Fair Dominating sets and Fair domination polynomial of a Wheel graph

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ABSTRACT

Let $G = (V, E)$ be a simple graph. A set $S \subseteq V$ is a fairdominating set of G, if every vertex not in S is adjacent to one or more vertices in S. A dominating set S of G is a fair dominating set if every two vertices $u, v \in V(G) - S$ are dominated by same number of verticesfrom S. The minimum cardinality taken over all fair dominating sets in G is called the fair domination number of G and is denoted by $\gamma_f(G)$. Let $W_{1,n}$be wheel graph of order $n + 1$. Let $W_{1,n}^i$ be the family of all fair dominating sets of a wheel$W_{1,n}$with cardinality i, and let $d_f(W_{1,n}, i) = |W_{1,n}^i|$. In this paper, we explore the fair domination polynomial of a wheel graph and also more properties are obtained in it.

Keywords:dominating sets, domination polynomial, fair dominating sets, fair domination polynomial.
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1. Introduction

Consider$G = (V, E)$be a simple, finite and undirected graph, where $|V(G)| = n$ denotes the number of vertices and $|E(G)| = m$ denotes the number of edges of G. For any undefined term in this paper we refer Harary[9].

A set $S \subseteq V(G)$ is a dominating set if every vertex not in S is adjacent to one or more vertices in S. The minimum cardinality taken over all dominating sets in G is called domination number of G and is called the domination number of G and is denoted by $\gamma(G)$. For complete review on the theory of domination and its related parameters we refer [10] and [11].

A dominating set$S$ is a fair dominating set if every two vertices $u, v \in V(G) - S$ are dominated by the same number of vertices from S. The minimum cardinality taken over all fair dominating sets in G is called the fair domination number of$G$ and is denoted by $\gamma_f(G)$. the concept of fair domination was first introduced by Yar Coro et al [6]. For more details on fair domination we refer [7].

A domination polynomial of a graph G is the polynomial $D(G, x) = \sum_{i=1}^{n} d(G, i)x^i$, where $d(G, i)$ is the number of dominating sets of G of cardinality i. For more details on domination polynomial we refer [3], [4] and [5].

Analogously, a fair domination polynomial of a graph G of order n is the polynomial$D_f(G, x) = \sum_{i=\gamma_f(G)}^{n} \mathrm{d}_f(G, i)x^i$, where $\mathrm{d}_f(G, i)$ is the number of fair dominating sets of G of cardinality i.

An element a is said to be a zero of a polynomial $f(x)$if $f(x) = 0$. An element a is called zero of a polynomial of multiplicity m if $(x - a)^m / f(x)$ and $(x - a)^{m+1}$ is not a divisor of $f(x)$.

The number of distinct subsets with $r$vertices that can be selected from a set with n vertices is denoted by $\binom{n}{r}$or $nC_r = \frac{n!}{(n-r)!r!}$. This number $\binom{n}{r}$ is called a binomial coefficient.

Let $W_{1,n}^i$ be the family offair dominating sets of a wheel graph$W_{1,n}$of cardinality i, and let $d_f(W_{1,n}, i) = |W_{1,n}^i|$. We call the polynomial $D_f(W_{1,n}, x) = \sum_{i=1 \, or \, \gamma_f(G)}^{n} \mathrm{d}_f(W_{1,n}, i)x^i$ the fair domination polynomial of wheel. Similarly the fair domination polynomial of star graph$S_{1,n}$ and cycle graph C_n are$D_f(S_{1,n}, x)$ and $D_f(C_n, x)$ respectively.

2. FairDominating sets of Wheel graph $(W_{1,n})$

Let $W_{1,n}, n \geq 3$ be the wheel graph with $n+1$ vertices $V(W_{1,n}) = \{0,1,2,\dots,n\}$and $E(W_{1,n}) = \{(0,1),(0,2),(0,3),\dots,(0,n),(1,2),(2,3),(3,4),\dots,(n-1,n),(n,1)\}$.

In this section, we investigate dominating sets ofwheels.

To prove our main results we need the following lemma:

Lemma: 2.1

For any cycle graph C_n with n vertices,

i. $d_f(C_n, i) = 0$ if $1 < \left\lceil \frac{n}{3} \right\rceil + 1$or $i > n$.
ii. $d_f(C_n, n) = 1$
iii. $d_f(C_n, n-1) = n$
iv. $d_f(C_n, n-2) = \binom{n}{2}$.

Theorem:2.2

For$n \geq 3$, a wheel graph$W_{1,3n}$ may not have a fair dominating set of cardinality $n+2$.

Proof:

Consider$W_{1,3n}$where $n \geq 3$. We shall find a fair dominating set S of cardinality $n+2$in $W_{1,3n}$. Since $n+2 < \left\lfloor \frac{n}{2} \right\rfloor$, not every element in $V(W_{1,n}) - S$ are independent. Then $V(W_{1,n}) - S$ contains at least two adjacent vertices. Since S is a fair dominating set of $W_{1,n}$, that $V(W_{1,n}) - S$ does not contain more than two adjacent vertices. We consider the following two cases:

Case (i): If every vertices in $V(W_{1,n}) - S$ forms induced union of path P_2. Then it is clear that S contains exactly $(n+1) -$ vertices. Hence this case fails.

Case (ii): If every vertices in $V(W_{1,n}) - S$need not forms induced union of path P_2. This means that $V(W_{1,n}) - S$ contains an induced path P_1. Assume v be the vertex of P_1. Then the vertices adjacent to $v$in $V(W_{1,n}) - S$ is dominated by three vertices of S and the remaining vertices in $V(W_{1,n}) - S$ are dominated by two vertices of S. So that S is not a fair dominating set.

Hence we cannot find a fair dominating set of cardinality $n+2$ for a wheel graph $W_{1,3n}$for $n \geq 3$.

Theorem: 2.3

For $n \geq 9$, a wheel graph $W_{1,n}$ not every power of x exists in a fair domination polynomial.

Proof:

Consider a wheel graph$W_{1,3n}$with$n \geq 9$ vertices. By Theorem 2.2, a wheel graph$W_{1,n}$ may not have a fair dominating set of particular cardinality. Hence the result follows.

Theorem: 2.4

For $i = 1$ or $\gamma_f(C_n) \leq i \neq n \leq n+1$, where $n \geq 5$, every centre vertex of $W_{1,n}$ lies in every fair dominating set of $W_{1,n}$ of cardinality i.

Proof:

Let v be a centre vertex of $W_{1,n}$ and let S be a fair dominating set of $W_{1,n}$ of cardinality i. To prove $r \in S$. If $i = 1$, then there is nothing to prove. Now assume $\gamma_f(C_n) \leq i \neq n \leq n+1$. Suppose $v \notin S$. Then $v \in V(W_{1,n}) - S$ has n neighbours in S. Since $i \neq n$, there exists a vertex v_j in C_n such that $v_j \notin S$.Clearly v_j has atmost three neighbours in$V(W_{1,n})$. Therefore v has atleast i neighbours in S and $v_j$has at most two neighbor verticesin$S$.Since $n \geq 5$, that $\gamma_f(C_n) \geq 3$ and so $i \geq 3$. It follows that S isnot a fair dominating set of $W_{1,n}$. Hence, $v \in S$.

Theorem: 2.5

Let$W_{1,n}$ be a wheel graph with $n \geq 4$ vertices. Then $d_f(W_{1,n}, i) \leq d_f(S_{1,n}, i) + d_f(C_n, i)$ for all i.

Proof:

Let $S_{1,n}$ be the star graph on $n+1$ vertices and let $v \in V(S_{1,n})$ be the centre vertex of $S_{1,n}$. Clearly $S_{1,n}$ be a spanning subgraph of $W_{1,n}$. Also $W_{1,n} - \{v\} = C_n$. Show that $W_{1,n} = S_{1,n} \cup C_n$. Therefore the number of dominating sets of $W_{1,n}$ with cardinality i is the sum of the number of dominating sets of $S_{1,n}$ with cardinality i and the number of dominating set of C_n with cardinality i. But in case of fair dominating sets, by theorem:2.2, $W_{1,n}$ does not have a fair dominating set of particular cardinality. Also by theorem:2.4, v lies on every fair dominating sets of $W_{1,n}$ for $i = 1$ or $\gamma_f(C_n) \leq i \neq n \leq n+1$. Therefore $d_f(W_{1,n}, i) \leq d_f(S_{1,n}, i) + d_f(C_n, i)$for $i < \gamma_f(C_n)$, $d_f(C_n, i) = 0$ and so $d_f(W_{1,n}, i) \leq d_f(S_{1,n}, i)$. For $i = n$, it is clear that $d_f(W_{1,n}, n) = n+1$. Also $d_f(S_{1,n}, n) = n+1$and $d_f(C_n, n) = 1$. Hence, for all i, $d_f(W_{1,n}, i) \leq d_f(S_{1,n}, i) + d_f(C_n, i)$.

Theorem: 2.6

For$n \geq 5$ and$1 < i \leq \gamma_f(C_n)$,there does not exists a fair dominating set of cardinality i.

Proof:

Let $n \geq 5$ and$1 < i \leq \gamma_f(C_n)$. Suppose there is a fair dominating set cardinality i. Let v be the certre vertex of $W_{1,n}$. We consider two cases:

Case (i)$v \in S$. Then by the choise of S, we choose $i-1$ vertices of S from $V(C_n)$. Since $i \leq \gamma_f(C_n)$, that $i-1$ vertices of C_n need not dominates every vertices in C_n. This shows that some vertices in $V(W_{1,n}) - S$ adjacent to these $i-1$ vertices are dominated by at most three vertices in S and not adjacent to these $i-1$ vertices are dominated by at most two vertices in S. So S is not a fair dominating set of $W_{1,n}$.

Case (ii)$v \notin S$. In this case the remaining $n-i$ vertices in C_n are dominated by at most two vertices in S or not dominated by any vertex of S and the vertex v is dominated by i vertices of S. Since $i \geq 2$, again that S is not a fair dominating set of $W_{1,n}$.

Hence there cannot be a fair dominating set of $W_{1,n}$ of cardinality $i$for $1 < i \leq \gamma_f(C_n)$.

Theorem: 2.7

Let$W_{1,n}$,$n \geq 5$ be the wheel graph with $V(W_{1,n}) = n+1$. Then,

i. $d_f(W_{1,n}, i) = 1$ if $i = 1$.
ii. $d_f(W_{1,n}, i) = 0$ if $1 < i < \left\lceil \frac{n}{3} \right\rceil + 1$ or $i > n+1$.
iii. $d_f(W_{1,n}, i) = d_f(C_n, i-1)$if $\left\lceil \frac{n}{3} \right\rceil + 1 < i \neq n \leq n+1$.
iv. $d_f(W_{1,n}, i) = 1 + d_f(C_n, i-1)$if $i = n$.

Proof:

Let v be a centre vertex of $W_{1,n}$.

i. For $i = 1$, it is clear that the centre vertex $\{v\}$ is the unique fair dominating set of cardinality i. Therefore,$d_f(W_{1,n}, i) = 1$if $i = 1$.
ii. Let $1 < i < \left\lceil \frac{n}{3} \right\rceil + 1$or $i > n+1$.
   The fair domination number of any cycle graph C_n, $n \geq 5$ is obtained as
$$\gamma_f(C_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil & if\ n \equiv 0\ or\ 1 (mod 3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 & if\ n \equiv 2\ (mod 3) \end{cases}$$
   If $1 < i < \gamma_f(C_n)$, then by theorem:6, we have, $d_f(W_{1,n}, i) = 0$. Moreover, there does not exist a fair dominating set of cardinality greater than $n+1$. Thus $d_f(W_{1,n}, i) = 0$ if $1 < i < \left\lceil \frac{n}{3} \right\rceil + 1$or $i > n+1$.
iii. Now, let $\left\lceil \frac{n}{3} \right\rceil + 1 < i \neq n \leq n+1$. Then by Theorem(4) v belongs to every fair dominating set of $W_{1,n}$. Let S be a fair dominating set of cardinality i. Since the remaining $i-1$ vertices of S for dominates the

vertices of C_n, every fair dominating set of $W_{1,n}$ of cardinality i contains the vertices which fair dominates the cycle C_n and $\{v\}$. Thus, if S' be a fair dominating set of C_n of cardinality $i-1$, then that $S = S' \cup \{v\}$.
Therefore, $d_f(W_{1,n}, i) = d_f(C_n, i-1)$.

iv. Further if $i = n$, then $d_f(W_{1,n}, i)$ contains a fair dominating set S of cardinality i with $v \notin S$. Therefore the number of fair dominating set of $W_{1,n}$ of cardinality i is one greater than the number of fair dominating set of C_n of cardinality $i-1$.
Hence $d_f(W_{1,n}, i) = 1 + d_f(C_n, i-1)$if $i = n$.

Theorem: 2.8

Let $W_{1,n}$, $n \geq 3$ be the wheel graph with$|V(W_{1,n})| = n+1$. Then the following properties are hold:

i. For $n \geq 3, d_f(W_{1,n}, n+1) = 1$.
ii. For $n \geq 3, d_f(W_{1,n}, n) = n+1$.
iii. For $n \geq 4, d_f(W_{1,n}, n-1) = \frac{n(n-1)}{2}$.
iv. For $k \geq 2, d_f(W_{1,3k}, k+1) = 3$.
v. For $n \geq 3, d_f(W_{1,3k}, k+2) = 0$.
vi. For $n \geq 3, d_f(W_{1,3k+1}, k+2) = 3k+1$.
vii. For $k \geq 3, d_f(W_{1,3k+2}, k+3) = 6k+4$
viii. $d_f(W_{1,n}, i)$is always a positive integer.

Proof

i. For any graph G with $n+1$ vertices. We have$d_f(G, n+1) = 1$. Hence $d_f(W_{1,n}, n+1) = 1$.

ii. For any graph G with $n+1$ vertices, and $\delta(G) \geq 1$, then we have $d_f(G, n) = 1$. Hence $d_f(W_{1,n}, n) = n+1$.

iii. By lemma:1,we have $d_f(C_n, n-2) = \binom{n}{2}$. Therefore by Theorem:2.7, we conclude $d_f(W_{1,n}, n-1) = \binom{n}{2} = \frac{n(n-1)}{2}$.

iv. Consider the wheelgraph $W_{1,3k}$, where$k \geq 2$. Then it has $3k+1$ vertices.The fair dominating sets of$W_{1,3k}$ of cardinality $k+1$ are $\{1,4,7,\ldots,3k-2\}, \{2,5,8,\ldots,3k-1\}and\{3,6,9,\ldots,3k\}$. Therefore we have3 fair dominating sets of $W_{1,3k}$ of cardinality $k+1$.
Hence $d_f(W_{1,3k}, k+1) = 3$.

v. This follow from Theorem:2.2.

vi. Consider the wheel graph $W_{1,3k+1}$. Then it has$3k+2$ vertices.The fair dominating set of $W_{1,3k+1}$of cardinality $k+2$ are$\{1,2,5,\ldots,3k-1\}, \{2,3,6,\ldots,3k\}$, $\{3,4,7,\ldots,3k+1\}, \ldots, \{3k+1,1,4,7,\ldots,3k-2\}$.
Therefore we have $3k+1$fair dominating sets of$W_{1,3k+1}$cardinality $k+2$. Hence $d_f(W_{1,3k+1}, k+2) = 3k+1$.

vii. Consider the wheel graph$W_{1,3k+2}$. Then it has$3k+3$ vertices.The fair dominating sets of$W_{1,3k+2}$ of cardinality $k+3$ are
$\{1,2,5,6,9,\ldots,3k\}, \{2,3,6,7,\ldots,3k+1\}, \{3,4,7,8,\ldots,3k+2\}, \ldots, \{3k+2,1,4,5,8,\ldots,3k-1\}, \{1,2,3,6,9,\ldots,3k\}, \{2,3,4,7,10,\ldots,3k+1\}, \{3,4,5,8,11,\ldots,3k+2\}, \ldots, \{3k+1,3k+2,1,4,7,\ldots,3k-2\}$.
Therefore we have$3k+2+3k+2$ fair dominating sets of cardinality $k+3$.Hence$d_f(W_{1,3k+2}, k+3) = 3k+2+3k+2 = 6k+4$.

viii. Clearly $d_f(W_{1,n}, i)$ is the cardinality of total collection of fair dominating sets of cardinality$i$. Hence $d_f(W_{1,n}, i)$ has to be a positive integer including zero.

3. Fair DominationPolynomial of a Wheel graph.

In this section we introduced and investigate the fair domination polynomial of wheels.

Definition: 3.1

Let$W_{1,n}^i$ be the family of fair dominating sets of a wheel graph $W_{1,n}$with cardinality i and let $d_f(W_{1,n}, i) = |W_{1,n}^i|$. Then the domination polynomial $D_f(W_{1,n}, x)$ of $W_{1,n}$is defined as $D_f(W_{1,n}, x) = \sum_{i=1 \ or\ \gamma_f(W_{1,n})}^{n} d_f(W_{1,n}, i) x^i$.

Example: 3.2

Consider the wheel graph $W_{1,5}$ in figure 1.

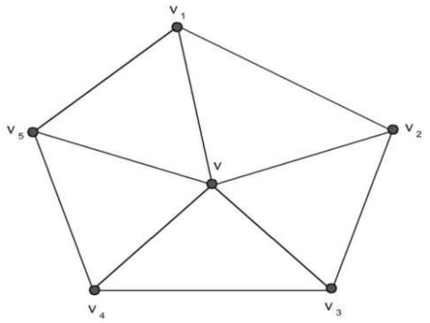

Figure 1

$W_{1,5}$

Here $W_{1,5}^1 = \{v\}$andso $d_f(W_{1,5},1) = 1$.

Now, $W_{1,5}^2 = \phi$. Therefore $d_f(W_{1,5},2) = 0$

Also, $W_{1,5}^3 = \phi$. Therefore $d_f(W_{1,5},3) = 0$

Here, $W_{1,5}^4 =$
$\{\{v_1,v_3,v_4,v\},\{v_2,v_4,v_5,v\},\{v_3,v_5,v_1,v\},\{v_4,v_1,v_2,v\},\{v_5,v_2,v_3,v\}, \{v_1,v_2,v_3,v\},\{v_2,v_3,v_4,v\},$
$\{v_3,v_4,v_5,v\},\{v_4,v_5,v_1,v\},\{v_5,v_1,v_2,v\}\}$.

Therefore $d_f(W_{1,5},4) = 10$.

Now, $W_{1,5}^5 = \{\{v_1,v_2,v_3,v_4,v_5\},\{v_1,v_2,v_3,v_4,v\},\{v_1,v_2,v_3,v_4,v\},\{v_2,v_3,v_4,v_5,v\},$

$\{v_1,v_3,v_4,v_5,v\},\{v_1,v_2,v_4,v_5,v\}\}$

Therefore $d_f(W_{1,5},5) = 6$.

Moreover, $W_{1,5}^6 = \{\{v_1,v_2,v_3,v_4,v_5,v\}$ and so $d_f(W_{1,5},6) = 1$.

Hence $D_f(W_{1,5},x) = x + 10x^4 + 6x^5 + 6$.

Theorem: 3.3

Let $W_{1,n}$ be a wheel graph with $n + 1 \geq 6$ vertices. Then

i. $D_f(W_{1,n},x)$has no constant term.
ii. $D_f(W_{1,n},x)$has x term, last no $x^2,x^3,\ldots,x^{\gamma_f(C_n)}$terms.
iii. $x = 0$ is a zero of $D_f(W_{1,n},x)$of multiplicity$\gamma_f(W_{1,n})$.

Proof:

Let $W_{1,n}$be a wheel graph with $n + 1 \geq 6$ vertices.

i. Since $D_f(W_{1,n},x) = \sum_{i=\gamma_f(W_{1,n})}^{n+1} d_f(W_{1,n},i)\,x^i$and $\gamma_f(W_{1,n}) \geq 1$, each term of $D_f(W_{1,n},x)$ has x in it. Hence $D_f(W_{1,n},x)$has no constant term
ii. We have$d_f(W_{1,n},1) = 1$, by theorem Hence$D_f(W_{1,n},x)$ has a x term. Also by Theorem:7 we have $d_f(W_{1,n},1) = 0$ if $1 < i < \left\lceil\frac{n}{3}\right\rceil + 1$.
Since$\left\lceil\frac{n}{3}\right\rceil \leq \gamma_f(C_n) \leq \left\lceil\frac{n}{3}\right\rceil + 1$, we conclude that $D_f(W_{1,n},x)$has no $x^2,x^3,\ldots,x^{\gamma_f(C_n)}$terms.
iii. By (i), $D_f(W_{1,n},x)$ has no constant term. This shows that $D_f(W_{1,n},x) = 0$ only if $x = 0$.
Hence $x = 0$ is zero of the given polynomial. Moreover the least positive of x in the expansion of $D_f(W_{1,n},x)$is$\gamma_f(W_{1,n})$.

Therefore the multiplicity of zero is $\gamma_f(W_{1,n})$.

Theorem: 3.4

Let $D_f(W_{1,n},x)$ and $D_f(C_n,x)$ be the fair domination polynomial of $W_{1,n}$ and C_n, respectively. Then, $D_f(W_{1,n},x) = x[1 + x^{n-1} + D_f(C_n,x)]$, $n \geq 5$.

Proof:

$$D_f(W_{1,n},x) = \sum_{i=1}^{n+1} d_f(W_{1,n},i)x^i$$

$$= \sum_{i=1}^{\gamma_f(C_n)-1} d_f(W_{1,n},i)x^i + \sum_{i=\gamma_f(C_n)}^{n-1} d_f(W_{1,n},i)x^i + d_f(W_{1,n},n)x^n + d_f(W_{1,n},n+1)x^{n+1}$$

$$= x + 0 + 0 + \cdots + 0 + \sum_{i=\gamma_f(C_n)}^{n-1} d_f(W_{1,n},i)x^i + (n+1)x^n + x^{n+1}$$

$$= x + \sum_{i=\gamma_f(C_n)}^{n-1} d_f(C_n,i-1)x^i + (n+1)x^n + x^{n+1}$$

$$D_f(W_{1,n},x) = x + x\left[\sum_{i=\gamma_f(C_n)}^{n-1} d_f(C_n,i-1)x^{i-1}\right] + (n+1)x^n + x^{n+1}$$

Put $i - 1 = j$

Therefore $i = j + 1$

$$D_f(W_{1,n},x) = x + x\left[\sum_{j+1=\gamma_f(C_n)}^{n-1} d_f(C_n,j)x^j\right] + (n+1)x^n + x^{n+1}$$

$$= x + x\left[\sum_{j=\gamma_f(C_n)}^{n-2} d_f(C_n,j)x^j\right] + (n+1)x^n + x^{n+1}$$

$$= x + x\left[\sum_{j=\gamma_f(C_n)}^{n} d_f(C_n,j)x^j - d_f(C_n,n-1)x^{n-1} - d_f(C_n,n)x^n\right] + (n+1)x^n + x^{n+1} \quad \text{[by lemma:1]}$$

$$= x + x[D_f(C_n,x) - nx^{n-1} - x^n] + (n+1)x^n + x^{n+1}$$

$$= x + xD_f(C_n,x) - nx^n - x^{n+1} + nx^n + x^n + x^{n+1}$$

$$= x + xD_f(C_n,x) + x^n$$

$$D_f(W_{1,n},x) = x[1 + x^{n-1} + D_f(C_n,x)].$$

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