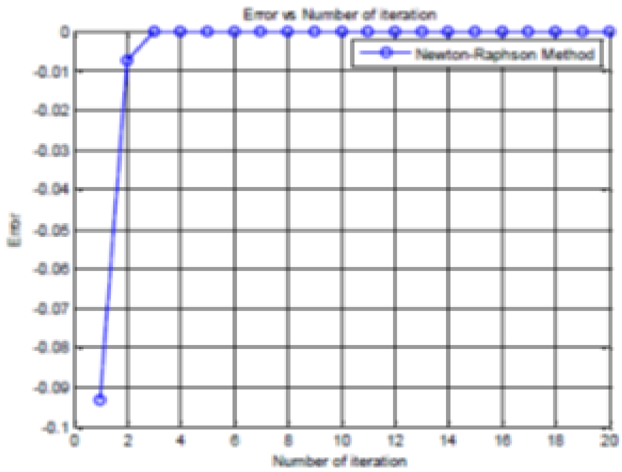

Figure 11. The graph of NRM

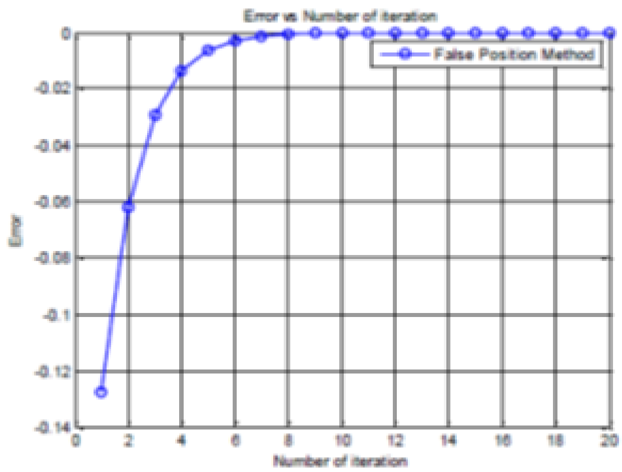

Figure 12. The graph of FPM

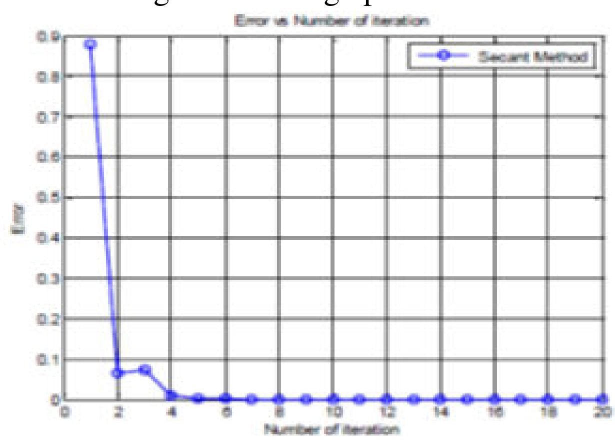

Figure 13. The graph of SM

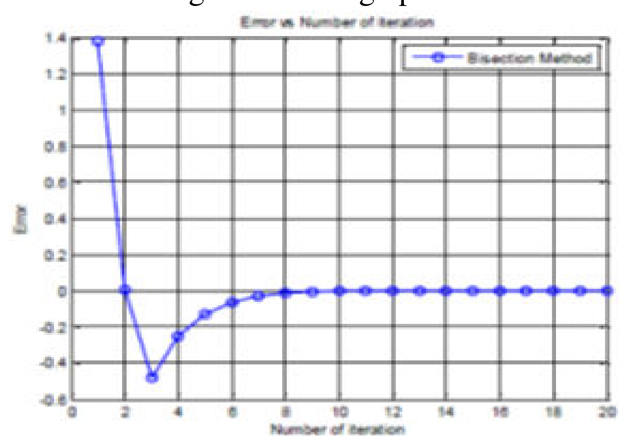

Figure 14. The graph of BM

The graph above shows that Newton-Raphson is preferred above others.

5. Conclusion

The purpose of this study was to examine and evaluate four iterative techniques for the resolution of non-linear equations in order to suggest a more effective strategy. The results of the MATLAB analysis are also given in order to confirm that the optimal technology is being used. By examining the estimated error chart, we can conclude that the Newton technique is the most dependable and competent solution to the nonlinear problem. In addition, when compared to other techniques, Newton's methodology requires fewer iterations and has a shorter handling time. Finally, Newton's technique is always suggested as the best and most accurate strategy for dealing with nonlinear issues since it is the most efficient and accurate.

References

1. Richard L. Burden, J.douglas Faires, Annette M. burden, “Numerical Analysis”, Tenth Edition, ISBN: 978-1-305-25366-7.
2. R. Vasishtha, Vipin Vasishtha, “Numerical Analysis”, Fourth Edition.
3. S.S. Sastry, “Introduction Methods of Numerical Analysis”, Fourth Edition, ISBN: 81-203-2761-6.
4. Shin Min Kang, Waqas Nazeer, “Improvements in Newton-Rapson Method for Nonlinear equations using Modified Adomain Decomposition Method”, International Journal of Mathematical Analysis, Vol. 9, 2015, no. 39, 1919.1928.
5. Muhammad Aslam Noor, Khalida Inayat Noor, Eisa Al-Said and Muhammad Waseem, “Some New Iterative methods for Nonlinear Equations, Hindawi Publishing Corporation.
6. Okorie Charity Ebelechukwu, ben Obakpo Johnson, Ali Inalegwu Michael, akuji Terhemba fideli, “Comparison of Some Iterative methods of Solving Nonlinear Equations”, International Journal of Theroretical and Applied Mathematics.
7. Ogbereyivwe Oghovese, Emunefe O. John, “New Three-Steps Iterative Method for Solving Nonlinear Equations, IOSR Journal of Mathematics.
8. Masoud Allame, Nafiseh azad, “A new method for solving nonlinear equations by Taylor expansion”, Application of Mathematics and Computer engineering.
9. Jishe Feng, “A New Two-step Method for solving Nonlinear equations”, International Journal of Nonlinear Science”, Vol.8(2009) No.1pp.40-44.
10. Masoud Allame and Nafiseh Azad, “On Modified Newton Method for solving a Nonlinear Algebraic equation by Mid-Point”, World applied science Journal 17(12): 1546-1548, 2012.
11. Mohammad Hani Almomani, Mahmoud Hasan Alrefaei, Shahd Al Mansour, “A method for selecting the best performance systems”, International Journal of Pure and Applied Mathematics.
12. Zolt ´an Kov´acs, “Understanding convergence and stability of the Newton-Raphson method”, https://www.researchgate.net/publication/277475242.
13. Fernando Brambila Paz, Anthony Torres Hernandez, Ursula Iturrarán-Viveros, Reyna Caballero Cruz, “Fractional Newton-Raphson Method Accelerated with Aitken’s Method”.
14. Robin Kumar and Vipan, “Comparative Analysis of Convergence of Various Numerical Methods”, Journal of Computer and Mathematical Sciences, Vol.6(6),290-297, June 2015, ISSN 2319-8133 (Online).
15. Nancy Velasco, Dario Mendoza, Vicente Hallo, Elizabeth Salazar-Jácome and Victor Chimarro, “Graphical representation of the application of the bisection and secant methods for obtaining roots of equations using Matlab”, IOP Conf. Series: Journal of Physics: Conf. Series 1053 (2018) 012026 doi :10.1088/1742-6596/1053/1/012026.

16. Kamoh Nathaniel Mahwash, Gyemang Dauda Gyang, “Numerical Solution of Non-linear Systems of Algebraic Equations”, International Journal of Data Science and Analysis2018; 4(1): 20-23 http://www.sciencepublishinggroup.com/j/ijdsa.
17. Farooq Ahmed Shah, and Muhammad Aslam Noor, “Variational Iteration Technique and Some Methods for the Approximate Solution of Non-linear Equations”, Applied Mathematics& Information Sciences Letters An International Journal, http://dx.doi.org/10.12785/amisl/020303.
18. M. A. Hafiz & Mohamed S. M. Bahgat, “An Efficient Two-step Iterative Method for Solving System of Non-linear Equations”, Journal of Mathematics Research; Vol. 4, No. 4; 2012 ISSN 1916-9795 E-ISSN 1916-9809.