Family of [0,1] Truncated Gompertz – Exponential DistributionWith Properties and Application

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Abstract:
In the paper we are introduce a new family of continuous distribution based on [0,1] truncated gompertz-G family distribution (TGO-GD) and [0,1] Truncated Gompertz-Exponential distribution (TGO-EXPD). We are discussed as special cases: Cumulative distribution function (CDF), probability density function (PDF), survival function (sf), hazard rate function (hrf), revers hazard rate function (rhrf), cumulative hazard rate function (chrf), the quntial function (Qf), moment generating function (M.G.F), the moments , the mean μ, the variance σ^2 ,the median M, the skewness S_K , the kurtosis K_u , the entropy, order statistics,Asymptotic behavior and maximum likelihood estimator for [0,1] Truncated Gompertz-Exponential Distribution (TGO-EXPD). We estimate of the model parameters by maximum likelihood and we apply empirically the potentiality of the new class by means of one real data set.
Keywords: Exponential distribution, Gompertz distribution, Maximum likelihood estimation,the Rényi entropy, the order statistics.

1. Introduction

The purpose of truncated distributions is to get a resultsfiner When a distribution is truncated the domain of truncated random variable is restricted based on the truncation points of interest and thus the shape of the distribution changes .A truncated distribution is a conditional distribution resulting when the domain of the parent distribution is restricted to a smaller region. It is occurs when we are unable to know about or record events occurring below or above a set threshold or outside a certain range. A truncated distribution was first introduced by Galton (1898)et al.[1] to analyzed speeds of trotting horses for eliminating records which was less than specific know time.

The Gompertz distribution is both skewed to the right and to the left. It is generalization of the exponential distribution, and is commonly .It is used in many applied problems. Particularly in lifetime data analysis. [2]. It is used in more applied in analysis of survival in some sciences such as computer.[3].

New families of distribution are produced time by time. From the new families we are obtain of distribution more flexible for adding parameters to all forms of probability distribution and are makes the resulting distribution more flexible for modeling heavily skewed dataset. The families of distribution are include the Gamma-G(type 3) by[4], Beta generalized family (Beta-G) by Eugene et al.[5].Exponential-G (EG) by Cordeiro et al.[6]. Lomax-G family by

Cordeiro et al.[7], a Beta Marshall – G family of distribution by Ailzadeh et al [8]. Marshall-olkin generated family (MO-G) by Marshall and olkin (2007),[9].

Following the introduction of the above listed families of probability distribution and the desire to add skewness and flexibility to classical distributions particularly the Gompertz distribution, many authors have proposed different extensions of the distribution and some of the recent and known studies include the generalized Gompertz distribution by El-Gohary and Al-Otaibi [10], which was based on an idea of Gupt and Kundu [11], the Beta Gompertz distribution by Jafaril et al. [12], and the Lomax-Gompertz distribution by Omale et al.[13].

Add of this Alizadeh et al.[14] in year (2017), using the Gompertz distribution(GOD) to generate a new family that has high flexibility in describing different data, Oguntunde et al.[15] in year (2019), using the Gompertz family, a new distribution called Gompertz Fréchet was found and applied to Engineering data, Eghwerido et al.[16] in year(2020), using the Gompertz family by finding a new distribution called

Gompertz-Alpha Power Inverted Exponential, also in the same year Khaleel et al.[17] (2020) by, using the Gompertz family by extending the Flexible Weibull distribution , and the application of these distributions to different types of data.

(1)

Truncated distribution has been derived from that of a parent distribution, for example Normal and exponential distributions, by bounding the random variable from either below or above (or both). Salah Abid et al. [18], in year (2017), he was using [0,1]Truncated of family distributions, by [0,1] Truncated Fréchet-Gamma and inverted gamma (TFIG) distributions. He was discussed as a special case. The cumulative distribution function, the moments, the mean, the variance, the skewnees, the kurtosis, the median, the characteristic function, the serviver function and the hazard rate function. Jumana A. Altawil et al. [19], in year (2019) , she was using [0,1] Truncated of family distribution, by [0,1] Truncated Lomax-Uniform distribution with properties. She was discussed properties of statistical above. The [0,1] Truncated Gompertz-G family (TGO-G) distribution is a parent Gompertz-G family distribution. This paper present introduced new [0,1]Truncated Gompertz-Exponential distribution (TGO-EXPD). The research aims to study some mathematical properties of the proposed (TGO-EXPD) and we show that the proposed distribution is more flexible than other distributions of real data.

2. [0,1] Truncated Gompertz–G family Distribution:

The Gompertz distribution is one a continuous probability distribution.Let X be a random variable it is has the Gompertz distribution with two parameters $\propto$ and β, it is denoted by:$X\sim GO(\propto,\beta)$. According to (Atanda, O. D., Mabur, T. M., &Onwuka, G. I. (2020)).by[20]. The cumulative distribution function (CDF) and the probability density function (PDF) of the Gompertz distribution are respectively.

$$F(x;\propto,\beta) = 1 - e^{-\frac{\propto}{\beta}(e^{\beta x}-1)} \quad (1)$$

$$f(x,\propto,\beta) = \propto e^{\beta x} e^{-\frac{\propto}{\beta}(e^{\beta x}-1)} \quad (2)$$

Such that $x \geq 0$,$\propto > 0$, $\beta > 0$ where $\propto$ is scale parameter and βIs shape parameter.

Suppose X is a random variable that is distributed according to some pdf g(x), withcdf G(x), both of which have infinite support. A random variable X lies within the interval $x \in [m,n]$ such that $(-\infty \le m \le x \le n \le \infty)$, under condition $(m \le x \le n)$ then the truncated distribution has (pdf) and (cdf), by(Singh, S. K., Singh, U., & Sharma, V. K. (2014). [21], is given by formula:

$$f(x\backslash m \le x \le n) = \frac{g(x)}{G(n)-G(m)} \quad \text{such that} \quad G(n)-G(m) \ne 0$$

$$F(x\backslash m \le x \le n) = \frac{G(x)-G(0)}{G(n)-G(m)} = \frac{G(x)}{G(n)-G(m)}$$
(*)

We can obtain of the (cdf) and (pdf) for [0,1] Truncated Gompertz distribution ([0,1] TGD), from above equation(*) are given as follows.

$$F_T(x,\propto,\beta) = \frac{1-e^{-\frac{\propto}{\beta}\left(e^{\beta x}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} \quad (3)$$

$$f_T(x,\propto,\beta,\xi) = \frac{\propto e^{\beta x} e^{-\frac{\propto}{\beta}(e^{\beta x}-1)}}{1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}} \quad \text{Where } (x > 0, \propto, \beta > 0) \ (4)$$

Now we are introduced of the new family proposed of distribution according to(Salah Abid in year (2017)) and (Altawil et al. in year (2019)),is called family [0,1] Truncated Gompertz-G distribution.We are obtain of (cdf) for a new family which we are suggested[0,1] Truncated Gompertz-G familydistribution by compensation the equation (3) with G such that G(x)with any parameters and let $x = t \rightarrow dx = dt$ then:

$$F(x;\propto,\beta) = \int_0^{G(x)} f(t;\propto,\beta)dt \text{such that} \quad (t \ge 0 \text{ and } \propto,\beta > 0)$$

$$F_T(x,\propto,\beta) = \frac{1-e^{-\frac{\propto}{\beta}(e^{\beta G(x)}-1)}}{1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}} \quad (5)$$

We are getting of (pdf) for ([0,1] (TGO-G)) by derived the equation (5).

(2)

$$f_T(x,\propto,\beta) = \frac{\propto e^{\beta G(x)} g(x,\xi) e^{-\frac{\propto}{\beta}(e^{\beta G(x)}-1)}}{1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}} \text{Where } (x > 0, \propto, \beta > 0) \ (6)$$

Now two the equations (5) and (6) are represented (cdf) and (pdf) of generalization for a new proposed family of distribution([0,1] TGO-G)

3- [0,1] Truncated Gompertz-Exponential Distribution:

In the section We are introduced a new family of distribution [0,1] TruncatedGompertz-Exponential ([0,1] TGO-EXP). Let the cumulative distribution function (CDF) and the probability density function (PDF) of the Exponential distribution (EXP) arerespectively with parameter (ξ), such that $(x \ge 0, \xi > 0)$ are given as:

$$G(x;\xi) = 1 - e^{-\xi x} \qquad (7)$$

$$g(x;\xi) = \xi e^{-\xi x} \quad (8)$$

Substituting the equation (7) in the equation (5) we obtain (CDF) for the a new distribution [0,1](TGO-EXP) of a random variable X given as:

$$F_T(x;\propto,\beta,\xi) = \frac{1-e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} \quad (9)$$

Substituting the equations (7) and (8) in the equation (6) we get (PDF) for a new proposed familydistribution[0,1] (TGO-EXP):

$$f_T(x;\propto,\beta,\xi) = \frac{\propto e^{\beta\left(1-e^{-\xi x}\right)}\xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} \quad (10)$$

Figure (1) is represents histogram of (cdf) is increasing and (pdf) is decreasing for [0,1] (TGO-EXPD)with different values of parameters by using software.

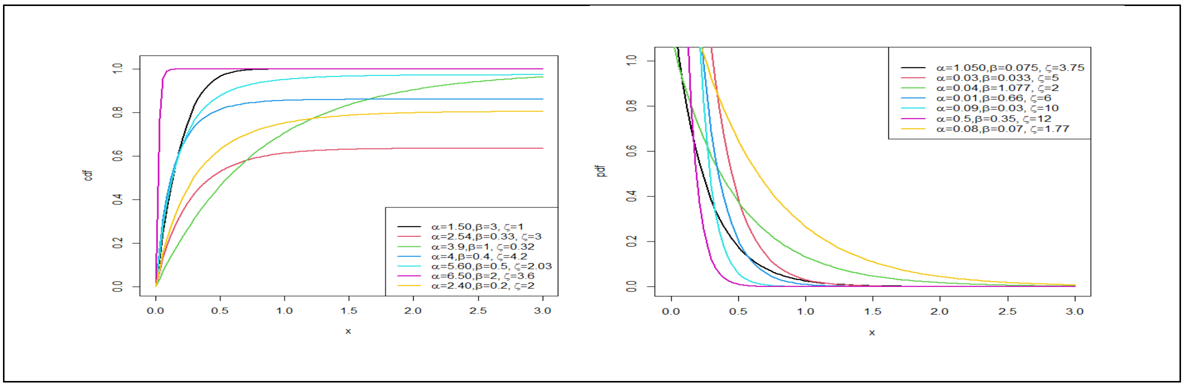


Figure (1) plot of (cdf) and (pdf) for [0,1] TGO-EXP distribution with different parameters by using R software.

4- Expansion Function:

In this section , the (PDF) and the (CDF) for the a new family (TGO-EXP) willbeexpanded in order to studying some of the statistical characteristics and characteristics of the new distribution . We can be taking equation (8) and simplifying itAccording to[22]: Exponentialexpansion and binomial series expansion multipletimes respectively as:

$$e^{-a} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} a^k, \qquad (1-z)^b = \sum_{j=0}^{\infty} (-1)^j \binom{b}{j} Z^j,$$

$(1-z)^{-b} = \sum_{m=0}^{\infty} \frac{\Gamma(b+m)}{m!\Gamma(b)} Z^m$, $\Gamma(b) = \int_0^{\infty} y^{b-1} e^{-y} dy$ Since in thelest second section the probability density function of a new proposed family .No.(8), is given as:

$$f_T(x,\propto,\beta,\xi) = \frac{\propto e^{\beta G(x,\xi)} g(x,\xi) e^{-\frac{\propto}{\beta}\left(e^{\beta G(x,\xi)}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}}$$

(3)

By using exponential series of $e^{-\frac{\propto}{\beta}(e^{\beta G(x,\xi)}-1)}$ we get:

$e^{-\frac{\propto}{\beta}(e^{\beta G(x,\xi)}-1)} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!}(\propto/\beta)^k(e^{\beta G(x,\xi)}-1)^k$(11)

And by using binomial exponential of $(e^{\beta G(x,\xi)}-1)^k$ we obtain:

$(e^{\beta G(x,\xi)}-1)^k = \sum_{j=0}^{\infty}(-1)^j\binom{k}{j}e^{\beta G(x;\xi)j}$(12)

Substituting the equation (12) in the equation (11) we get:

$e^{-\frac{\propto}{\beta}(e^{\beta G(x,\xi)}-1)} = \sum_{k=j=0}^{\infty}\frac{(-1)^{k+j}}{k!}(\propto/\beta)^k\binom{k}{j}e^{\beta G(x;\xi)j}$ (13)

Now substituting the equation (13) in equation (8) we obtain: $f_T(x,\propto,\beta,\xi) = \sum_{k=j=0}^{\infty}\frac{\propto^{k+j}(-1)^{k+j}\binom{k}{j}e^{\beta G(x,\xi)}e^{\beta G(x;\xi)j}\ g(x,\xi)}{\beta^k k!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}$(14) substituting the equations (5) and (6) in equation (12) we get:

$$f_T(x;\propto,\beta,\xi) = \sum_{k=j=0}^{\infty}\frac{\propto^{k+j}\ (-1)^{k+j}\binom{k}{j}}{\beta^k k!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}e^{\beta(1-e^{-\xi x})}e^{\beta(1-e^{-\xi x})j}\left(\xi e^{-\xi x}\right)$$

$= \sum_{k=j=0}^{\infty}\frac{\propto^{k+j}(-1)^{k+j}\binom{k}{j}}{\beta^k k!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}e^{\beta(1+j)}e^{-\beta(1+j)e^{-\xi x}}(\xi e^{-\xi x})$(15)

By using exponential series of $e^{-\beta(1+j)e^{-\xi x}}$ we get:

$e^{-\beta(1+j)e^{-\xi x}} = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!}(\beta(1+j)^m e^{-(\xi x)m}$ (16)

Now substituting the equation (16) in equation (15) we obtain:

$$= \sum_{k=j=m=0}^{\infty}\frac{\propto^{k+j}\ (-1)^{k+j+m}\binom{k}{j}}{\beta^k k!\,m!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}e^{\beta(1+j)}(\beta(1+j))^m e^{-(\xi x)m}(\xi e^{-\xi x})$$

$$= \sum_{k=j=m=0}^{\infty}\frac{\propto^{k+j}\ (-1)^{k+j+m}\binom{k}{j}}{\beta^k k!\,m!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}e^{\beta(1+j)}(\beta(1+j))^m\xi e^{-\xi(m+1)x}$$

Let $L_{k,j,m} = \frac{\propto^{k+j}(-1)^{k+j+m}\binom{k}{j}}{\beta^k k!m!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}(\beta(1+j))^m\xi e^{\beta(1+j)}$

Then the expansion of probability density function for [0,1] truncated gompertz– exponential distribution is given by formula:

$f_T(x;\propto,\beta,\xi) = \sum_{k=j=m=0}^{\infty} L_{k,j,m}\, e^{-\xi(m+1)x}$(17)

5. The properties of [0,1] (TGO-EXPD):

In this section we show some properties of the[0,1] Truncated Gompertz _ Exponential Distribution.

5-1. The survival function(sf): We can find by formula.

$S(x;\propto,\beta,\xi) = 1 - F_T(x;\propto,\beta,\xi) = p_r(X > x)$ (18)

Substituting equation (9) in equation (18) we get.

(4)

$$s(x;\propto,\beta,\xi) = \frac{e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)} - e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} \quad (19)$$

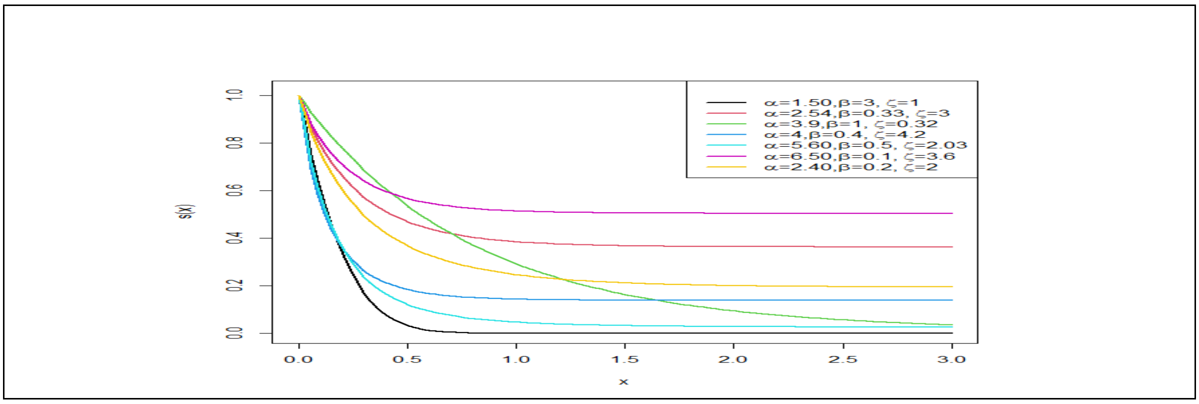


Figure (2) plot of the survival function for [0,1] TGO-EXP distribution with different values of a parameters by using R software.

The figure (4) is represent the histogram of the survival function, for [0,1] TGO-EXP distribution when we are taking different values of parameters for this model, then the function is degreasing by using R software.

5-2. The hazard rate function (hrf): Is given by.

$h(x;\propto,b,\xi) = \frac{f_T(x;\propto,\beta,\xi)}{s(x;\propto,\beta,\xi)}$(20) Substituting the equations (10) and (19) in equation (20) we get.

$$h(x;\propto,\beta,\xi) = \frac{\propto e^{\beta\left(1-e^{-\xi x}\right)}\left(\xi e^{-\xi x}\right) e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)}}{e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)} - e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} \quad (21)$$

The figure (3) is represents histogram of hazard rate function for [0,1] Truncated Gompertz-Exponential Distribution, it is monotonically degreasing and monotonically increasing with different values of a parameters for this model by using R software.

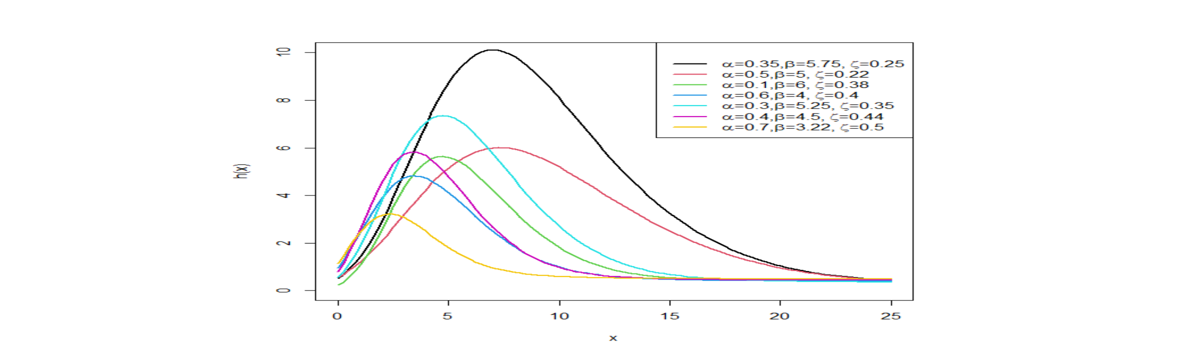


Figure (3) plot of hazard rate function for [0,1] TGO-EXP distribution with different values parameters by using R software.

5-3. The cumulative hazard rate function (chrf): The (chrf)for [0,1] truncated Gompertz-Exponential Distribution Is given by:

$H(x;\propto,\beta,\xi) = -\,log[S(x;\propto,\beta,\xi)]$ (22)

$$= \boldsymbol{log}\left(\mathbf{1} - \boldsymbol{e}^{-\frac{\propto}{\beta}(\boldsymbol{e}^{\beta}-\mathbf{1})}\right) - \boldsymbol{log}(\boldsymbol{e}^{-\frac{\propto}{\beta}\left(\boldsymbol{e}^{\beta\left(1-\boldsymbol{e}^{-\xi x}\right)}-\mathbf{1}\right)} - \boldsymbol{e}^{-\frac{\propto}{\beta}(\boldsymbol{e}^{\beta}\,-\mathbf{1})})\quad(23)$$

5-4. The reverse hazard rate function (rhrf): We can get the rhrf of anew proposed family by formula.

$r(x;\propto,\beta,\xi) = \frac{f_T(x;\propto,\beta,\xi)}{F_T(x;\propto,\beta,\xi)}$ (24) Substituting the equations (9) and (10) in equation (24) we get:

(5)

$r(x;\propto,\beta,\xi) = \frac{\propto e^{\beta\left(1-e^{-\xi x}\right)}(\xi e^{-\xi x})\; e^{-\frac{\propto}{\beta}(e^{\beta\left(1-e^{-\xi x}\right)}-1)}}{1-e^{-\frac{\propto}{\beta}(e^{\beta\left(1-e^{-\xi x}\right)}-1)}}$ (25)**5-5. the quntial function (Qf)**: We can obtain of (Qf) for anew proposed distribution by formula: $Q(P) = F^{-1}(P) = inf\{x; F(x), P\}$ such that $0 < P < 1$, then

Put $A = (1 - e^{-\frac{\propto}{\beta}(e^{\beta}-1)})^{-1}$ then

$X_q = -\frac{1}{\xi}\left\{ln\left[1 - \frac{1}{\beta}\,ln\left(\left(\frac{\beta}{\propto}ln\frac{A}{A-P}\right)+1\right)\right]\right\}$(26)**5-6. The$r^{th}$ moments:** We can calculated the r-degree moment of anew proposed family (TGO-EXP) distribution.

$E(X^r) = \int_0^{\infty} x^r f(x;\propto,\beta,\xi)dx$(27)

Substituting the equation (17) in equation (27) we get:

$E(X^r) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \int_0^{\infty} x^r\, e^{-\xi(m+1)x} dx$(28) Now let$y = \xi(m+1)x \;\; \rightarrow \;\; x = \frac{y}{\xi(m+1)} \;\; \rightarrow \;\; dx = \frac{dy}{\xi(m+1)}$ substituting of each $\xi(m+1)x$, x and dx in equation (28) we get:

$$E(X^r) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \int_0^{\infty} (\frac{y}{\xi(m+1)})^r\, e^{-y}\, \frac{dy}{\xi(m+1)}$$

$$E(X^r) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} (\frac{1}{\xi(m+1)})^{r+1} \int_0^{\infty} y^r\, e^{-y} dy$$

Using integration Gammar$(b) = \int_0^{\infty} y^{b-1}\, e^{-y} dy$ then

$$E(X^r) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} (\frac{1}{\xi(m+1)})^{r+1} (\Gamma(r+1))$$

$E(X^r) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(r+1)}{(\xi(m+1))^{r+1}}$(29)

Where $L_{k,j,m} = \frac{\propto^{k+j}(-1)^{k+j+m}\binom{k}{j}}{\beta^k k!m!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)} (\beta(1+j))^m \xi e^{\beta(1+j)}$

The first, second, third and fourth moment for [0,1] Truncated Gompertz-Exponential Distribution are respectively by substituting r=1, r=2, r=3 and r=4 in equation (29) we are obtain:

$$E(X) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(2)}{(\xi(m+1))^2} = \mu$$

$$E(X^2) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(3)}{(\xi(m+1))^3}$$

$E(X^3) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(4)}{(\xi(m+1))^4}$ $E(X^4) = \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(5)}{(\xi(m+1))^5}$**5-6-1. The moment generating function (M.G.F):**we can calculated of (M.G.F) by formula:$M_{(x;\propto,\beta,\xi)}(t) = E(e^{tx}) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(x^r)$(30)

Substituting equation (29) in equation (30) we obtain:

$$M_{(x;\propto,\beta,\xi)}(t) = \sum_{k=j=m=r=0}^{\infty} V_{k,j,m,r} \frac{\Gamma(r+1)}{(\xi(m+1))^{r+1}}$$

(6)

Where $V_{k,j,m,r} = \frac{\propto^{k+j}(-1)^{k+j+m}\binom{k}{j}t^r}{\beta^k k!m!r!\left(1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}\right)}(\beta(1+j))^m \xi e^{\beta(1+j)}$

5-6-2. The mean (μ): Wecan be calculated by.

$$\mu = E(X) = \sum_{k=j=m=0}^{\infty} L_{k,j,m}\frac{\Gamma(2)}{(\xi(m+1))^2}$$

5-6-3. The variance (σ^2) can be calculated by$\sigma^2 = E(X^2) - (E(X))^2$

$$\sigma^2 = \sum_{k=j=m=0}^{\infty} L_{k,j,m}\frac{\Gamma(3)}{(\xi(m+1))^3} - (\sum_{k=j=m=0}^{\infty} L_{k,j,m}\frac{\Gamma(2)}{(\xi(m+1))^2})^2$$

5-6-4. The median(M): is one measures of tendency the central. It hasknown value on the X-axis we can calculated median by:

$\int_{-\infty}^{M} f(x)dx = 0.5$such that$\int_{-\infty}^{M} f(x)dx = p_r(x \le M) = F(M)$.

$\int_{-\infty}^{M} f_T(x; \propto, \beta, \xi)dx = 0.5$ (31) substituting equation (10) in equation (31) we are get:

$$\int_0^M \frac{\propto e^{\beta(1-e^{-\xi x})}(\xi e^{-\xi x})e^{-\frac{\propto}{\beta}(e^{\beta(1-e^{-\xi x})}-1)}}{1-e^{-\frac{\propto}{\beta}(e^{\beta}-1)}}dx = 0.5$$

Put $(1 - e^{-\frac{\propto}{\beta}(e^{\beta}-1)})^{-1} = \rho$

$$M = -\frac{1}{\xi}ln\left\{1 - \frac{1}{\beta}ln\left[\frac{\beta}{\propto}ln\left(\frac{2\rho}{2\rho-1}\right) + 1\right]\right\}$$

5-6-5. The Skewness(Sk): The skewness of distribution $(S_k)$by[23], Kerl Pearsons measure of skewness is given by formula:$S_k = \frac{\mu_3}{\mu_2^{(3/2)}}$, Where (μ_2) is the second central moment and (μ_3) is the third central moment then.

$S_k = \frac{E(X-\mu)^3}{(\sigma^2)^{(3/2)}} = \frac{E(X^3)-3E(X)E(X^2)+2(E(X))^3}{(\sigma^2)^{(3/2)}}$ (32)

Put $E(X), E(X^2), E(X^3), \sigma^2$ in equation (32))we get.

$$S_k = \frac{\begin{cases} \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(4)}{(\xi(m+1))^4} \\ -3\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(2)}{(\xi(m+1))^2} \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(3)}{(\xi(m+1))^3}\right) \\ +2\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(2)}{(\xi(m+1))^2}\right)^3 \end{cases}}{\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(3)}{(\xi(m+1))^3} - \left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{\Gamma(2)}{(\xi(m+1))^2}\right)^2\right)^{3/2}}$$

5-6-6. The kurtosis(Ku): The (Ku) of distribution by[23], can be calculated as. $K_u = \frac{\mu_4}{\mu_2^2}$ Where μ_2, μ_4 are the second and fourth central moments, such that $\mu_2 = (X-\mu)^2 = \sigma^2$, $\mu_4 = (X-\mu)^4$ then.

$$K_u = \frac{E(X-\mu)^4}{(\sigma^2)^2} = \frac{E(X^4) - 4E(X)E(X^3) + (E(X))^2 E(X^2) - 3(E(X))^4}{(\sigma^2)^2} \quad (33)$$

Put $E(X), E(X^2), E(X^3), E(X^4)$ in (33),we obtain:

(7)

$$K_u = \frac{\begin{cases} \sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{24}{((1+m)\xi)^5}\right] \\ -4\left[\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{1}{((1+m)\xi)^2}\right]\right)\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{6}{((1+m)\xi)^4}\right]\right)\right. \\ +\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{1}{((1+m)\xi)^2}\right]\right)^2 \left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{2}{((1+m)\xi)^3}\right]\right) \\ -3\left(\sum_{k=j=m=0}^{\infty} T_{k,j,m} \left[\frac{1}{((1+m)\xi)^2}\right]\right)^4 \end{cases}}{\left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{2}{((1+m)\xi)^3}\right] - \left(\sum_{k=j=m=0}^{\infty} L_{k,j,m} \left[\frac{1}{((1+m)\xi)^2}\right]\right)^2\right)^2}$$

Note: If there is the distribution of a particular and this it is has probability density function (PDF), then if $f(x)$ is symmetrical around μ then $S_K = 0$, if S_K is negative then the skewness to the left from $f(x)$ and if S_K is positive then the skewness to the right from $f(x)$.

6- The otherstatistical properties: In the section we show some of statistical properties for anew proposed family (TGO-EXP) distribution.

6-1. The entropy: The entropy of a random variable X is a measure of variation of the uncertainty then high entropy is means high uncertainty. It has been used in various situations science and engineering.

6-1-1. The Rényi entropy: The Rényientropyof a random variable X is defined by the formula:

$$I_\delta(x) = \frac{1}{1-\delta} log\left[\int_{-\infty}^{\infty} f(x; \propto, \beta, \xi)^\delta \, dx\right] \quad (34)$$

Where $\delta > 0$ $and\, \delta \neq 1$. The Rényi entropy of the a new [0,1] Truncated Gompertz-Exponential distribution we can be obtained it is by substituting the equation (17) in the equation (34).

$$I_\delta(x) = \frac{1}{1-\delta} log\left[\int_0^\infty \sum_{k=j=m=0}^{\infty} L_{k,j,m} \, [e^{-\xi(m+1)x}]^\delta \, dx\right]$$

$I_\delta(x) = \frac{1}{1-\delta} log \sum_{k=j=m=0}^{\infty} L_{k,j,m} \int_0^\infty e^{-\xi\delta(m+1)x}\, dx$ (35)

Where $L_{k,j,m} = \frac{\propto^{k+j}(-1)^{k+j+m}\binom{k}{j}}{\beta^k k!m!\left(1-e^{-\frac{\propto}{\beta}(e^\beta-1)}\right)}(\beta(1+j))^m \xi e^{\beta(1+j)}$ Then.

$I_\delta(x) = \frac{1}{1-\delta} log \sum_{k=j=m=0}^{\infty} L_{k,j,m} \frac{1}{\xi\delta(m+1)}$ (36)Is the form of the Rényi entropy of the [0,1] Truncated Gompertz-Exponential distribution.

6-2. The order statistics(OS): let$X_1, X_2, \dots, X_n$ be a random sample of size n from Truncated Gompertz – Exponential Distribution which has a cumulative distribution function $F(x;\propto,\beta,\xi)$ and probability density function $f(x;\propto,\beta,\xi)$ we are find the probability density function for order statistics by formula:

$f_{k:n}(x) = \frac{n!}{(k-1)!(n-k)!} f(x;\propto,\beta,\xi)[1 - F(x;\propto,\beta,\xi)]^{n-k}[F(x;\propto,\beta,\xi)]^{k-1}$ (37) By using binomial exponential of $[1 - F(x;\propto,\beta,\xi)]^{n-k}$ we get

$[1 - F(x;\propto,\beta,\xi)]^{n-k} = \sum_{r=0}^{n-k}(-1)^r \binom{n-k}{r}[F(x;\propto,\beta,\xi)]^r$ (38) Substituting the equation (38)in the equation (37) we obtain:

$f_{k:n}(x) = \frac{n!}{(k-1)!(n-k)!}\sum_{r=0}^{n-k}(-1)^r \binom{n-k}{r} f(x;\propto,\beta,\xi)\, [F(x;\propto,\beta,\xi)]^{k+r-1}$ (39)

Where $l = \frac{n!}{(k-1)!(n-k)!}$

substituting for each l , (9)and(10), in (39) we obtain:

$f_{k:n}(x) =$

$l\sum_{r=0}^{n-k}(-1)^r \binom{n-k}{r}\left[\frac{\propto e^{\beta(1-e^{-\xi x})}\xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta(1-e^{-\xi x})}-1\right)}}{1-e^{-\frac{\propto}{\beta}(e^\beta-1)}}\right]\left[\frac{1-e^{-\frac{\propto}{\beta}\left(e^{\beta(1-e^{-\xi x})}-1\right)}}{1-e^{-\frac{\propto}{\beta}(e^\beta-1)}}\right]^{k+r-1}$ (40) When $k = 1$ we get lowest ranked statistic:

(8)

$f_{1:n}(x) = n\sum_{r=0}^{n-1}(-1)^r \binom{n-1}{r}\left[\frac{\propto e^{\beta(1-e^{-\xi x})}\xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta(1-e^{-\xi x})}-1\right)}}{1-e^{-\frac{\propto}{\beta}(e^\beta-1)}}\right]\left[\frac{1-e^{-\frac{\propto}{\beta}\left(e^{\beta(1-e^{-\xi x})}-1\right)}}{1-e^{-\frac{\propto}{\beta}(e^\beta-1)}}\right]^{r}$ When $k = n$ we get largest statistic in order

$$f_{n:n}(x) = n\left[\frac{\propto e^{\beta(1-e^{-\xi x})}\xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta(1-e^{-\xi x})}-1\right)}}{1-e^{-\frac{\propto}{\beta}(e^\beta-1)}}\right]\left[\frac{1-e^{-\frac{\propto}{\beta}\left(e^{\beta(1-e^{-\xi x})}-1\right)}}{1-e^{-\frac{\propto}{\beta}(e^\beta-1)}}\right]^{n+r-1}$$

6-3. The Asymptotic behavior: In the section we are studying the behavior of the [0,1] Truncated Gompertz-Exponential Distribution model in the equation (10) $as\ x \to 0\ and\ as\ x \to \infty$ then:

$$\lim_{x\to 0} f(x,\propto,\beta,\xi) = \lim_{x\to 0} \frac{\propto e^{\beta\left(1-e^{-\xi x}\right)}\xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}}$$

$= \frac{\propto\xi}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} > 0\ lim_{x\to\infty} f(x,\propto,\beta,\xi) = lim_{x\to\infty} \frac{\propto e^{\beta\left(1-e^{-\xi x}\right)}\xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)}}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}} = 0$, the behavior of the probability density function of ([0,1] TGO-EXP) distribution is degreasing.

6-4.The estimation: The**maximum likelihood** is one method measure of central tendency. It is use in estimation the parameters of statistical distributions. The maximum likelihood estimators (M.L.E),of the unknown parameters for the Truncated Gompertz-Exponential Distribution, are calculated based on complete samples. Let X_1 , X_2 , … , X_n be observed values from the (TGO-EXPD) model with set of parameters$(\propto,\beta,\xi)$,by[37],is given by: $L = \Pi_{i=1}^{n} f_T(x;\propto,\beta,\xi)$(41) substituting equation (10) in equation (41).

$$L = \Pi_{i=1}^{n}\left\{\frac{1}{1-e^{-\frac{\propto}{\beta}\left(e^{\beta}-1\right)}}\left[\propto e^{\beta\left(1-e^{-\xi x}\right)}\ \xi e^{-\xi x} e^{-\frac{\propto}{\beta}\left(e^{\beta\left(1-e^{-\xi x}\right)}-1\right)}\right]\right\}$$

$$L = \propto^{n} e^{\beta\left(1-e^{-\xi\sum_{i=1}^{n}x_i}\right)}\left(\xi^{n} e^{-\xi\sum_{i=1}^{n}x_i}\right) e^{-\frac{\propto}{\beta}\left(e^{\beta\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)}-1\right)}\left(1-e^{-\propto\left(e^{\beta}-1\right)}\right)^{-n}$$

We are take ln of both sides

$$lnL = nln\propto + \beta\left(1-e^{-\xi\sum_{i=1}^{n}x_i}\right) + nln\xi - \xi\sum_{i=1}^{n}x_i - \frac{\propto}{\beta}\left(\left(e^{\beta\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)}-1\right)\right)$$

1- If $\beta\ and\ \xi$ are known: $\frac{\partial lnL}{\partial\propto} = \frac{n}{\propto} - \frac{1}{\beta}\left(e^{\beta\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)}-1\right)$

2- If $\propto\ and\ \xi$ are known:

$$\frac{\partial lnL}{\partial\beta} = \frac{n}{\beta} + \left(1-e^{-\xi\sum_{i=1}^{n}x_i}\right) - \propto\left[\frac{\beta\left(\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)\right)\left(e^{\beta\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)}\right) - \left(e^{\beta\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)}-1\right)}{\beta^{2}}\right]$$

3- If $\propto,\beta$ are known:

$$\frac{\partial lnL}{\partial\xi} = \beta\sum_{i=1}^{n}x_i\, e^{-\xi\sum_{i=1}^{n}x_i} + \frac{n}{\xi} - \sum_{i=1}^{n}x_i - \propto\left[\left(e^{\beta\sum_{i=1}^{n}\left(1-e^{-\xi x_i}\right)}\right)\left(\sum_{i=1}^{n}x_i\, e^{-\xi\sum_{i=1}^{n}x_i}\right)\right.$$

The (M.L.E) of the formula parameters are calculated by solution numerically of the equation (9)

$\frac{\partial lnL}{\partial\propto} = 0, \frac{\partial lnL}{\partial\beta} = 0,$ and $\frac{\partial lnL}{\partial\xi} = 0$

7- Application:

In this a section we are apply of real data sets of model [0,1] Truncated Gompertz-Exponential Distributionin order to get on result better of through comparison the values statistical standards(-LL, AIC, CAIC, BIC, HQIC) with some models such as Beta Exponential distribution (BE), Kumaraswamy Exponential distribution (KuE), Exponential Generalized Exponential distribution (EGE), Weibull Exponential distribution (WeE), Gompertz Exponential distribution (GoE), Exponential distribution (E), by using R software.

We consider the data set, such that it is the life of fatigue of Kevlar 373/epoxy that are subject to constant pressure at the 90% stress level until all had failed. Alizadeh et al ,(2017),[24]. The data set are given as:

(0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960)

Var	n	mean	sd	median	Min	Max	Skew	Kurtosis
	76	1.96	1.57	1.74	0.03	9.1	1.94	4.95

Table (1) is represented description arithmetic data.

We are note in the table(1) the value of the skewness is positive, and this indicates that thedata is skewed to the right of the function . As for kurtosis, its value is positive, and this means that the data with thin flatness.

Model	Est para	-LL	AIC	CAIC	BIC	HQIC
[0,1] TGO-E	0.060 3.832 0.623	120.2	246.4	246.8	253.4	249.2
BE	1.1679 1.523 0.480	122.2	250.4	250.7	257.4	253.2
KuE	1.556 2.441 0.328	122.0	250.1	250.5	257.1	252.9
EGE	0.498 1.709 1.410	122.2	250.4	250.8	257.4	253.2
WE	1.325 2.431 1.139	122.5	251.0	251.3	258.0	253.8
	0.742					

GoE	0.219 0.554	125.3	256.7	257.0	263.7	259.5
E	0.510	127.1	256.2	256.2	258.5	257.1

Table(2)is represented the values of statistically criteria (-LL,AIC,CAIC BIC,HQIC).

Also we have analyzed the data by using R software in order to expense the statistically standards , in the table (2) we got to results through the values of the statistically criteria (-LL, AIC, CAIC, BIC, HQIC), it is shows the distribution of the proposed showed more matching data being achieved less the value of these statistical standards.Figure (4) show is histogram and estimated of (pdf) and it is show empirical and estimated of (cdf) for [0,1] Truncated Gompertz-Exponential Distribution.

(10)

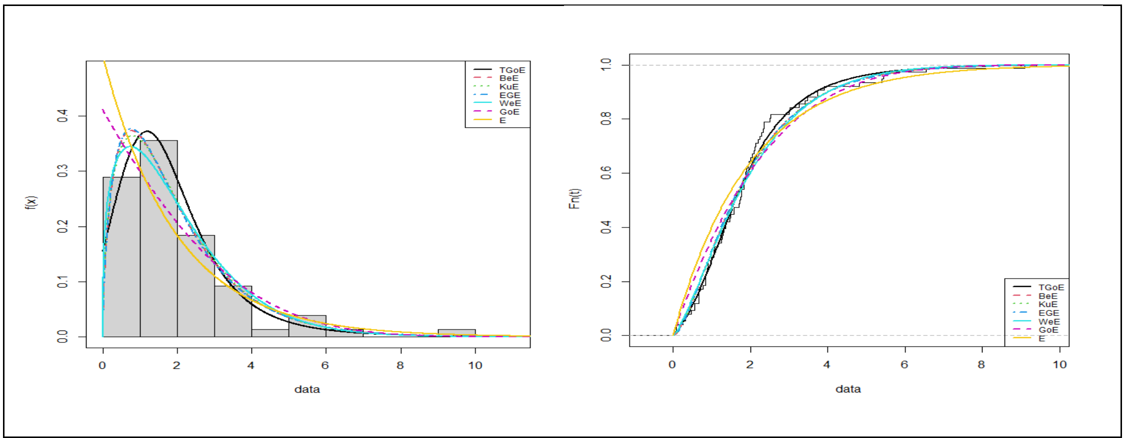
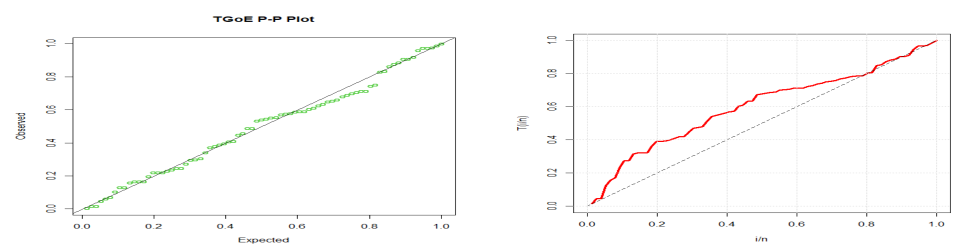


Figure (4) plot of estimated (pdf) and(cdf) for [0,1] TGO-EXP Distribution for the real data by using R software.

Figure (5) plot of estimated (5-i) estimated of observed and (5-ii) plot of total time on test (TTT) is shows path of the data an upside of the linegraph is represented failure rate function but we make clear how [0,1] Truncated Gompertz-Exponential distribution absorption (accommodating) the failure rate of the data. by using R software which support the results in above table.



(i) (ii)

Figure (5) plot (i) observed (ii) (TTT) for [0,1] TGO-EXP distribution for the real data by using R software.

8- Conclusion:

In this paper, a new family distribution of [0,1] TGO-EXP has been introduced The MLE method was used for estimating the three parameters of distribution. It was introduced some statistical properties of [0,1] TGO-EXP distribution such as hazard ratefunction, quantile function, moments,moment generating functions,skewness, kurtosis, entropy,Asymptotic behavior and order statistics. The a new family distribution has less the value of each the statistically criteria(LL, AIC, CAIC, BIC and HQIC). So it is shown that it offers better fit and more flexibility than many other comparative models such as Beta Exponential distribution (BE), Kumaraswamy Exponential distribution (KuE), Exponential Generalized Exponential distribution (EGE), Weibull Exponential distribution (WE), Gompertz Exponential distribution (GoE), Exponential distribution (E).It is use in various fields of application, because the flexibility

9- References:

[1]. Galton, F. (1898). An examination into the registered speeds of american trotting horses, with remarks on their value as hereditary data. Proceedings of the Royal Society of London, 62(379-387), 310-315

[2]. Johnson, N. L., Kotz, S., & Balakrishnan, N. (1995). Continuous univariate distributions, volume 2 (Vol. 289). John wiley& sons.

[3].Ohishi K, Okamura H, Dohi T. Gompertz software reliability model: Estimation algorithm and empirical validation. J. of Syst. And Software. 2009;82(3):535–543.

[4].Torabi H, Montazari NH. The gamma-uniform distribution and its application. Kybernetika.2012;48:16– 30.

(11)

[5].Eugene N, Lee C, Famoye F. Beta-normal distribution and it applications. Comm. In Stat.: Theo. And Meth. 2002;31:497-512.

[6].Cordeiro GM, Ortega EMM, da Cunha DCC. The exponentiated generalized class of distribution. J. of Data Sci. 2013;11:1–27.

[7].Cordeiro, G. M., Ortega, E. M., Popović, B. V., &Pescim, R. R. (2014). The Lomax generator of distributions: Properties, minification process and regression model. Applied Mathematics and Computation, 247, 465-486.

[8].Alizadeh, M., Cordeiro, G. M., De Brito, E., &Demétrio, C. G. B. (2015). The beta Marshall-Olkin family of distributions. Journal of Statistical Distributions and Applications, 2(1), 1-18.

[9].Yousof, H. M., Afify, A. Z., Nadarajah, S., Hamedani, G., &Aryal, G. R. (2018). The Marshall-

Olkin generalized-G family of distributions with Applications. Statistica, 78(3), 273-295.
[10]. El-Gohary, A., Alshamrani, A., & Al-Otaibi, A. N. (2013). The generalized Gompertz distribution. Applied mathematical arginal, 37(1-2), 13-24.
[11]. Gupta, R. D., & Kundu, D. (1999). Theory & methods: Generalized exponential distributions. Australian & New Zealand Journal of Statistics, 41(2), 173-188.
[12]. Jafari, A. A., Tahmasebi, S., & Alizadeh, M. (2014). The beta-Gompertz distribution. Revista
Colombiana de Estadística, 37(1), 141-158.
[13]. Omale, A., Asiribo, O. E., & Yahaya, A. (2019). On properties and applications of Lomax-
Gompertz distribution. Asian Journal of Probability and Statistics, 1-17.
[14]. Alizadeh, M., Cordeiro, G. M., Pinho, L. G. B., & Ghosh, I. (2017). The Gompertz-G family of
distributions. Journal of Statistical Theory and Practice, 11(1), 179-207.
[15].Oguntunde, P. E., Khaleel, M. A., Ahmed, M. T., &Okagbue, H. I. (2019). The Gompertz
fréchet distribution: properties and applications. Cogent Mathematics & Statistics, 6(1),1568662.
[16]. Eghwerido, J., Zelibe, S., &Efe-Eyefia, E. (2020). Gompertz-alpha power iverted exponential
distribution: Properties and applications. Thailand Statistician, 18(3), 319-332.
[17]. Khaleel, M. A., Oguntunde, P. E., Ahmed, M. T., Ibrahim, N. A., &Loh, Y. F. (2020). The
Gompertz Flexible Weibull Distribution and its Applications. Malaysian Journal of Mathematical
Sciences, 14(1), 169-190.
[18]. Abid, S. H., & Abdulrazak, R. K. (2017). International Journal of Scientific World.
[19]. Altawil, J. A. (2019). [0, 1] Truncated Lomax–Uniform distribution with properties. Journal of
Interdisciplinary Mathematics, 22(8), 1415-1431.
[20].Atanda, O. D., Mabur, T. M., &Onwuka, G. I. (2020). A New Odd Lindley-Gompertz Distribution: Its Properties and Applications. Asian Journal of Probability and Statistics, 29-47.
[21]. Singh, S. K., Singh, U., & Sharma, V. K. (2014). The truncated Lindley distribution: Inference
and application. Journal of Statistics Applications & Probability, 3(2), 219.
[22]. Boshi, M. A. A., Abid, S. H., & Al-Noor, N. H. (2020, July). Generalized Gamma–Generalized Gompertz Distribution. In Journal of Physics: Conference Series (Vol. 1591, No
[23]. KHALEEL, M. A. (2017). GENERALIZED BURR TYPE X DISTRIBUTION AND ITS
PROPERTIES AND APPLICATIONS.
[24]. Alizadeh, M., Merovci, F., &Hamedani, G. G. (2017). Generalized transmuted family of

distributions: properties and applications. Hacettepe Journal of Mathematics and Statistics, 46(4), 645-667.