Training math teachers in Panamá: A mixed research

Dr. L. Morales Maure[a], V. Font[b], R. Duran Gonzalez[c], E. Garcia Vasquez[d]

[a]Department of Mathematics, Universidad de Panamá, SNI-I, SENACYT, Panama, e-mail: luisa.morales@up.ac.pa

[b]Departament ofLinguistic,Literatic, Didactic, CCEE and Mathematics Education, University of Barcelona, Spain,

e-mail:vfont@ub.edu

[c]Academic Area of Education Science, Autonomous Universityof Hidalgo State, Mexico, e-mail:rdurango@uaeh.edu.mx

[d] Faculty of Education Sciences, University of Panama, Panama, email:evelyn.garciav@up.ac.pa

Abstract: The results of an evaluation process of a degree in teaching strategies implemented with elementary school mathematics teachers in Panama are presented. The degree course involved activities developed over 4 months, based mainly on the problem-solving approach, contextualized activities and processes of reflection on teaching practice. The purpose was to provide teachers with tools for the design, implementation and improvement of their didactic sequences. To evaluate the impact of this degree, the logical model of evaluation was used and the results were analyzed in quantitative and qualitative areas. As a general result, reflective processes were presented and teachers were able to evaluate their teaching abilities, as well as their attitudes and motivations regarding mathematics teaching, strengthening their disciplinary knowledge and didactic reflection.

Key words: diploma, mathematicsteaching,evaluation.

1. Introduction

Many students fail to be successful in mathematics, not because of a lack of innate abilities, but because of the inadequacy of the pedagogical practices used in their education and the climate generated in the classroom (**Zevenbergen, Mousley& Sullivan, 2004**; **Marzano & Pickering, 2009).** According to **Štech (2006)**, mathematics taught in schools is a "highly abstract exercise of the mind that serves toclassify children as *talented* or not" and exhibits a lack of strategies that are useful in daily life. This situation contravenes the goal of the school as a socializing agent, as it affects its contribution to the development of mental functions and even more, to the personality of children. The process of transformation of mental functions, required in mathematics, merits an approach to daily life where students observe, discover and argue. This proximity is achieved when there is a teacher with a plan, strategies and a curricular logic that supports his action.

In this perspective, teacher training actions become relevant. Training enhances teaching skills, encourages the desire to teach, generates satisfaction with the learning experience and serves as a multiplying agent in the teaching of their students (**Lieff et al., 2012**). Cognitive empowerment contributes to strengthen teacher effectiveness, motivation and commitment and allows them to have a significant impact in the classroom (**Flores & Day, 2006**).

In view of the devaluation of mathematical knowledge on the part of teachers and even the educational system, the incentive to study and, therefore, to strengthen mental functions from the primary level decreases, a fact that is exacerbated for students in inclusive education (**Dewsbury, 2017**; **Kahn &Behlol, 2014**). Thus, the training of primary school teachers, in terms of mathematics didactics and the strategies they implement, continues to be a repetitive and persistent topic within the educational field.

To this end, it is essential to develop an external programmatic evaluation that qualitatively and quantitatively evidences the achievement of the general objective and the specific objectives proposed in the programs of competence in mathematics didactics offered to active teachers in the profession. In addition to the above, as part of the commitment to accountability, evaluation is one of the processes that allows us to know the scope, achievements and challenges of teacher training in Panama and to influence decision making for teacher training in mathematics.

The fact that socio-cultural factors are exerting a strong influence on the pedagogical conception and practice of teaching would partly explain the increase in the number of research studies on the conditioning sciences of education (biological, psychological, social, economic, among others) in recent decades. These sciences are dedicated to the study of the characteristics of educators and their students as well as the conventions, norms and organization of educational institutions (**Ball, 2000**; **Lewis, 2002**; **Fernández & Yoshida, 2004**; **Wang-Iverson & Yoshida, 2005**; **Ribeiro, 2011**; **Pochulu, Font & Rodríguez, 2016**; **Hummes, Font & Breda, 2019**; **Godino, Batanero& Font, 2020**, **Vanegas, Font & Pino-Fan, 2019**, **García Marimón et al, 2021**).

In this contribution, we characterize the evaluation of the Program: Didactic Strategies for the Teaching of Mathematics (DSTM), which was implemented in 2018 with 150 in-service teachers of basic education in Panama.

2. Background

DSTM Diploma Course, the target of evaluation of this contribution, was a program developed and implemented for the year 2018 with the institutional support of the University of Panama, the Ministry of Education-MEDUCA (Aval 1097 and 038/PC/2018 respectively) and the National Secretariat of Innovation and Technology (SENACYT).

This initiative was based on international and national guiding documents that seek full compliance with the principles of progress, equity and diversity. It is especially aligned with Agenda 2030 objective 8, approved in September 2015 by the General Assembly of the United Nations. This Agenda capitalizes on Quality Education, whose Sustainable Development Goals (SDGs) aim to Ensure inclusive, equitable, quality education and promote lifelong learning opportunities for all. It also advocates the development of values such as: dignity, justice, respect and human rights, as well as the full realization to contribute to an empowered, tolerant, open and inclusive society where the needs of the most vulnerable are met (**UnitedNations, 2015**).

In a pilot study conducted during the period from March 9 to June 19, 2017, a multi-case methodology was used with nine (9) teachers from the Omar Torrijos Herrera Elementary School, located in Panama Centre-Educational Region. With prior approval from school authorities and the MEDUCA, participants were exposed to a wide range of pedagogical activities and mathematical solution tasks involving the active use of technology, under a blended learning modality. The determination to implement a pilot study as a preliminary phase aims to identify the real needs of the population to be impacted and to plan strategies in line with their interests, motivation and didactic needs. This methodology makes it possible to make core actions feasible when formally developing the Diploma Course, which increases the possibility of success, reduces desertion among participants and increases the achievement of the project's objectives.

As part of the pilot study, a content analysis was made of the graduate profiles of two of the main higher education institutions offering teacher training in Panama. The teaching competencies established by MEDUCA in its guiding documents were also analyzed. This analysis gave rise to a Conceptual Thematic Matrix (**Morales-Maure, 2019**) aligned to six (6) areas of adequacy that teachers should have when teaching mathematics. An important limitation was found due to the little or non-existent emphasis given to the areas of: foreign language, digital competence, mathematical competence, innovation and initiation to research.

In the same context, an analysis of the statistics on professional training offered by MEDUCA to teachers yielded very revealing results on the priority given to mathematics didactics in the Panamanian territory. The evidence analyzed shows a distribution of training by subject with barely any teacher training offered in the mathematics area.

3. Theoretical frameworks

The theoretical framework of this follow-up work is the model of mathematical didactic competencies and knowledge of the mathematics teacher known as the DMKC model, which is based on constructs of the Onto-semiotic Approach - OSA (**Godino, Batanero, Font &Giacomene, 2016**; **Godino, Giacomone, Batanero& Font, 2017, Esqué de los Ojos& Breda, 2021, Morales-Maure, Durán-González y GarcíaVázquez, 2019; Morales-Maure, Durán-González, Pérez-Maya y Bustamante, 2019, Pochulu, Font y Rodríguez, 2016;Seckel, 2016; Seckel y Font, 2020**).

As a result of this approach, some theoretical tools have been developed that allow the analysis of mathematical activities in the processes of teaching and learning mathematics. Based on these tools, a typology of interconnected knowledge and competences has been set up.

In the DMKC model, competencies are understood as effective actions performed in a given context, or in relation to the design of content, in order to achieve a given purpose (see Fig. 1).

Figure 1 shows how competencies are developed and evaluated. Let us look at an example: to characterize the problem-solving competency, a level of development is marked and indicators are placed for each of the levels of development. In order to develop and evaluate it, tasks must be proposed, which in this case would be problems. The problems activate the competencies (see Figure 1) and can manifest the level of competency through actions or practices that solve the proposed tasks.

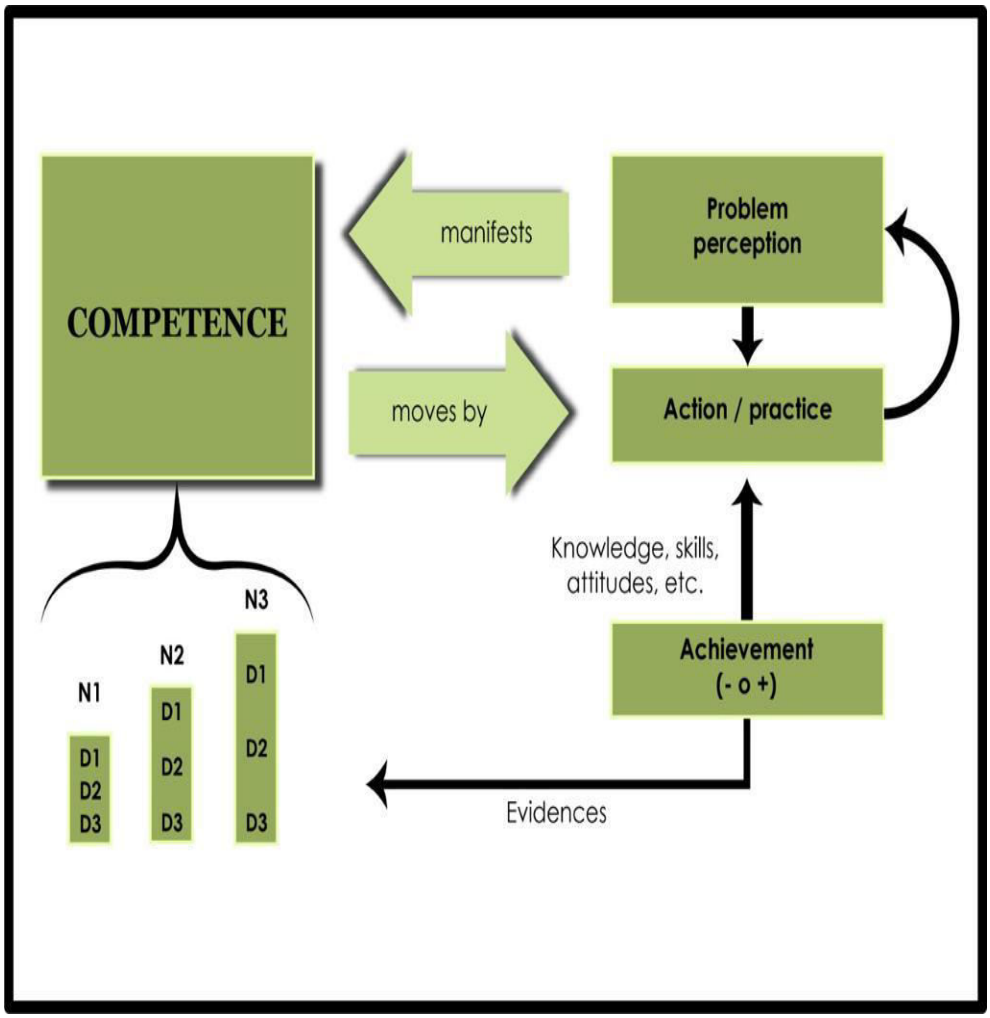


Figure 1. Competencies under the DMKC model. Source: **Seckel & Font, 2015**

It is important to note that this model provides a theoretical tool for the interpretation of mathematics teacher competencies and knowledge.

5. Materials and methods

About the program's contents

The DSTM program was designed to offer training in mathematics didactics to 150 active teachers at the elementary and preschool extended day levels of the official Panamanian education system, who work in 12 schools in six educational regions. The development activities were implemented over a period of four months, for a total of 240 hours with a ratio of 170 virtual hours and 70 classroom hours.

The degree program, which is the object of this evaluation and follow-up analysis, is based on strategies that promote problem solving as the main source of mathematical knowledge generation. To this end, reflection on practice and the presentation of educational activities that motivate the approach, resolution and design of problems as the axis that articulates the contents are privileged. It also focuses on the scope and depth of the contents and the understanding obtained when they are modeled in different contexts. The Program holds that these strategies of reflection and contextualized activities lead to the understanding of the problem situations that occur in the classroom and give meaning to the mathematical contents that are worked on in elementary school.

As a relevant strategy, the course encourages participants to develop activities and didactic sequences applicable to their classrooms, using as a generator the access obtained by being exposed to materials with a new didactic approach to the teaching of mathematics in elementary school.

The modality of this program was of a blended learning nature with sessions in which the participant attended classes for a period of approximately two (5) weeks per module, with the purpose of clarifying doubts, whether technological or of content. This way of study was complemented with an interactive modality that allowed access from the participant's computer equipment, in the time and space convenient for the teacher. In this case, the program recommended three (3) hours of study per day. Technological support was provided through the Moodle learning platform, developed by the Virtual Campus of the University of Panama, with the purpose of responding to the real needs of the participants.

Each participant and institution gave full and voluntary informed consent to the activities and purposes of the research. Informed consent was given after the researcher provided sufficient information to allow teachers to understand the implications of their participation and to reach a fully informed, considered and free decision about whether or not to participate in the study, without any pressure or coercion.

The instructional module strategy was a key factor within the course. A total of four (4) modules made up the mathematical content of the Program. The modules were structured by teachers specialized in mathematics didactics, resources that in turn offered virtual mentoring to the participants. The sessions included the topics shown in Chart 1. Each module contributed a particular perspective on mathematics in terms of knowledge, skills and abilities, and attitudes towards mathematics teaching. The purpose of each module and its mathematical content is presented below.

Chart 1. Modules that made up the program.

Modules				Points
	total	theoretical	practical	
Module I- Introduction to the Maths Didactics	48	8	40	1
Module 2- Mathematics and its Didactics I	48	8	40	1
Module 3- Mathematics and its Didactics II	60	10	50	1
Module 4- Intervention and reflection on one's own practice	48	8	40	1
Total	204	34	170	4

The general objective of the modules that make up the Degree program is that participating teachers improve their level of mastery of the mathematical content that is the subject of study in elementary school, and develop competencies to design and implement increasingly better teaching and learning processes in mathematics.

It is assumed that, with a better preparation in mathematics and a better understanding of the approaches to learning and teaching proposed in the current basic education curriculum of **MEDUCA (2014)**, teachers will improve their teaching practice and, consequently, will achieve a significant improvement in the quality of mathematics education received by children in school.

One of the cores of the training in mathematics didactics is the analysis of the teaching and learning process of mathematics for elementary school, according to the current curriculum that allows its planning, implementation, evaluation and improvement. For this reason, this Degree is organized in four modules. The first, brief and general, is of an introductory nature (Introduction to Educational Mathematics), which presents what didactics is and what it deals with, the purpose of teaching mathematics and the mathematics learning process, as well as the

role of the teacher and teaching models; this block ends with the presentation of the curriculum and its concretion in sequences of tasks. The second and third modules aim to provide the most relevant mathematical complements for teaching mathematics in elementary school, as well as didactic knowledge for teaching and learning the different blocks of mathematical contents of the elementary school curriculum.

The mathematical complements are taught, on one hand, with emphasis on mathematical processes and, on the other hand, in the deepening in the study of the contents of the elementary curriculum with a historical perspective. The didactics of the contents analyzes the key concepts and their construction, as well as the methodologies for their teaching and learning. It also deals with the use of appropriate contexts, the connections between each block and the rest of the mathematical contents of the curriculum, the presentation made in textbooks and other materials, as well as the learning difficulties and the most frequent mistakes made by students. Reflection on the material and technological resources and on a good evaluation is also present in order to reduce cognitive obstacles, get closer to the students' reality, understand the value of mathematics and allow for interdisciplinarity.

It is relevant to note that, for the selection and adaptation of the contents, the strategic plans of MEDUCA2015-2019 (axes 1 and 5) (**MEDUCA, 2014a**), the strategic plan of the University of Panama 2016-2021 (**UP, 2016**), and the national plan PENCYT 2015-2019 (**SENACYT, 2015**) were considered, specifically in the program for the development of science and scientific capabilities. Based on this alignment, the strategies of the program and the objective of the degree program were designed, which focused on providing preschool and elementary school teachers with professional training to enable them to design, implement, evaluate and improve didactive sequences, adapted to the different contexts in which mathematics teaching is carried out at the elementary level, in order to provide suitable teaching.

About the evaluation sampling process

For the qualitative phase, a politically relevant sample of participating teachers was used in the Educational Regions of Cocle, Azuero and Veraguas, in order to collect data from primary sources that would allow listening to the voices, testimonies, events, perceptions, and transformations resulting from the improvement of mathematics teaching, and the impact on student learning from the teacher's perspective. Three of the six provinces were selected because they have the largest number of participating teachers per school, and because they are diverse in terms of multi-grade schools, regular schools and educational centers. These particularities make it possible to make the voices of each sector visible.

It should be noted that the qualitative approach is also quantifiable, as it uses the code unit of analysis, a hermeneutic and interpretative methodology that exposes data saturation and serendipity in a quantifiable way, as well as providing evidence of the impact and transformation of classroom teaching through evidences, datas and testimonies. Given this fact, the quantitative phase used convenience sampling in the educational regions of Cocle, Azuero and Veraguas with the purpose of enriching the data. The established methodology stipulated the collection of 5 teachers per region for a total of 30 teachers. Due to the fact that the criterion for the selection of schools was subject to those with the greatest number of participants, a politically relevant sample of 18 teachers was finally obtained. This particularity does not affect the result, since the participating voices are those with the greatest knowledge of the event being evaluated.

Stages of the external evaluation

It covers 4 stages: planning, development, implementation and delivery of the final report. It includes the participation of external peers and project collaborators for field work.

The stages followed are detailed below: stage 1 involved the construction of methodological references and the design of instruments, which were later approved by the Bioethics Committee of the University of Panama. The evaluation team was trained and the logistics of the fieldwork were planned. For stage 2, which consisted of fieldwork, data collection was carried out on site, corresponding to the 3 provinces selected under the criterion of the greatest number of teachers participating in the campus. Four focus groups were implemented with the participation of 3 to 9 teachers per province. In stage 3, the battery of qualitative and quantitative data was organized, systematized and analyzed using EXCEL software. This phase included socialization among peers about the training and the recovery of information from primary sources. In stage 4, the final report was written.

Techniques and instruments applied

The focus group technique and documentary review were used to collect qualitative data. A guide of 12 open questions was structured based on the general objective and the specific objectives of the course. The focus group technique was conducted in each educational region with groups of primary school teachers participating in the program. A total of four focus groups were conducted. In harmony with the fundamentals of qualitative research, the size of the groups ranged from 3 to 9 participants. The technique was conducted by two external evaluators in

charge of collecting the participants' input. Each focus group was audio-recorded with the consent of the teachers and supplemented with the notes taken in manuscript, which subsequently resulted in four official transcripts. All transcripts were coded (**Saldaña, 2013**) and subjected to data reduction using the Wolcott method (**Lucca & Berrios, 2009**) and from there categories and subcategories emerged. The research included methodological triangulation and triangulation of sources, so that the data obtained in both procedures were susceptible to contrast and enrichment. All focus group interventions were conducted in private spaces to ensure the confidentiality of the process.

The documentary review technique was used in the process of examining videos and fundamental documents of the Degree. All the instruments, including the protocol, the bioethics qualifications and certifications of the evaluators and the informed consent document, were submitted to the Bioethics Committee of the University of Panama prior to the implementation of the external evaluation.

For the quantitative phase, a questionnaire consisting of 23 closed-ended items divided into three variables was used to collect data on: mathematical knowledge, mathematical skills and abilities and attitude towards mathematics teaching, aligned with the general and specific objectives of the Degree. The instrument included a five-point Likert Scale with degrees of agreement to disagreement and scores ranging from 5 to 0. The questionnaire includes a section for the collection of sociodemographic data that allowed characterizing the profile of the participants. These include variables such as: age, gender, educational region, years of service, ethnic group to which they belong and motivation for taking the refresher course, among other variables. The data were analyzed using descriptive statistics and the results were illustrated with histograms and pie charts. All the techniques employed were conducted under an agreement of voluntariness, anonymity and confidentiality.

6. Results

Population attended

A total of 150 teachers from 6 educational regions attended the course: Cocle, Colon, Azuero, Veraguas, Central Panama and West Panama. Their academic preparation is shown in the Figure 2.

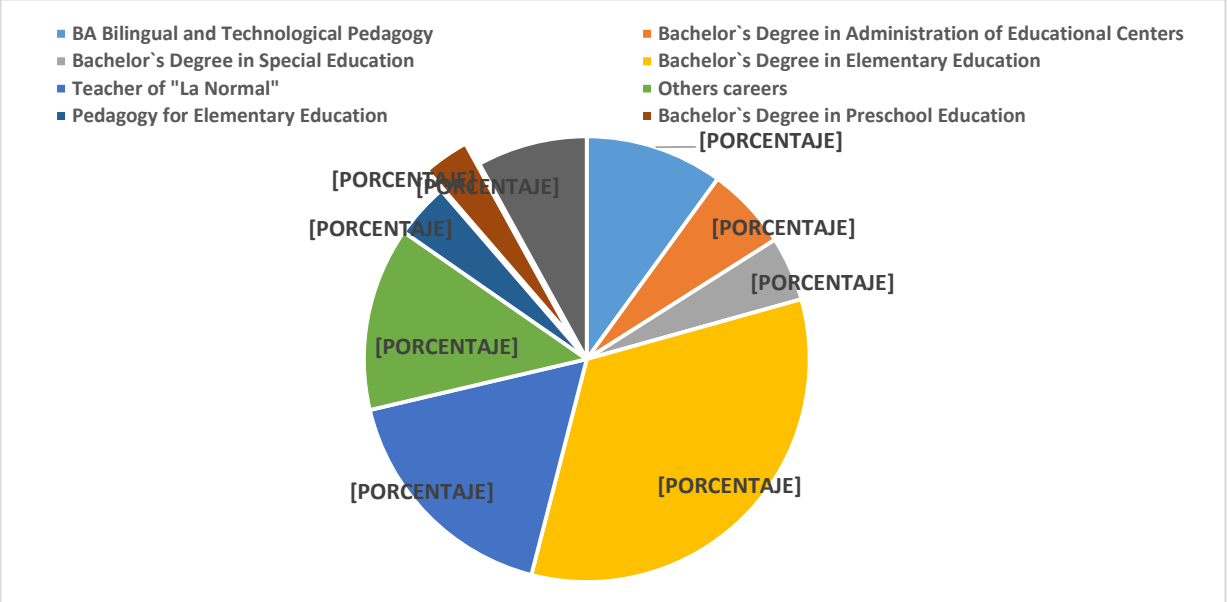


Figure 2. Frequency distribution by career of participant teachers.

It is important to point out that the largest percentage of participants is theBachelor's degree in Elementary Education (34%), followed by Teacher of "La Normal" (17%) and training related to the education profile, who have the required pedagogical and didactic theoretical background. On the other hand, the following graph shows the distribution of the careers to which the participants' teachers belong. It was identified that 13% of the teachers had non-pedagogical academic preparation and were far from what we consider mathematical didactic knowledge. This finding is worrying due to the impact it can have on children's education.

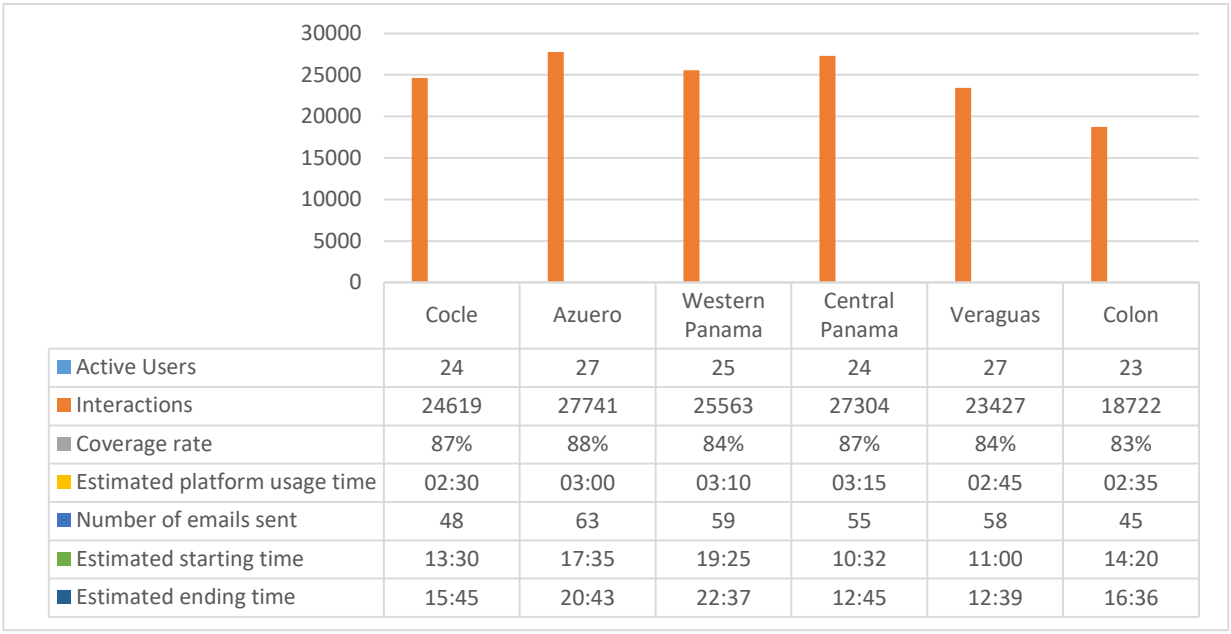
This device was accessible to all preschool and elementary school teachers in order to strengthen their competencies in information technologies and with the idea of responding to the real needs of the participants. The gain in the teachers' technological mastery was evident during the implementation of the course, which can be seen in the delivery of assignments through forums and e-mails. Graph 1 illustrates the use of the platform by region, which contributes to an updated teacher training in line with contemporary education.

Even though there is evidence of an increase in the use of technology (Graph 1), some educational regions showed limited interaction on the platform, but they did comply with the number of mandatory assignments. This is due to the fact that there are still pedagogical practices based on traditionalism, where the teacher requires the printed document in order to execute the assignment. The coverage rate (72 tasks or instructions) is due to the non-mandatory instructions (15 tasks) in which the Azuero region fulfilled these tasks more consistently than the other regions.

The face-to-face sessions were held at the Center for Innovation, Technological Development and Entrepreneurship (CIDETE by its Spanish acronym), at the end of each module the participant teachers attended class where they developed tasks from a computer, in their own time and space (3 hours per day on average).

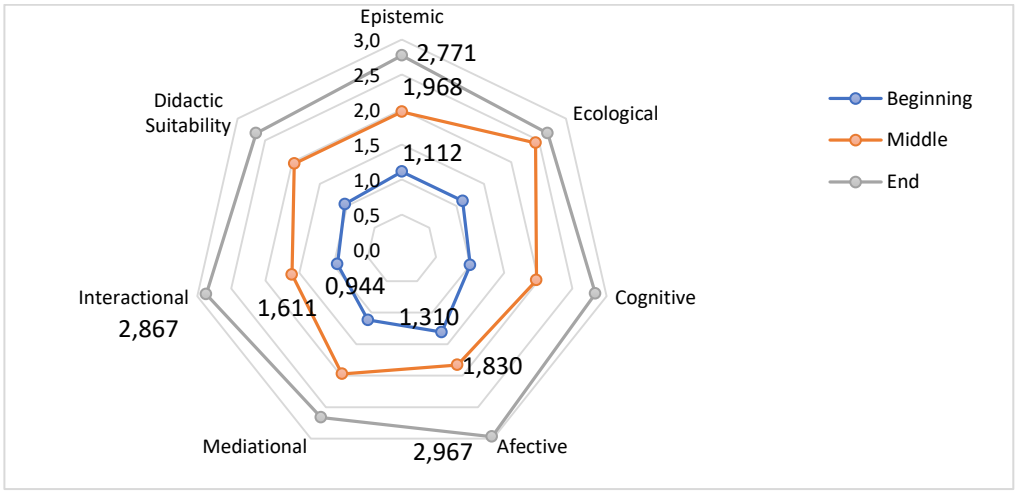
Graph 1. Frequency distribution of the use of the platform by Educational Region.

The face-to-face sessions were held at CIDETE, at the end of the module, the participant teachers attended class where they developed work from a computer, in their own time and space (3 hours per day on average).



	Cocle	Azuero	Western Panama	Central Panama	Veraguas	Colon
Active Users	24	27	25	24	27	23
Interactions	24619	27741	25563	27304	23427	18722
Coverage rate	87%	88%	84%	87%	84%	83%
Estimated platform usage time	02:30	03:00	03:10	03:15	02:45	02:35
Number of emails sent	48	63	59	55	58	45
Estimated starting time	13:30	17:35	19:25	10:32	11:00	14:20
Estimated ending time	15:45	20:43	22:37	12:45	12:39	16:36

Development of innovative educational strategies and practices aligned to the six (6) adaptations that enhance the strengthening of mathematical didactics. Graph 2 illustrates the pre- and post-diploma competencies of the participants.



Graph 2. Semantic representation of the Didactic Suitability before and after the degree course.

Graph 2 shows the teaching competencies before taking the degree course. They show that the teachers implemented didactic units in which the enhancement of motivation, the use of manipulative and technological resources and unidirectional interaction (blue hexagon) were considered very limited. At the end of the degree course, their didactic units incorporated motivating tasks, in which manipulative and technological resources were

used and where interaction was no longer centered on the teacher but on the child, a situation that contributes to an active and meaningful learning.

The evaluation model

The World Bank establishes references for the global evaluation of projects and offers a methodological guide with general and suitable proposals for educational projects. For the purposes of this Program, the Evaluation Logic Model was used to determine the impact of the DSTM Degree Program during 2018 (see figure 3).

This model requires identifying inputs from the participants, determining the activities, the process and the impact or final result. For this, it was necessary to have evidence on the results and transformations in the classroom, specifically in the strategies for teaching mathematics implemented by preschool and elementary school teachers in the participating schools. This implies a review of both, the achievement of the objectives of the degree course and the final effect on the beneficiaries.

The Logic Model (LM) is a tool, in visual format, that illustrates how various elements are connected to each other to generate the complete picture of the evaluation. There is convergence with Donald Stufflebeam's CIPP Evaluation Model, as it consists of: Context, Input Process and Output. However, the ML emphasizes the activity and synthesizes the process, emphasizing the product or what it calls impact or final value. It is a model widely used when evaluating programs subsidized by external sources, as is the case of this Degree.

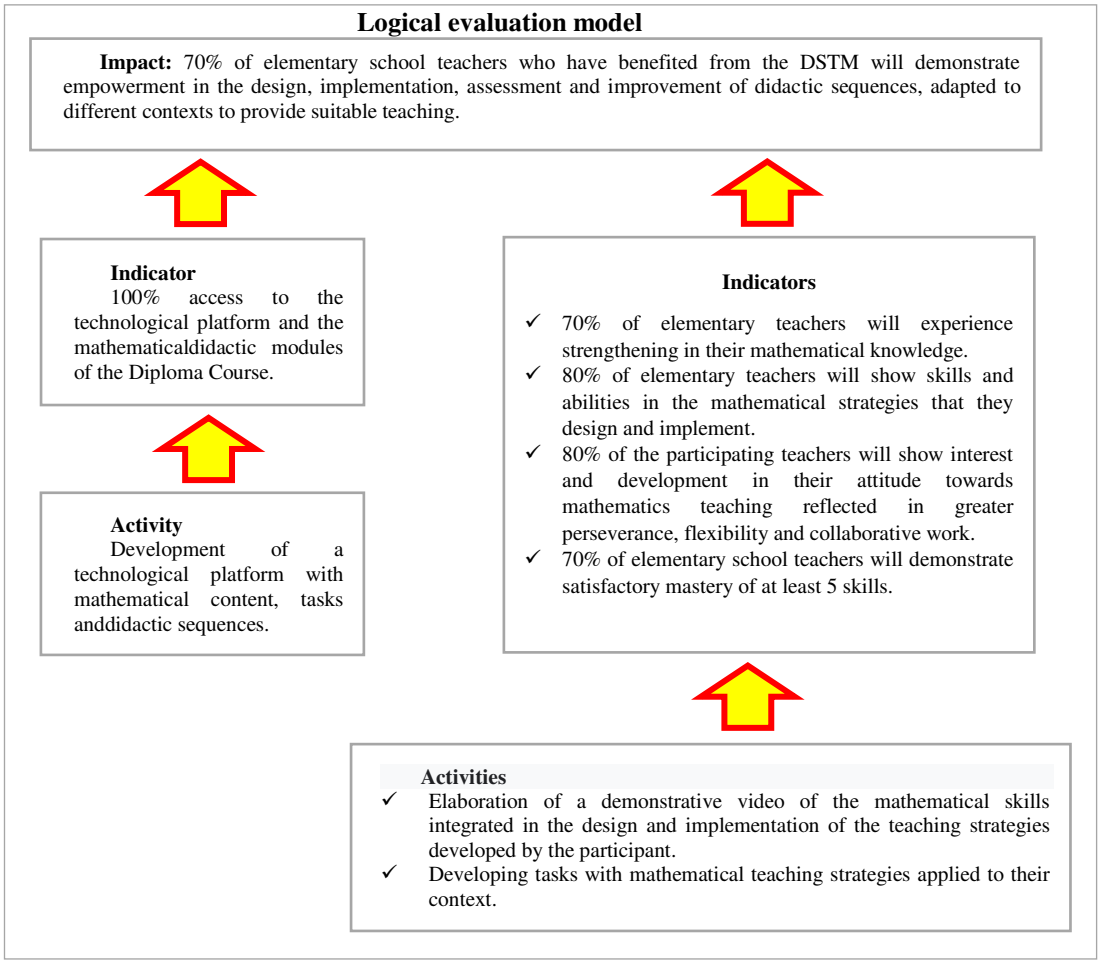


Figure 3. Logical Evaluation Model. Source: **Cohen & Franco (1992); Baker (2000).**

Discoveries of the quantitative study

The following are the findings obtained from the administration of the questionnaire to a sample of 18 teachers participating in the Degree program that was the object of this external evaluation. The data revolved around the three priority areas presented in the objectives of the Degree, namely: Mathematical content, Teacher skills and abilities, and Attitude towards teaching mathematics.

Dimension 1: mathematical knowledge

Teacher training would not fulfill its purposes unless the acquired disciplinary knowledge can be significant in teaching practice. The appropriation of mathematical knowledge allows mastery of the content and enables reflection and transformation of teaching practice.

Table 2a presents results referring to the level of appropriation of mathematical knowledge that teachers obtained in the areas of arithmetic, algebra, geometry, metrics, statistics and probability. In another section we also inquired about the appropriation and transfer of theoretical-methodological references, didactic resources and mathematical difficulties and obstacles. In the population studied, 77.78% expressed complete agreement in the acquisition of arithmetic topics, 22.22% agreed, so we can affirm that in 100% the appropriation of knowledge was positive (see table 2a).

In geometry topics we found that 61.11% completely agree in strengthening the knowledge, followed by 38.39% agree, that is, 100% positive response to the knowledge acquired. In statistics content we found 61.11% completely agree, 27.78% agree and 11.11% did not answer. Finally, in algebra knowledge, we found 55.56% of teachers completely agree in having acquired it and 44.44% agree (see Table 2a).

In the knowledge about didactic strategies, 66% completely agreed to have acquired this knowledge and 27.28% agreed, only 6% disagreed. On the topic of didactic resources, 55.56% completely agreed and 27.78% agreed. Only 6% disagreed. A fundamental aspect is the identification and analysis of the difficulties and obstacles in the children's learning, since it makes it possible to make adjustments and differentiated attention to the children in terms of their learning, thus avoiding failure and mathematical backwardness. In this aspect, we found that in the identification of obstacles, 55.56% were in complete agreement, while 38.89% agreed. Only 6% disagreed that they had acquired this knowledge.

Table 2a. Basic statistics obtained from the questionnaire

Dimension surveyed	Sub-dimensions	Disagree	I agree	Completely agreed
Mathematical knowledge	Up-to-date teaching resources	6	38.39	55.56
	Theories for design and implementation	11	27.78	61.11
	Identifying difficulties and obstacles in mathematical learning	6	38.89	55.56
	Analysis of difficulties and obstacles in mathematics learning		38.89	61.11
	Updated didactic strategies at the elementary level	6	27.78	66.67
	Methodologies for the design and implementation of mathematics teaching	6	33.33	61.11
	Elementary Mathematics: Geometry and mean		38.89	61.11
	Elementary mathematics: arithmetic		22.22	77.78
	Elementary mathematics: algebra		44.44	55.56
	Elementary mathematics: statistics and probability		27.78	61.11

Dimension 2: Mathematical skills and abilities.

The mathematical skills and abilities acquired in the diploma course allow teachers to transform their teaching practice and achieve learning in elementary school children. We corroborated this through the processing of information that shows that teachers implement procedural knowledge that allows them to design, apply and adapt the curriculum to the needs of the students (see Table 2b).

Table 2b. Basic statistics obtained from the questionnaire

Dimension surveyed	Sub-dimensions	Disagree	I agree	Completely agreed
Math skills and abilities	Mathematical proficiency needed to teach at the elementary level	6	44.44	50
	Ability to design educational strategies in		44.44	55.56

mathematics		
Skills to apply the mathematical strategies I designed	50	50
Ability to diversify the curriculum from the cultural realm	55.56	44.44
Ability to address the mathematical evolution of my students	61.11	38.89
Ability to address my students' mathematical functionality	61.11	38.89
Skills to analyze my pedagogical practice in mathematics	66.67	33.33
Skill in the selection of assessment techniques and instruments for mathematical learning.	66.67	33.33
Skill in the application of assessment techniques and instruments for mathematical learning.	66.67	33.33

In the study we found to a greater extent more skills to design and apply strategies, diversifying the curriculum in addition to the mathematical domain, more than half of the teachers affirm to be in total agreement 55.56% and another percentage of 44.44% agree in these assertions, (see table 2b). In this study, we found that teachers were able to address mathematical function and evolution; 38.89% completely agreed and 61.11 agreed. These skills and abilities are related to the systematization and follow-up of the process; therefore, we found it advisable to systematize evolutionary stages to the implementation.

Finally, the skills for the selection and application of techniques and instruments for mathematical implementation were on a smaller scale; 33.33% said they completely agreed and 66.67% agreed. Only 6% affirmed not having the mathematical mastery to teach at the basic level, contrary to 94% who affirmed being in complete agreement in 50% and 44.44% agreeing in the mathematical mastery acquired in the degree course.

Dimension3: Attitudes towards the teaching of mathematics.

Finally, among the teachers in the provinces studied, we found that 98% agreed with a favorable attitude towards the teaching of mathematics. The degree has generated synergies in the schools for collaborative work, to propose and develop projects, as well as 98% completely agree that there is greater interest in improving teaching performance (see Table 2c).

Table 2c. Basic statistics obtained from the questionnaire

Dimension surveyed	Sub-dimensions	Disagree	I agree	Completely agreed
Attitudes towards the teaching of mathematics	Interest in improving my teaching performance			25
	Interest in designing and implementing learning projects		1	24
	Interest in working collaboratively		1	24
	Flexible and tolerant attitude towards divergent views		1	24

Qualitative study findings

The qualitative phase of this external evaluation was supported by focus group and documentary analysis techniques. Four (4) focus groups were held with participants from Azuero, Cocle and Veraguas. From this technique, categories and subcategories were developed, using a combination of Holistic and Axial Coding, proposed by **Saldaña (2013)** and **Corbin & Strauss (2008)**, respectively. Finally, two (2) categories and seven (7) subcategories emerged with their corresponding subthemes (See Matrix 1 in Table 3).

Table 3. Matrix 1 of emerging categories and subcategories

Category	Subcategories	Category	Subcategories
Contextual	• Diplomate's Speech • Methods and content • Design • Planning • Didactic strategies • Educational environment • Social cultural	Personal	• Perception of their capabilities • Empowerment • Attitude • Skills • Motivation and challenges • Impact of Experience • Expectations Cognitive-Pedagogical • Follow-up

The emerging categories group ideas and expressions that are interrelated and that offer a broader view of the concept and that in turn respond to the following assumptions: *1) Teachers have transformed the teaching of mathematics with the tools of the degree and design from the context and characteristics of their students , 2) Teachers design didactic sequences with trends in the teaching of mathematics, problem solving, and evaluation; 3) Teachers consider the conditions of the context, cultural and social aspects and 4) Teachers emphasize motivation and encourage creativity and innovation in the teaching of Mathematics*. The subcategories make visible and describe the concept.

Within the contextual category, 7 subcategories emerged. Some findings included the following: participants identified the degree course as a "forward-looking" program, which included the use of participatory and group strategies, the constructivist approach and active learning. An analysis of some videos presented by the participants during the *Pedagogical Workshop on Didactic Strategies in Mathematics* showed the variety of designs and contextualized planning that the participants carried out as a result of their learning. These videos highlighted the primary discourse of the Degree that aims at the development of contextualized, creative and innovative strategies that illustrate the capacity of design and planning considering the didactic suitability.

The playful aspect in the mathematics didactics emerges as a core element within the strategies developed and was highly valued by the teachers. The methods and contents of the Degree have enhanced the design and implementation of other strategies aimed at the development of metacognition skills. The following comment points towards the integration of subject-specific language and high level cognitive skills.

> *"We have revisited learning theories and students have learned to analyze and experiment. Fraction has been tedious, but they already know mathematical terms and language. Same with angles, perimeters and areas."*

It is evident the relevance of mentoring and coaching when designing, implementing and assessing new mathematical didactic strategies. To this end, the participants highlighted the presence and support of the Degree facilitators. This comment is significant because it highlights the element of interactional and media suitability when implementing a teacher training program. Even though, for some participants, the course should have adopted only the face-to-face strategy, the combination with technology is attractive and pertinent, due to location and costs, to the extent that there is coaching.

We note in some cases the reluctance to use technology and the urgency to receive direct strategies for rapid implementation. This is understandable in andragogic education since the adult wishes to quickly implement the acquired knowledge. However, this should not marginalize the theory that frames the action. The integration of more practical face-to-face activities could mitigate this need.

For the subcategory of Methods and Content, we found that the Degree Course alignment exhibited patterns characteristic of Piaget's cognitive development, marked playful approaches characteristic of Vigotsky, and Dewey's and Brunner's active and meaningful meaning, respectively. This theoretical amalgam is combined with aspects of multiple intelligences by privileging multisensory didactic strategies to promote mathematical learning.

The need for coaching or scaffolding as a support for the participant was also evidenced, so that this method should prevail and even be reinforced to cover all teachers and collect data on results of individual and collective progress.

Regarding the mathematical content of the course, divergent opinions emerge on the adequacy of the topics and the scope and depth of the course, based on the levels it is intended to impact. The teachers who show that they feel at a disadvantage are those who offer the low level (preschool, 1-3rd level).

Within the personal category, some relevant findings were also obtained, which can be classified in the subcategories shown in matrix 1 of Table 2.

Regarding the subcategory of perception of their abilities, the participants presented contrasting arguments regarding the way they visualize their ability in didactics of mathematics. This difference is associated with the grade they teach at the elementary level. Those who are categorized as belonging to the lower level (preschool, 1-3rd level) demonstrate a greater sense of urgency to achieve greater capacity. To these ends a participant indicated:

"(Math) was an area I felt I needed. I am not a math specialist and I needed more information, besides, the students don't like math. Since we work with concrete multilevel and mathematical content is introduced"(sic).

In general, most of the participants of the higher grades (4-5-6th grade), considered that they had no difficulties in the handling of the mathematical content addressed. This perception is consistent with that identified under the subcategory of methods and contents, where teachers in higher grades questioned the need to reinforce educational theories and the math history.

In reference to the attitudes shown, we note that the participants exhibited a positive and collective attitude towards group work. Contemporary research clearly establishes that, traditionally, teaching has been based on an intellectual egocentrism that limits interaction and disciplinary exchange among teachers. However, we found that when the participants evaluated their mathematical abilities, they adopted a marked attitude of collaboration, given the need to offer and receive support from other teaching colleagues.

In the subcategory motivation and challenges, motivation appears as an element linked to teacher and student achievement. To these ends, a participant states:

> *"The desire to want more (to learn more) and to research. To look for more strategies, more (sic) motivating for them to acquire knowledge. It is important to fix the knowledge to have a base for the next levels".*

Teachers also point out as a challenge for mathematical learning, the limited motivation of children and procedural aspects in problem solving. This points to limitations in the use of didactic strategies that encourage active and meaningful learning, even when they are offered in the Degree. The lack of knowledge of the practical and daily use of mathematics strengthens the resistance to its learning.

In the subcategory impact of the experience, the diploma course generated an integral impact on the professional and personal capacities of the teachers, according to the comments of several participants. These point out:

> *"There were oral presentations. Individual and group data research. (The Degree) enhances analysis, observation, self-evaluation, communication, leadership and critical thinking. Students are lacking motivation".*

This statement not only highlights the strengthened areas, but also establishes the importance of the modeling that is transmitted in the absence of student motivation. This answer, as well as others in this line, identifies the collaborative environment and reciprocity among colleagues as a reiterative impact. This relationship strengthens professional communication and the quality of the materials developed, as well as allows validating the adequacy of the strategies designed and implemented.

The development of creativity formed part of a specific objective within this Program. In the opinion of the participants, the course encouraged their creativity and innovation in the development of didactic strategies aimed at mathematics. In this same sense, we found that the strategies implemented strengthened competencies in the design of didactic material among the participants' students. This serendipity illustrates the transfer of knowledge and attitudes from the teacher to the students and the development of a modeling that encouraged creativity in the classroom.

In the opinion of the participants, in addition to the fact that the Degree encouraged their creativity and innovation, it also fostered didactics and real integration of mathematical knowledge. A participant stated the following:

> "*The Degree serves to teach and for children to learn well. It is a permanent learning".*

The notion of permanence of knowledge and its perpetuity emerged in several instances. Teachers emphasized the need for mathematical knowledge to transcend time without being limited to temporary learning without real internalization. The concern for the permanence of acquired knowledge turns out to be reiterative and revealing of the teacher's commitment to the student.

In the subcategory of expectations, two subcategories were identified, namely: cognitive-pedagogical and follow-up. Under the cognitive-pedagogical theme, the following are identified:

"We need methods to teach tables at elementary 1-2 and 3rd grade level".

"*We need to work specifically, what didactic material do we need to work on the fraction."*

"The teacher should be given the tools so that the teacher (sic) learns to use these bases. This way, when the child reaches 4th-5th grade, it is easier for him/her".

An example within the subcategory of follow-up expectations was the following:

"An example within the subcategory of follow-up expectations was the following:"

7.Final reflections and conclusions

The sample of male and female teachers claimed to have acquired mathematical knowledge, the most difficult to acquire were algebra, probability and statistics.

Acquiring mathematical knowledge is a precedent for articulating the mathematics didactics, favoring reflection, analysis and the design of strategies.

Reflective didactics generates collaborative synergies with other teachers to propose mathematical learning projects in the classroom.

The positive attitude towards mathematics on the part of teachers is a strength for the continuous improvement and attitude of children.

When teachers take improvement courses due to training needs, the courses are meaningful to the teaching practice and the contents and strategies have a better chance of reaching the classroom as they would otherwise be institutionally imposed.

After assessing the Didactic-Mathematics competencies of the graduate in Panama and the specific generic and professional competencies of the mathematics teacher, we look specifically, from the field of mathematics education, at the knowledge and characteristics required by a teacher who teaches mathematics at different educational levels. The importance of the knowledge of mathematics teachers is highlighted by the following statement of the National Council of Teachers of Mathematics (NCTM) in a document called Principles and Standards for Mathematics Education, corresponding to the year 2000:

Students learn mathematics through the experiences teachers provide. Thus, students' understanding of mathematics, their ability to use mathematics to solve problems, their confidence in, and their disposition toward mathematics are shaped by the forms of instruction they encounter in school. Improving mathematics education for all students requires effective teaching in all classrooms (**NCTM, 2000, p.16**).

All teachers would recommend this Diploma to other colleagues.

It was evident that the participants felt motivated enough to leave their comfort zone and experiment with new strategies and didactical sequences. This willingness was enhanced by the new techniques presented in the course, which required giving greater prominence to the classroom climate through the creation of collaborative environments, the development of relaxed and fun environments conducive to the promotion of leadership and social values, and the modeling of the teacher in his or her role as a teacher.

The possibilities of designing and redesigning didactic strategies and procedures gave the teacher the conceptual empowerment and motivation necessary to investigate with other colleagues and on the Internet to strengthen their processes. This amalgam of situations impacted the teacher's attitudes and the vision he has of himself, which is expected to strengthen the quality of his performance in mathematics teaching and create motivation and interest in his students.

The technological platform with mathematical content, tasks and didactic sequences was developed, implemented and was accessible to most of the participants.

The strategies included in the Degree point to constructivist and pragmatic dimensions in harmony with the competencies that a teacher of the 21st century should have.

Creativity and innovation were teaching competencies that emerged both in the teacher and in specific groups of students.

The official norms and policies, as well as the offering of improvement courses disjointed from the teaching interest, constitute a factor that could limit the development of new strategies and mathematical teaching.

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