Fixed point Results on a CompleteSoftUsual MetricSpace

RamakantBhardwaj[a,b]

[a]Department of Mathematics, APS University Rewa(M.P.)
[b]Department of Mathematics, AMITY University Kolkata W.B.
Email: rkbhardwaj100@gmail.com

Abstract
The present work is related to the theory of fixed point in complete soft usual metric space, which are extension of well-known results on complete metric space as well as complete soft metric space. The obtained results can be used in decision making as well as based on uncertainty problems. Soft weak contractive mapping and soft generalized weak contractive mapping are used to obtained the results.

Keywords: - Soft usual metric space, soft weak contractive mapping, soft fixed point.

Mathematics Subject Classification: - 47H10, 54H25.

1. Introduction and preliminaries

Most of the real-world problems cannot be solved by the theory of crisp sets. Some of them can be solved by theory of probability or by theory of fuzzy sets.In 1999, Molodtsov [4] initiated a novel concept of soft set theory, which is a new mathematical tool for dealing with uncertainties. Soft set is a parameterized general mathematical tool which deals with a collection of approximate descriptions of objects.The detail about soft sets, soft fixed point, soft complete metric can be seen in [1-3, 5-8]

Definition 1.1: Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if X is a mapping from E into the set of all subsets of the set X, i. e. $F: E \to P(X)$, where $P(X)$ is the power set of X.

Definition 1.2: The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.This is denoted by $(F, A) \cap (G, B) = (H, C)$.

Definition 1.3: The union of two soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F, A) \cup (G, B) = (H, C)$.

Definition 1.4: The soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $\varepsilon \in A, F(\varepsilon) = \phi$ (null set).

Definition 1.5: A soft set (F, A) over X is said to be an absolute soft set, if for all $\varepsilon \in A, F(\varepsilon) = X$.

Definition 1.6: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c: A \to P(X)$ is mapping given by $F^c(\alpha) = X - F(\alpha), \forall \alpha \in A$.

Definition 1.7: Let $\mathfrak{R}$ be the set of real numbers and $B(\mathfrak{R})$ be the collection of all nonempty bounded subsets of $\mathfrak{R}$ and E taken as a set of parameters. Then a mapping $F: E \to B(\mathfrak{R})$ is called a soft real set. It is denoted by (F, E). If specifically,(F, E) is a singleton soft set, then identifying (F, E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

Definition 1.8: For two soft real numbers

(i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(e) \leq \tilde{s}(e)$, for all $e \in E$.

(ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(e) \geq \tilde{s}(e)$, for all $e \in E$.

(iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(e) < \tilde{s}(e)$, for all $e \in E$.

(iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(e) > \tilde{s}(e)$, for all $e \in E$.

Definition 1.9: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \phi, \forall e' \in E\backslash\{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 1.10: Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = e'$ and $P(e) = P(e')$ i.e. $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 1.11: A mapping $|.|: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$, is said to be a usual soft metric on the soft set $\tilde{X}$If $|.|$ satisfies the following conditions:

(M1) $\tilde{|}(\tilde{x}_{e_1} - \tilde{y}_{e_2})| \tilde{\geq} \bar{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M2) $|(\tilde{x}_{e_1} - \tilde{y}_{e_2})| = \bar{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

(M3) $|(\tilde{x}_{e_1} - \tilde{y}_{e_2}) = \tilde{|}(\tilde{y}_{e_2} - \tilde{x}_{e_1})|$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M4) $\tilde{|}(\tilde{x}_{e_1} - \tilde{z}_{e_3})| \tilde{\leq} \tilde{|}(\tilde{x}_{e_1} - \tilde{y}_{e_2})| + \tilde{|}(\tilde{y}_{e_2} - \tilde{z}_{e_3})|$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with a soft mod ($|.|$) on $\tilde{X}$ is called a soft usual metric space and denoted by $(\tilde{X}, |.|, E)$.

Definition 1.13 (Cauchy Sequence in usual soft metric): A sequence $\{\tilde{x}_{\lambda,n}\}_n$ of soft points in $(\tilde{X}, \tilde{|}|, E)$ is considered as a Cauchy sequence in $\tilde{X}$if corresponding to every $\tilde{\varepsilon} \tilde{\geq} \bar{0}, \exists m \in N$ such that $|(\tilde{x}_{\lambda,i} - \tilde{x}_{\lambda,j})| \tilde{\leq} \tilde{\varepsilon}, \forall\, i, j \geq m$, i.e. $|(\tilde{x}_{\lambda,i} - \tilde{x}_{\lambda,j})| \to \bar{0}$, as $i, j \to \infty$.

Definition 1.14 (Complete soft Usual Metric Space): A soft usual metric space $(\tilde{X}, ||, E)$ is called soft complete usual , if every Cauchy Sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.

2. **Main Results:** Some soft fixed-point theorems are established for Usual soft metric space

Theorem 2.1: Let $(f, \varphi) : (\tilde{X}, ||, E) \to (\tilde{X}, ||, E)$, where$(\tilde{X}, |, |, E)$is ausual soft metric space. if for all $\tilde{x}_\lambda, \tilde{y}_\mu \in SP(X)$,

$$\tilde{|}\left((f,\varphi)(\tilde{x}_\lambda) - (f,\varphi)(\tilde{y}_\mu)\right)| \leq \frac{1}{2}\left[\left(\tilde{x}_\lambda - (f,\varphi)(\tilde{y}_\mu)\right)| + |\left(\tilde{y}_\mu - (f,\varphi)(\tilde{x}_\lambda)\right)|\right]$$

$$-\psi\left(|\left(\tilde{x}_\lambda - (f,\varphi)(\tilde{y}_\mu)\right)|, |\left(\tilde{y}_\mu - (f,\varphi)(\tilde{x}_\lambda)\right)|\right) \quad (2.1.1)$$

Where $\psi : [0,\infty)^2 \to [0,\infty)$ is a continuous mapping such that $\psi(\tilde{x}_\lambda, \tilde{y}_\mu) = 0$ if and only if $\tilde{x}_\lambda = \tilde{y}_\mu = 0$.Then (f, φ) has a unique soft fixed point.

Proof: Let $\tilde{x}_\lambda^0$ be any soft point in $SP(X)$.

Set $\tilde{x}_{\lambda_1}^1 = (f, \varphi)(\tilde{x}_\lambda^0) = \left(f(\tilde{x}_\lambda^0)\right)_{\varphi(\lambda)}$

– – – – – – – – – – – – – – –

$$\tilde{x}^{n+1}_{\lambda_{n+1}} = (f,\varphi)\left(\tilde{x}^{n}_{\lambda_n}\right) = \left(f^{n+1}(\tilde{x}^{0}_{\lambda})\right)_{\varphi^{n+1}(\lambda)}, --$$

If $\tilde{x}^{n}_{\lambda_n} = (f,\varphi)\left(\tilde{x}^{n-1}_{\lambda_{n-1}}\right)$ then $\tilde{x}^{n}_{\lambda_n}$ is a fixed point of (f,φ).

So we assume $\tilde{x}^{n}_{\lambda_n} \neq \tilde{x}^{n+1}_{\lambda_{n+1}}$

Putting $x = \tilde{x}^{n-1}_{\lambda_{n-1}}$ and $y = \tilde{x}^{n}_{\lambda_n}$ in (2.1.1) we have for all $n = 0,1,2, \ldots ....$

$$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| = \tilde{l}\left((f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)|$$

$$\leq \frac{1}{2}\left[|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)| + |\left(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\right)|\right]$$

$$-\psi\left(|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)|, |\left(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\right)|\right)$$

$$\leq \frac{1}{2}\left[|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| + |\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n}_{\lambda_n}\right)|\right] - \psi\left(|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, |\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n}_{\lambda_n}\right)|\right)$$

$$\leq \frac{1}{2}\left[|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| + 0\right] - \psi\left(|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0\right)$$

$\psi\left(\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0\right) = 0$, and $\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq \frac{1}{2}\left[|\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|\right]$

$$|\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq S\,\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n}_{\lambda_n}\right)|$$

$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq S^{n}\tilde{l}\left(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{1}_{\lambda_1}\right)|$ Where $S = \frac{1/2}{1-1/2}$

That is

$$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq S^{n}\tilde{l}\left(\tilde{x}^{0}_{\lambda_0} - \tilde{x}^{1}_{\lambda_1}\right)|$$

For any $n > m, \ m,n \in N$

$$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{m}_{\lambda_m}\right)| \leq \frac{S^{m}}{1-S}\, |\left(\tilde{x}^{0}_{\lambda_0} - \tilde{x}^{1}_{\lambda_1}\right)|$$

Since $0 \leq S < 1$ and as $n \to \infty$ then

$$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n}, \tilde{x}^{m}_{\lambda_m}\right)| = 0.$$

That is $\{\tilde{x}^{n}_{\lambda_n}\}$ is a soft Cauchy sequence, By the soft completeness of $\tilde{X}$, there is $\tilde{x}^{*}_{\lambda} \in \tilde{X}$ such that $\tilde{x}^{n}_{\lambda_n} \to \tilde{x}^{*}_{\lambda}, n \to \infty$.Let $\tilde{x}^{n}_{\lambda_n} \to \tilde{x}^{*}_{\lambda}$ as $n \to \infty$

Then

$$|(\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{x}^{*}_{\lambda})| \leq \left(\tilde{x}^{*}_{\lambda} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right) + \tilde{l}\left(\tilde{x}^{n+1}_{\lambda_{n+1}} - (f,\varphi)\tilde{x}^{*}_{\lambda}\right)|$$

$$\leq \tilde{l}\left(\tilde{x}^{*}_{\lambda} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| + \tilde{l}\left((f,\varphi)\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{*}_{\lambda}\right)|$$

$$\leq \tilde{l}(\tilde{x}^*_\lambda - \tilde{x}^{n+1}_{\lambda_{n+1}})| + \frac{1}{2}[|(\tilde{x}^n_{\lambda_n} - (f,\varphi)\tilde{x}^*_\lambda)| + |(\tilde{x}^*_\lambda - (f,\varphi)\tilde{x}^n_{\lambda_n})|]$$

$$-\psi(|(\tilde{x}^n_{\lambda_n} - (f,\varphi)\tilde{x}^*_\lambda)|, |(\tilde{x}^*_\lambda - (f,\varphi)\tilde{x}^n_{\lambda_n})|)$$

$\Rightarrow \tilde{l}(\tilde{x}^*_\lambda - (f,\varphi)\tilde{x}^*_\lambda)| \leq \frac{1}{2}\tilde{l}(\tilde{x}^*_\lambda - (f,\varphi)\tilde{x}^*_\lambda)|$

This contradicts the assumption. So $\tilde{x}^*_\lambda = (f,\varphi)\tilde{x}^*_\lambda$. $\tilde{x}^*_\lambda$ is a soft fixed point of (f,φ) for usual soft metric space

Uniqueness: If $\tilde{y}^*_\mu$ is anothersoft fixed point of (f,φ) in $\tilde{X}$ such that $\tilde{x}^*_\lambda \neq \tilde{y}^*_\mu$,

$$\tilde{l}(\tilde{x}^*_\lambda - \tilde{y}^*_\mu) = \tilde{l}\left((f,\varphi)\tilde{x}^*_\lambda - (f,\varphi)\tilde{y}^*_\mu\right)|$$

$$\leq \tilde{l}\left((f,\varphi)\tilde{x}^*_\lambda - (f,\varphi)\tilde{x}^n_{\lambda_n}\right)| + \tilde{l}\left((f,\varphi)\tilde{x}^n_{\lambda_n} - (f,\varphi)\tilde{y}^*_\mu\right)|$$

$$\leq \tilde{l}\left((f,\varphi)\tilde{x}^*_\lambda - (f,\varphi)\tilde{x}^n_{\lambda_n}\right) + \frac{1}{2}[|(\tilde{x}^n_{\lambda_n} - (f,\varphi)\tilde{y}^*_\mu)| + |(\tilde{y}^*_\mu - (f,\varphi)\tilde{x}^n_{\lambda_n})|]$$

$$-\psi(|(\tilde{x}^n_{\lambda_n} - (f,\varphi)\tilde{y}^*_\mu)|, |(\tilde{y}^*_\mu - (f,\varphi)\tilde{x}^n_{\lambda_n})|)$$

$n \to \infty$, $\tilde{l}(\tilde{x}^*_\lambda - \tilde{y}^*_\mu)| \leq 0$.This contradict the assumption so$\tilde{x}^*_\lambda = \tilde{y}^*_\mu$ that is, $\tilde{x}^*_\lambda$ is unique soft fixed point of (f,φ) in usual soft metric space.

Theorem 2.2: Let $(f,\varphi) : (\tilde{X}, \widetilde{|.|}, E) \to (\tilde{X}, \widetilde{|.|}, E)$, (where$(\tilde{X}, |\,.\,|, E)$is a complete usual soft metric space) be a soft generalized weak contraction that is the following condition is true

if for all $\tilde{x}_\lambda, \tilde{y}_\mu \in SP(X)$,

$$\left|\left((f,\varphi)(\tilde{x}_\lambda) - (f,\varphi)(\tilde{y}_\mu)\right)\right| \leq \rho\left[\max\left\{\begin{matrix} |(\tilde{x}_\lambda, (f,\varphi)(\tilde{x}_\lambda)) - d\left(\tilde{y}_\mu, (f,\varphi)(\tilde{y}_\mu)\right)|, \\ |\left(\tilde{x}_\lambda, (f,\varphi)(\tilde{y}_\mu)\right)| - |\left(\tilde{y}_\mu, (f,\varphi)(\tilde{x}_\lambda)\right) - (\tilde{x}_\lambda, \tilde{y}_\mu)| \end{matrix}\right\}\right]$$

$$-\psi\left(\begin{matrix} |(\tilde{x}_\lambda - (f,\varphi)(\tilde{x}_\lambda))|, |\left(\tilde{y}_\mu - (f,\varphi)(\tilde{y}_\mu)\right)|, |\left(\tilde{x}_\lambda - (f,\varphi)(\tilde{y}_\mu)\right)|, \\ |\left(\tilde{y}_\mu - (f,\varphi)(\tilde{x}_\lambda)\right)|, |(\tilde{x}_\lambda, \tilde{-}\tilde{y}_\mu|) \end{matrix}\right)$$

(2.2.1)

Where $\boldsymbol{\rho} \in [0,1), \psi : [0,\infty)^5 \to [0,\infty)$ is a continuous mapping :

$\psi(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5) = 0$ if and only if one of $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5$ is equal to 0.Then (f,φ) has a uniquesoft fixed point.

Proof: Let $\tilde{x}^0_\lambda$be any soft point in usual soft metric space

Set $\tilde{x}^1_{\lambda_1} = (f,\varphi)(\tilde{x}^0_\lambda) = \left(f(\tilde{x}^0_\lambda)\right)_{\varphi(\lambda)}$

$$--------------$$

$$\tilde{x}^{n+1}_{\lambda_{n+1}} = (f,\varphi)(\tilde{x}^n_{\lambda_n}) = \left(f^{n+1}(\tilde{x}^0_\lambda)\right)_{\varphi^{n+1}(\lambda)}, --$$

If $\tilde{x}^n_{\lambda_n} = (f,\varphi)(\tilde{x}^{n-1}_{\lambda_{n-1}})$ then $\tilde{x}^n_{\lambda_n}$ is a fixed point of (f,φ).

Taking $\tilde{x}^{n}_{\lambda_n} \neq \tilde{x}^{n+1}_{\lambda_{n+1}}$

Putting $x = \tilde{x}^{n-1}_{\lambda_{n-1}}$ and $y = \tilde{x}^{n}_{\lambda_n}$, for all $n = 0,1,2,\ldots\ldots$

$$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| = l\left(\left((f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)\right|$$

$$\leq \rho\left[\max\left\{\begin{matrix}\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\right)|, \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)|, l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)|, \\ \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\end{matrix}\right\}\right]$$

$$-\psi\left(\begin{matrix}\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\right)|, l\left(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)|, \\ \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{n-1}_{\lambda_{n-1}}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\end{matrix}\right)$$

$$\leq \rho\left[\max\left\{\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right), \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\right\}\right]$$

$$-\psi\left(\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|, \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)\right)$$

(2.2.2)

By the condition of the theorem of generalized week contraction

$\psi\left(l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|, l\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\right) = 0$ And

$$l\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq \alpha\left[\max\left\{l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} -, \tilde{x}^{n}_{\lambda_n}\right)|, \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\right\}\right] \quad (2.2.3)$$

Case I: If we choose

$$\max\left\{\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|, \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} -, \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0, l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\right\} = \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|$$

Now by (2.2.3)

$$l\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq \rho\, l\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|$$

Similarly, we can write, $\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)| \leq \rho\, \tilde{l}\left(\tilde{x}^{n-2}_{\lambda_{n-2}} - \tilde{x}^{n-1}_{\lambda_{n-1}}\right)|$

$$l\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq \rho^n \tilde{l}\left(\tilde{x}^{0}_{\lambda_0} - \tilde{x}^{1}_{\lambda_1}\right)|$$

For any $n > m,\ m,n \in N$

$$\tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{m}_{\lambda_m}\right)| \leq \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n-1}_{\lambda_{n-1}}\right)| + \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n-2}_{\lambda_{n-2}}\right)| + \ldots\ldots\ldots + \tilde{l}\left(\tilde{x}^{m+1}_{\lambda_{m+1}} - \tilde{x}^{m}_{\lambda_m}\right)|$$

$$l\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{m}_{\lambda_m}\right)| \leq \frac{\rho^m}{1-\rho}\tilde{l}\left(\tilde{x}^{0}_{\lambda_0} - \tilde{x}^{1}_{\lambda_1}\right)|$$

Since $0 \leq \rho < 1$ and as $n \to \infty, \Rightarrow \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{m}_{\lambda_m}\right)| = 0.$

$\left\{\tilde{x}^{n}_{\lambda_n}\right\}$ is a soft Cauchy sequence, By the completeness of $\tilde{X}$, there is $\tilde{x}^{*}_{\lambda} \in \tilde{X}: \tilde{x}^{n}_{\lambda_n} \to \tilde{x}^{*}_{\lambda}, n \to \infty.$

Case – 2: If

$$\max\left\{\tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|, \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|, 0, \tilde{l}\left(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n}\right)|\right\} = \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|$$

$\Rightarrow \quad \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)| \leq \rho\, \tilde{l}\left(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}\right)|$, Since $0 \leq \rho < 1$, this is contradiction.

Case-3: If

$$\max\{\tilde{l}(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n})|, \tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}})|, \tilde{l}(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}})|, 0, \tilde{l}(\tilde{x}^{n-1}_{\lambda_{n-1}}, -\tilde{x}^{n}_{\lambda_n})|\} = |(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n+1}_{\lambda_{n+1}})|$$

Then 2.2.3 can be written as $|(\tilde{x}^{n}_{\lambda_n} -, \tilde{x}^{n+1}_{\lambda_{n+1}})| \leq \rho\, \tilde{d}(\tilde{x}^{n-1}_{\lambda_{n-1}}, \tilde{x}^{n+1}_{\lambda_{n+1}})$

Using the property of Usual soft metric space as triangular inequality

$$\tilde{l}(\tilde{x}^{n}_{\lambda_n} - x\lambda^{n+1}_{n+1})| \leq \rho\, [|(\tilde{x}^{n-1}_{\lambda_{n-1}} - \tilde{x}^{n}_{\lambda_n})| + \tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}})|]$$

$\Rightarrow \tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}}|) \leq \delta^n \tilde{d}(\tilde{x}^{0}_{\lambda_0}, \tilde{x}^{1}_{\lambda_1})$ Let $\frac{\rho}{1-\rho} = \delta$

$\Rightarrow \{\tilde{x}^{n}_{\lambda_n}\}$ is a soft Cauchy sequence $\dashrightarrow \tilde{x}^{*}_{\lambda} \in \tilde{X}$.

It is clear that $\{\tilde{x}^{n}_{\lambda_n}\}$ is a soft Cauchy sequence and converges to $\tilde{x}^{*}_{\lambda} \in \tilde{X}$because $\tilde{X}$is taken to be complete. Let $\tilde{x}^{n}_{\lambda_n} \to \tilde{x}^{*}_{\lambda}$ as $n \to \infty$.

$$\tilde{l}(\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{x}^{*}_{\lambda})| \leq \tilde{l}(\tilde{x}^{*}_{\lambda} - \tilde{x}^{n+1}_{\lambda_{n+1}})| + \tilde{l}(\tilde{x}^{n+1}_{\lambda_{n+1}} - (f,\varphi)\tilde{x}^{*}_{\lambda})|$$

$$\leq \tilde{l}(\tilde{x}^{*}_{\lambda} - \tilde{x}^{n+1}_{\lambda_{n+1}})| + \tilde{l}\left((f,\varphi)\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{*}_{\lambda}\right)|$$

$$\leq \tilde{l}(\tilde{x}^{*}_{\lambda} - \tilde{x}^{n+1}_{\lambda_{n+1}})|$$

$$+\rho\left[\max\left\{\begin{matrix}\tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}})|, \tilde{l}(\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{x}^{*}_{\lambda})|, |(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{*}_{\lambda})|, \\ \tilde{l}(\tilde{x}^{*}_{\lambda}, \tilde{-}x^{n+1}_{\lambda_{n+1}})|, \tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{*}_{\lambda})|\end{matrix}\right\}\right]$$

$$-\psi\begin{pmatrix}\tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{n+1}_{\lambda_{n+1}})|, \tilde{l}(\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{x}^{*}_{\lambda})|, \tilde{l}(\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{x}^{*}_{\lambda}), \\ \tilde{l}(\tilde{x}^{*}_{\lambda} - \tilde{x}^{n+1}_{\lambda_{n+1}})|, \tilde{l}(\tilde{x}^{n}_{\lambda_n} - \tilde{x}^{*}_{\lambda})|\end{pmatrix}$$

$$|(\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{x}^{*}_{\lambda})| \leq \rho\tilde{l}(\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{x}^{*}_{\lambda})|$$

This is a contradiction. So $\tilde{x}^{*}_{\lambda} = (f,\varphi)\tilde{x}^{*}_{\lambda}$.That is $\tilde{x}^{*}_{\lambda}$ is a fixed soft point of(f,φ).
Uniqueness: If$\tilde{y}^{*}_{\mu}$ is another soft fixed point of (f,φ) in$\tilde{X}$:such that $\tilde{x}^{*}_{\lambda} \neq \tilde{y}^{*}_{\mu}$
$|(\tilde{x}^{*}_{\lambda} - \tilde{y}^{*}_{\mu})| = \tilde{l}\left((f,\varphi)\tilde{x}^{*}_{\lambda} - (f,\varphi)\tilde{y}^{*}_{\mu}\right)|$

$\leq |\left((f,\varphi) -, (f,\varphi)\tilde{x}^{n}_{\lambda_n}\right)| + |\left((f,\varphi)\tilde{x}^{n}_{\lambda_n} - (f,\varphi)\tilde{y}^{*}_{\mu}\right)|$

Taking $n \to \infty \Rightarrow |(\tilde{x}^{*}_{\lambda} - \tilde{y}^{*}_{\mu})| \leq 0$.This contradicts the assumption $\Rightarrow \tilde{x}^{*}_{\lambda} = \tilde{y}^{*}_{\mu}$ that is, $\tilde{x}^{*}_{\lambda}$ is unique soft fixed point of (f,φ).

REFERENCES

1. Ali, MI, Feng, F. Liu, XY, Min, WK, Shabir, M., On some new operations in soft set theory, Math. Appl. 62, 351-358(2011).
2. Chen, D., the parameterization reduction of soft sets and its applications, Comput. Math. Appl. 49,, 757-763(2010).
3. Das,SSamanta, SK, Soft real sets, soft real numbers and their properties. J.Fuzzy Math. 20(3),551-576(2012).
4. Molodtsov, D. A., Soft set theory – first results, Comput. Math. Appl. 37, 19-31(1999)
5. Roy A., P.K. Maji and R. Biswas, An application of soft sets in a decision making problem, “Comput. Math. Appl.” 44,1077–1083(2002)
6. Roy A., P.K. Maji and R. Biswas, Soft set theory, “Computers & Mathematics with Applications” 45no. 4-5, 555–562 (2003)

7. Sujoy Das and S. K. Samanta, "Soft metric", Annals of Fuzzy Mathematics and Informatics, 6(1) ,77-94 (2013)
8. WadkarBalajiRaghunath Rao, Ramakant Bhardwaj, Rakesh Mohan Sharraf, " Couple fixed point theorems in soft metric spaces" Materials Today Proceedings 29 P2,617-624, (2020)
9. Rhoades, B.E, Some theorems on weakly contractive maps, Nonlinear Analysis, 47 , 2683-2693(2001)