METHODS OF PROVING SOME TRIGONOMETRIC IDENTITIES AND INEQUALITIES USING THE PROPERTIES OF GEOMETRICAL FORMS

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Annotation. In this scientific article, methods of solving some trigonometric inequalities and proofs of identities using geometrical methods are shown. Teaching in such a style frames the reader with the beauty, charm, simplicity and ease of the procedure for solving the issue. Several variants of the proof of trigonometric identities are presented in the article, as a result of which the student's knowledge, skills and ability to solve algebraic issues using the properties of geometrical forms are further formed. In addition to the Pythagorean theorem, the formula of the surface of a triangle, the inner and outer drawn circles of a triangle, as well as the Karno formula, were used to prove trigonometric identities. It is possible to recommend the following teaching methods in the article so that they can be used as a methodological guide in school and academic high school lessons.
Keywords. Trigonometric functions, identity, theorem, proof, angle, triangle, circle, straight rectangle, example and solution.

Introduction. Today, we require urgent tasks such as developing new methods of teaching and learning from pedagogues, connecting science(integration), educating creative and free, as well as young people who can think independently in all respects. What kind of education should be given to the modern student. It is better to teach them in any way, to find answers to the questions of how to teach them mathematics, we will impose on the teacher-educators a huge set of tasks. The modern educator must first of all be able to get the students interested in knowledge, show the student the right way in the manifestation of his abilities, form logical and free thinking in children, the ability to understand the thoughts of others and correctly convey his thoughts.

Literature review. Looking at the history of mathematics, the famous Greek geographer Euclid explained algebraic expressions in his work "bases"(in some sources it was called "bases" or "beginnings"), the actions between them by intersections, that is, he used geometric algebra. The methods of solving some types of quadratic equations in a geometrical way were demonstrated by the great middle Asian mathematician Muhammad ibn Musa al-Khorezmi in his work "a brief book on the calculation of Al-jabr Al-Muqabala". [5]

A group of scientists from the CIS countries also used some methods in their research. In particular, Russian mathematicians I.F.Your Partner, N.B.Alfutova, G.Z.Genkin, V.L.Kryukova, Ukrainian mathematician I.A.Kushnir, Tajik mathematician A.In their research, sofievs used geometrical methods in solving some algebraic issues. [9,10]

V on the subject.L.Kryukova conducted a number of studies in her candidate's dissertation. In particular, he used the sciencelararo integration of algebra and geometry in solving some algebraic equations and inequalities with the help of triangular inequality, the length of a broken line, the distance from point to straight line, the theorem of cosines, the properties of a regular Triangle, the forms of an internal drawing of a circle, as well as vectors. [3]

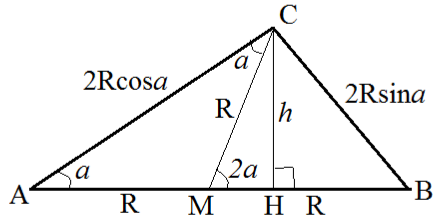
Research-methodology. It is known that the science of algebra and geometry is considered one of the important branches of mathematics. They have always filled each other, without one of them it is difficult to imagine the other. In geometry, there are such issues that it will be very difficult to calculate them by geometrical methods that we know. For example, algebraic methods are very useful to us in solving such issues as finding the largest surface Polygon among polygons with the same perimeter, finding the largest geometrical bodies with the largest volume, finding the volume of non-standard rotational bodies. Now there are such cases that it is more convenient to solve some algebraic equations and inequalities in a geometrical way than to solve them in the usual algebraic methods, more precisely to say that the support training of geometrical methods. This article shows the science of integration of algebra and geometry, that is, some trigonometric calculations, as well as the support of geometrical methods of proof of identities, to be more convenient than to solve them in algebraic methods, which are familiar to us. Support for geometric methods in this case is convenient to teach, the reader indirectly follows the algorithm of solving the problem through geometric drawings, sees it in a live state. These observations help the reader to more deeply master the science of algebra, to more deeply understand the meanings of formulas, as well as to gain a certain level of experience. As a result, the reader independently develops the ability to take algebraic-type issues with the support of geometrical methods, his self-confidence increases. Teaching in such a method will lead to a further increase in students ' spatial ideas (spatial representations), geometrical knowledge, skills and skills.

Analysis and results. In this scientific article we have shown some methods of proving trigonometric identities and inequalities using the properties of geometrical forms. When trained with the help of these methods, we tried to show that understanding the techniques of proving trigonometric identities would be very convenient for the masses of students. In addition to the Pythagorean theorem in proving trigonometric identities, we also benefited from the formulas of the surface of the Triangle, the inner and outer drawn circles of the Triangle, as well as the Karno formula. We also used the help of vectors in proving a single trigonometric inequality. We have shown several methods of proving trigonometric identities so that the reader can choose the most optimal solution for himself. We usually use the properties of a rectangular triangle so that the masses of readers can understand the definitions of trigonometric functions in depth. Taking advantage of this tradition in today's article, we will look at the integration of algebra-geometrics in addition to it. Teaching with the help of integrated techniques will frame the reader with the beauty, charm, simplicity and ease of the procedure for solving the issue.

1-example. $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$ prove identity.

Solution. We will see the proof of this disorder in the 3 method below.

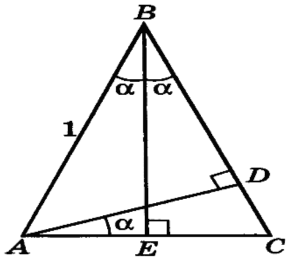
1-method. First we make a right-angled ABC triangle, from the end of its right angle we draw a height and a median to the hypotenuse(Figure 1).



1-figure. The form corresponding to the condition of the issue.

As you know, the center of the outer drawn circle of this triangle lies at the point M, then it will be AM=MC=MB=R (R is the outer drawn circle). AMC is an equilateral triangle $\angle MAC = \angle ACM = \alpha$ assuming, is(based on the central angular property) $\angle CMB = 2\alpha$. From the definition of cosine and sine, we find that AC=2Rcos$a$ and BC=2Rsin$a$. From the formula of finding the surface of a triangle $S = \frac{1}{2} \cdot 2R\sin\alpha \cdot 2R\cos\alpha = 2R^2 \sin\alpha\cos\alpha$, on the other hand $S = \frac{1}{2} \cdot AB \cdot h = \frac{1}{2} \cdot 2R \cdot R\sin 2\alpha = R^2 \sin 2\alpha$. From these two formulas $R^2 \sin 2\alpha = 2R^2 \sin\alpha\cos\alpha$ mold, and from this it turns out: $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$.

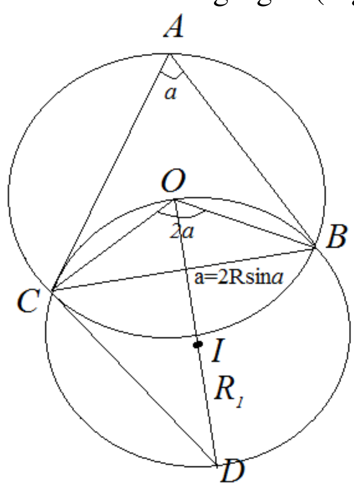
<u>2-method.</u> We look at an equilateral triangle with an angle on the tip 2α and an ABC equal to 1 on the side. We pass the ad and BE heights of this triangle(Figure-2).



2-figure. The form corresponding to the condition of the issue.

As we can see from the drawing, $AD = \sin 2\alpha$, $AE = EC = \sin\alpha$. Also from the fact that the ABE and CAD triangles are similar, $\frac{AB}{AC} = \frac{BE}{AD}$. From these equations $\frac{1}{2\sin\alpha} = \frac{\cos\alpha}{\sin 2\alpha}$. And from this: $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$. [1, 2]

<u>3-method.</u> To begin with, we draw the following figure(Figure 3).



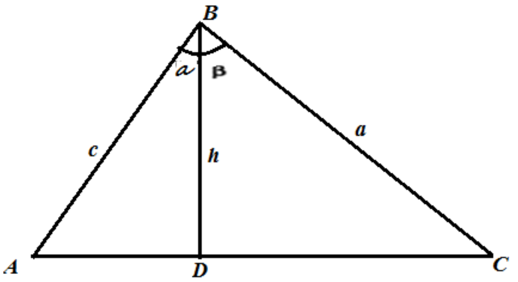
3-figure. The form corresponding to the condition of the issue.

O point ABC let the center of the circle drawn outside the Triangle. In this case, from the properties of the central angle $\angle COB = 2\angle CAB = 2\alpha$. Since the I point is the center of the circle drawn outside the OCB triangle, the OCD triangle is rectangular(OC=OB=R, OD=2R$_1$). Here R is the circle drawn externally to the ABC triangle, R$_1$ is the circle drawn externally to the OBC triangle. Without it, we can write the following formulas: $a = 2R\sin\alpha,\ R = 2R_1\cos\alpha,\ a = 2R_1\sin 2\alpha$. From these formulas we can write the following equation: $2R\sin\alpha = 2\cdot 2R_1\cdot\cos\alpha\cdot\sin\alpha = 2R_1\sin 2\alpha$.

It turns out as a result: $\sin 2\alpha = 2\sin\alpha\cdot\cos\alpha$. [8]

2-example. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ prove identity.

Solution. **1-method.** ABC can make a triangle in such a way that the following relationship is fulfilled: $BD \perp AC,\ \angle ABD = \alpha,\ \angle CBD = \beta$. D – point is the basis of the perpendicular, α and β - sharp corners(Figure 4).



4-figure. The form corresponding to the condition of the issue.

$BC = a,\ AC = b,\ AB = c$ and $BD = h$ we get that. In this case, according to the formula of finding the surface of the triangle

$$S_{\Delta ABC} = \frac{1}{2}ac\sin(\alpha+\beta),\quad S_{\Delta ABC} = S_{\Delta ABD} + S_{\Delta CBD}$$

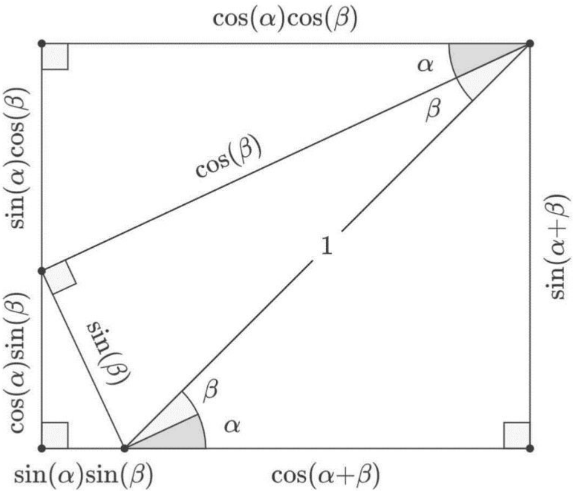
$$S_{\Delta ABD} = \frac{1}{2}ch\sin\alpha = \frac{1}{2}c\sin\alpha\cdot a\cos\beta = \frac{1}{2}ac\sin\alpha\cos\beta$$

$$S_{\Delta CBD} = \frac{1}{2}ah\sin\beta = \frac{1}{2}a\sin\beta\cdot c\cos\alpha = \frac{1}{2}ac\cos\alpha\sin\beta$$

and from these formulas comes the given nausea, that is,

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta. \qquad [7]$$

<u>2-method.</u> Draw the following figure in such a way that the hypotenuse of a right-angled triangle inside the rectangle is equal to 1(Figure 5).



5-figure. The form corresponding to the condition of the issue.

If we take advantage of the sine and cosine definitions for the resulting 4 rectangular triangles, then the proof of identities arises, namely

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

In this we have come to prove the formula of addition not only for sinuses, but also for cosines.

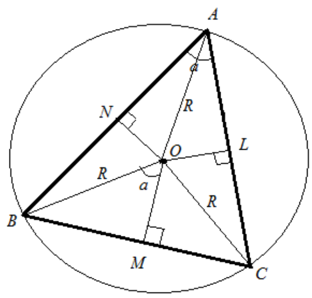
3-example. If α, β and γ are the angles of an arbitrary triangle, then the following $\cos\alpha+\cos\beta+\cos\gamma\le\frac{3}{2}$ prove inequality.

Solution. **1-method.** Exiting a point in the plane $\vec{r}_1, \vec{r}_2, \vec{r}_3$ we look at the unit vectors. The corners between them $180^0-\alpha,\ 180^0-\beta,\ 180^0-\gamma$ let there be. These $(\vec{r}_1+\vec{r}_2+\vec{r}_3)^2\ge 0$ odds are known to be reasonable. According to him

$$\vec{r}_1^{\,2}+\vec{r}_2^{\,2}+\vec{r}_3^{\,2}+2\vec{r}_1\vec{r}_2+2\vec{r}_1\vec{r}_3+2\vec{r}_2\vec{r}_3\ge 0,$$
$$3+2\cos(180^0-\alpha)+2\cos(180^0-\beta)+2\cos(180^0-\gamma)\ge 0,$$
$$3-2(\cos\alpha+\cos\beta+\cos\gamma)\ge 0.$$

The last is due to inequality $\cos\alpha+\cos\beta+\cos\gamma\le\frac{3}{2}$. [4, 6]

2-method. We draw an outer circle centered O on a triangle with an acute angle of ABC(Figure 6).



6-figure. The form corresponding to the condition of the issue.

Central corner properties $\angle COB=2\angle CAB=2\alpha$ the formula is reasonable. $\angle BAC=\angle BOM=\alpha$ and since BOM is a right angle of a triangle $\cos\alpha=\frac{OM}{R}$. The same is true $\cos\beta=\frac{OL}{R},\ \cos\gamma=\frac{ON}{R}$. Without it, we can write the following equation $\cos\alpha+\cos\beta+\cos\gamma=\frac{OM+OL+ON}{R}$. As you know, from the Karnoformula $OM+OL+ON=R+r$ relationship is appropriate, such R and r are respectively the outer and inner drawn circles of the triangle. From the second side $r\le\frac{R}{2}$ if we take advantage of the fact that,

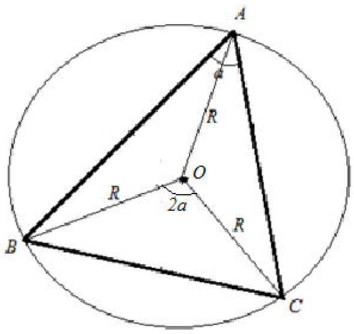
$$\cos\alpha+\cos\beta+\cos\gamma=\frac{OM+OL+ON}{R}=\frac{R+r}{R}\le\frac{R+\frac{R}{2}}{R}=1+\frac{1}{2}=\frac{3}{2}$$

it turns out that. Inequality proved.

5-example(M.I.Skanavi. Collection of questions from mathematics for those entering technical higher education institutions: 3.370-example). Prove identity.

$$\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4\sin\alpha \cdot \sin\beta \cdot \sin\gamma$$

Solution. We make a triangle ABC with an acute angle and draw on it an outer circle with a rhombus(Figure 7).



7-figure. The form corresponding to the condition of the issue.

From the surface formulas of triangles, we can write the following equations:

$$S_{AOB} + S_{BOC} + S_{COA} = \frac{1}{2}R^2 \sin 2\alpha + \frac{1}{2}R^2 \sin 2\beta + \frac{1}{2}R^2 \sin 2\gamma =$$

$$= \frac{1}{2}R^2(\sin 2\alpha + \sin 2\beta + \sin 2\gamma)$$

$$S_{ABC} = \frac{1}{2}ab\sin\gamma = \frac{1}{2}\cdot 2R\sin\alpha \cdot 2R\sin\beta \cdot \sin\gamma = \frac{1}{2}R^2 \cdot 4\sin\alpha \cdot \sin\beta \cdot \sin\gamma$$

As you know, the sum of the surfaces of triangles AOB, BOC and COA is equal to the surface of the triangle ABC, that is, $S_{AOB} + S_{BOC} + S_{COA} = S_{ABC}$. And from this comes the confirmation of identity $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 4\sin\alpha \cdot \sin\beta \cdot \sin\gamma$.

Conclusion and recommendation. Some of the trigonometric calculations mentioned in the article, Proof of identities, as well as the methods of teaching using the geometric methods of solving trigonometric equations, we can say that the recommended hook of options is a higher level of imaginative thinking than the familiar algebraic solutions. Using geometric methods, teaching becomes visually appealing, that is, close to Real life and aesthetically attractive.

It is known that mathematics abstract is among the sciences, however, in many there is an understanding that a large part of the knowledge in mathematics curriculum is far from life. When we teach algebraic equations to support geometrical methods, we get rid of this abstract a little bit, bringing mathematical knowledge closer to Real life for a while.

The interest of students in the subject of mathematics is further increased if these teaching methods mentioned in the article are used as a methodological guide in school and academic high school lessons.

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