SUPPLY CHAIN MODEL FOR DETERIORATING ITEMS WITH FUZZY ENVIRONMENT

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ABSTRACT

The players (vendors, retailers, distributors, etc.) in a supply chain may belong to different corporate entities and be more interested in minimizing their own cost rather than that of the chain as a whole. This kind of single-sided optimal strategy is not suitable for today's global competitive environment. In real life, there are many examples in which a manufacturer (here as vendor) has own set of direct outlets (here as buyers) to route the produced item to end customers. A production model for deteriorating items with the fuzzy costs has been developed taking into account the views of both the vendor and the multi-buyers.

Keywords: Supply chain, deteriorating, vendor, Buyers, Shortages.

NOMENCLATURE		p_v	the unit production cost for vendor
θ	the deterioration rate	$\tilde{p}_v$	fuzzified unit production cost for vendor
N	number of buyers	p_b	the unit price for buyer
d_i	the demand rate per year for buyer i, i = 1,2,3......N	$\tilde{p}_b$	fuzzified unit price for buyer
p	the production rate per year	C_v	the setup cost of each production cycle for vendor
T	time length of each cycle, where $T = T_1 + T_2$	$\tilde{C}_v$	fuzzified setup cost of each production cycle for vendor
T_1	the length of production time in each production cycle T	C_b	the setup or ordering cost per order for vendor
T_2	the length of non-production time in each production cycle T	h_v	the holding cost per dollar per year for vendor
$I_{v1}(t_1)$	inventory level for vendor when t_1 is between 0 and T_1	$\tilde{h}_v$	fuzzified holding cost per dollar per year for vendor
$I_{v2}(t_2)$	inventory level for vendor when t_2 is between 0 and T_2	h_b	the holding cost per dollar per year for buyer
$I_{bi}(t)$	inventory level for buyer i when t is between 0 and T/n_i	$\tilde{h}_b$	fuzzified holding cost per dollar per year for buyer
n_i	delivery times per period T for buyer i where i = 1,2,3,……N	VC	the cost of vendor per unit time
I_{mv}	the maximum inventory level of vendor	BC	the cost of all buyers per unit time
I_{mb}	the maximum inventory level of buyer	TC	the integrated cost of vendor and all buyers per unit time
		$\widetilde{TC}$	fuzzified integrated total cost

1. INTRODUCTION

A supply chain may be defined as an integrated process wherein a number of various business entities work together in an effort to: (1) acquire raw materials, (2) convert these raw materials into specified final products, and (3) deliver these final products to retailers. Several authors have studied the integrated policies of the manufacturers and the retailers. Most of the research in this area is based on the classic work of Clark and Scarf (1960). The idea of joint total cost of the supplier and the customer was first introduced by Goyal (1976). Later, Cohen and Lee (1988), determined material requirement for all materials at every stage in a supply chain. Pake and Cohen (1993), extended the above study to consider for stochastic sub systems. Gyana and Bhabha (1999), explored a single manufacturing system for procurement of raw materials with a multi-ordering policy that minimized the total inventory costs of both the raw materials and the finished goods. Sarkar *et al.* (2000) explored a supply chain model for determining an optimal ordering policy under inflation and allowable shortages. Goyal and Nebebe (2000), considered a problem of determining economic production and shipment policy of a product from a vendor to a buyer. Woo *et al.* (2001), considered an integrated inventory system where a vendor purchases and processes raw materials and delivered the finished items to multiple buyers. Rau *et al.* (2003) developed a multi-echelon inventory model for a deteriorating item and derived an optimal joint total cost from an integrated perspective among the supplier, the producer, and the buyer. Yang and Wee (2003), developed an integrated inventory model with constant rate of deterioration and multiple deliveries. Chien and Lin (2004), investigated the optimal order interval and discount price such that the joint total cost was minimized during a finite planning horizon. Banerjee (2005), developed a model where a supplier makes an agreement with a buyer, and determines the inventory policies of the supplier. Hans *et al.* (2006), presented a methodology to obtain the joint economic lot size in distribution system. Ahmed *et al.* (2008). have coordinated a two level supply chain in which they considered production interruptions for restoring of the quality of the production process. Jha and Shanker (2009) considered a two-echelon supply chain inventory problem consisting of a single-vendor and a single-buyer. In the system under study, a vendor produced a product in a batch production environment and supplied it to a buyer. Also, buyer's lead time was controllable which can be shortened at an added cost and all shortages were backordered. Uthayakumar and Geetha (2009). presented an inventory model for non-instantaneous deteriorating items. In their study, they considered stock-dependent demand rate and shortages were partially backlogged with single storage facility.

2. METHODOLOGY

❖ GRADED MEAN INTEGARTION REPRESENTATION METHOD

Chen and Hsieh (1998, 1999, 2000b) introduced Graded Mean Integration Representation Method based on the integral value of graded mean h-level of generalized fuzzy number for defuzzifing generalized fuzzy number. They also found this method is better than the methods of Adamo (1980), Campos and Verdegay (1989), Yager (1981), Kaufmann and Gupta (1991), Lee and Yao (1998), Liou et al (1992), Heilpern 1997). Here, we describe generalized fuzzy number as follows.

Suppose Ã is a fuzzy number it is described as a fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions.

- ❖ μ_A is a continuance mapping from R to the closed interval [0, 1],
- ❖ $\mu_A = 0, \ -\infty < x \le a_1$,
- ❖ $\mu_A = L(x)$ is strictly increasing on $[a_1, a_2]$,
- ❖ $\mu_A = 1, a_2 \le x \le a_3$,
- ❖ $\mu_A = R(x)$ is strictly decreasing on $[a_3, a_4]$,

- $\mu_A = 0, a_4 \le x < \infty$,

Where a_1, a_2, a_3 and a_4 are real numbers s.t. $a_1 \le a_2 \le a_3 \le a_4$.

Also this type of fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$.

By Graded Mean Integration Representation method, L^{-1} *and* R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}$ is $h(L^{-1}(h) + R^{-1}(h))/2$ as Figure1. Then the graded mean integration representation of Ã is P(Ã) where

$$P(A) = \int_0^1 h \frac{(L^{-1}(h) + R^{-1}(h))}{2} dh \Big/ \int_0^1 h dh \qquad (I)$$

With $0 < h \le 1$.

Throughout this chapter, we only use normal trapezoidal fuzzy number as the type of all fuzzy parameters in our proposed inventory models. Let $\bar{B}$ be a trapezoidal fuzzy number, and be denoted as $\tilde{B} = (b_1, b_2, b_3, b_4)$. Then we can get the graded mean integration representation of $\bar{B}$ by formula (I) as

$$P(\tilde{B}) = \int_0^1 h \left(\frac{b_1 + b_4 + (b_2 - b_1 - b_4 + b_3)h}{2}\right) dh \Big/ \int_0^1 h dh$$

$$= \frac{b_1 + 2b_2 + 2b_3 + b_4}{6} \qquad (II)$$

- **THE FUZZY ARITHMETICAL OPERATION UNDER FUNCTION PRINCIPLE:**

Function principle is introduced by Chen (1985) to treat the fuzzy arithmetical operations by trapezoidal fuzzy numbers. We will use this principle as the operation of addition, multiplication, subtract, division of trapezoidal fuzzy numbers, because (1) Function principle is easier to calculate than Extension Principle, (2) Function Principle will not change the shape do trapezoidal fuzzy number after the multiplication of two trapezoidal fuzzy numbers, but the multiplication of two trapezoidal fuzzy numbers will become drum like shape fuzzy number by using Extension Principle, (3) If we have to multiple more than four trapezoidal fuzzy numbers then the Extension Principle cannot solve the operation, but Function Principle can easy to find the result by point wise computation.

3. MATHEMATICAL MODELS

The mathematical model in this study is developed on the basis of the following assumptions:

- A single item with constant deterioration rate of the on –hand inventory is considered.
- Single-vendor multi-buyers with one item is assumed.
- Shortage is not allowed.
- There is no replacement or repair of deteriorated units.

- Holding cost is applied to good units only.
- The production rate is finite and is greater than the sum of all buyer's demand.

Here we suppose that $\tilde{c}_v = (c_{v1}, c_{v2}, c_{v3}, c_{v4})$, $\tilde{p}_b = (p_{b1}, p_{b2}, p_{b3}, p_{b4})$, $\tilde{p}_v = (p_{v1}, p_{v2}, p_{v3}, p_{v4})$ $\tilde{h}_v = (h_{v1}, h_{v2}, h_{v3}, h_{v4})$ are non-negative trapezoidal fuzzy number.

4. INVENTORY MODELS

- **EACH BUYER'S INVENTORY MODEL** The inventory system depicted in Fig.1 is represented by the following differential equations:

$$I_{bi}'(t) + \theta I_{bi}(t) = -d_i \qquad 0 \le t \le \frac{T}{n_i},\ i = 1,2,3,........., N \qquad (1)$$

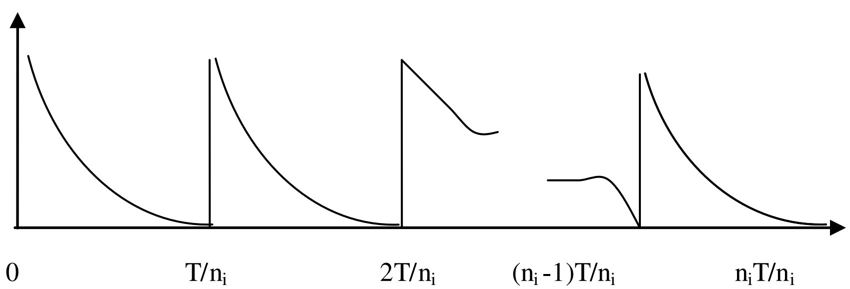


Figure 1. Buyer's Inventory level

On using the boundary condition $I_{bi}(T) = 0$, solution of the above differential equation is

$$I_{bi}(t) = \frac{d_i}{\theta}\left[e^{\theta(\frac{T}{n_i} - t)} - 1\right] \qquad 0 < t < \frac{T}{n_i},\ i = 1,2,3,....., N \qquad (2)$$

Maximum inventory of each buyer is

$$I_{mi} = I_{bi}(0)$$

$$\Rightarrow I_{mi} = \frac{d_i}{\theta}\left[e^{\theta T/n_i} - 1\right], \qquad i = 1,2,3,................N \qquad (3)$$

$$HC_b = \frac{p_b h_b}{T}\sum_{i=1}^{N} n_i \int_o^{T/n_i} I_{bi}(t)dt$$

The yearly holding cost for all buyers is

$$HC_b = \frac{p_b h_b}{T}\sum_{i=1}^{N} \frac{n_i d_i}{\theta^2}\left\{e^{\frac{\theta T}{n_i}} - \frac{\theta T}{n_i} - 1\right\} \qquad (4)$$

The annual deteriorated costs for all buyers is

$$DC_b = \frac{p_b}{T}\sum_{i=1}^{N} n_i\left(I_{mi} - \frac{Td_i}{n_i}\right)$$

$$DC_b = \frac{p_b}{T}\sum_{i=1}^{N} \frac{n_i d_i}{\theta}\left\{e^{\frac{\theta T}{n_i}} - \frac{\theta T}{n_i} - 1\right\} \qquad (5)$$

The setup cost per year for all buyers is

$$SC_b = \frac{c_b}{T}\sum_{i=1}^{N} n_i \qquad (6)$$

The buyer's total cost is the sum of the Holding cost, deteriorated cost and the setup cost

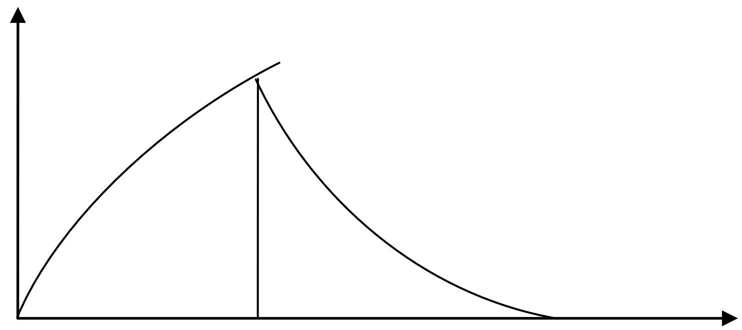
$$BC = HC_b + DC_b + SC_b \qquad (7)$$

❖ **VENDOR'S INVENTORY MODEL**

The inventory system depicted in Fig.2 is represented by the following differential equations

$$I_{v1}^{'}(t_1) + \theta I_{v1}(t_1) = p - \sum_{i=1}^{N} d_i \qquad 0 \le t_1 \le T_1 \qquad (8)$$

$$I_{v2}^{'}(t_2) + \theta I_{v2}(t_2) = -\sum_{i=1}^{N} d_i \qquad 0 \le t_2 \le T_2 \qquad (9)$$



0 T1 T2 Time

--------------------------T----------------------

Figure 2. Vendor's Inventory Level

Using the boundary conditions $I_{v1}(0) = 0$ and $I_{v2}(T_2) = 0$ the solutions of the above differential equations are

$$I_{v1}(t_1) = \frac{p - \sum_{i=1}^{N} d_i}{\theta}[1 - e^{-\theta t_1}], \qquad 0 \le t_1 \le T_1, \qquad (10)$$

$$I_{v2}(t_2) = \frac{\sum_{i=1}^{N} d_i}{\theta}[e^{\theta(T_2 - t_2)} - 1], \qquad 0 \le t_2 \le T_2, \qquad (11)$$

Maximum inventory level of the vendor is

$$I_{mv} = \frac{\sum_{i=1}^{N} d_i}{\theta}[e^{\theta T_2} - 1] \qquad (12)$$

By the boundary condition $I_{v1}(T_1) = I_{v2}(0)$, we have

$$\left(p - \sum_{i=1}^{N} d_i\right)[1 - e^{-\theta T_1}] = \sum_{i=1}^{N} d_i [e^{\theta T_2} - 1]$$

$$T_1 = \frac{\sum_{i=1}^{N} d_i}{p - \sum_{i=1}^{N} d_i} T_2 (1 + \frac{1}{2}\theta T_2) \qquad (13)$$

We know that $T = T_1 + T_2$, thus

$$T = \frac{T_2}{p - \sum_{i=1}^{N} d_i}\left\{p + \frac{1}{2}\theta T_2 \sum_{i=1}^{N} d_i\right\} \quad (14)$$

Holding cost for the vendor is

$$HC_v = \frac{p_v h_v}{T}\left[\int_0^{T_1} I_{v1}(t_1)dt_1 + \int_0^{T_2} I_{v2}(t_2)dt_2 - \sum_{i=1}^{N} n_i \int_o^{T/n_i} I_{bi}(t)dt\right]$$

$$HC_v = \frac{p_v h_v}{T\theta^2}\left[p\left\{e^{-\theta T_1} + \theta T_1 - 1\right\} + \sum_{i=1}^{N} d_i\left\{e^{\theta T_2} - e^{-\theta T_1} - \theta T\right\} - \sum_{i=1}^{N} \frac{n_i d_i}{\theta^2}\left\{e^{\frac{\theta T}{n_i}} - \frac{\theta T}{n_i} - 1\right\}\right] \quad (15)$$

The annual deteriorated cost for the vendor is

$$DC_v = \frac{p_v}{T}\left[pT_1 - \sum_{i=1}^{N} n_i I_{mi}\right]$$

$$= \frac{p_v}{T}\left[pT_1 - \sum_{i=1}^{N}\left\{n_i \frac{d_i}{\theta}\left[e^{\theta T/n_i} - 1\right]\right\}\right] \quad (16)$$

The setup cost per year for the vendor is

$$SC_v = \frac{c_v}{T} \quad (17)$$

The vendor's total cost is the sum of the holding cost, deteriorated cost and the setup cost as

$$VC = HC_v + DC_v + SC_v \quad (18)$$

The integrated total cost of the vendor and the buyers, TC, is the sum of (7) and (18). By (13) TC is a function of T_2 for a fixed value of n_i, thus

$$TC = BC + VC \quad (19)$$

$$TC = \frac{c_v}{T} \oplus \frac{c_b}{T} \otimes \sum_{i=1}^{N} n_i \oplus \frac{p_b \otimes (h_b + \theta)}{T\theta^2} \sum_{i=1}^{N} n_i d_i \left(e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right) \oplus \frac{p_v \otimes h_v}{T\theta^2}$$

$$\left\{ p\left\{ e^{-\theta T_1} + \theta T_1 - 1 \right\} + \sum_{i=1}^{N} d_i \left\{ e^{\theta T_2} - e^{-\theta T_1} - \theta T \right\} - \sum_{i=1}^{N} \tfrac{n_i d_i}{\theta^2} \left\{ e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right\} \right\}$$

$$\oplus \frac{p_v}{T} \otimes \left\{ pT_1 - \sum_{i=1}^{N} \left\{ n_i \frac{d_i}{\theta} \left(e^{\theta T / n_i} - 1 \right) \right\} \right\} \qquad (20)$$

Where $\oplus, \otimes$ are the fuzzy arithmetic operations under Function principle.

Firstly, we get the fuzzy total integrated total cost in the form of trapezoidal fuzzy number as below

$$TC = \left[\frac{c_{v1}}{T} \oplus \frac{c_{b1}}{T} \otimes \sum_{i=1}^{N} n_i \oplus \frac{p_{b1} \otimes (h_{b1} + \theta)}{T\theta^2} \sum_{i=1}^{N} n_i d_i \left(e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right) \oplus \frac{p_{v1} \otimes h_{v1}}{T\theta^2} \right.$$

$$\left\{ p\left\{ e^{-\theta T_1} + \theta T_1 - 1 \right\} + \sum_{i=1}^{N} d_i \left\{ e^{\theta T_2} - e^{-\theta T_1} - \theta T \right\} - \sum_{i=1}^{N} \tfrac{n_i d_i}{\theta^2} \left\{ e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right\} \right\} \oplus \frac{p_{v1}}{T}$$

$$\otimes \left\{ pT_1 - \sum_{i=1}^{N} \left\{ n_i \frac{d_i}{\theta} \left(e^{\theta T / n_i} - 1 \right) \right\} \right\}, \frac{c_{v2}}{T} \oplus \frac{c_{b2}}{T} \otimes \sum_{i=1}^{N} n_i \oplus \frac{p_{b2} \otimes (h_{b2} + \theta)}{T\theta^2}$$

$$\sum_{i=1}^{N} n_i d_i \left(e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right) \oplus \frac{p_{v2} \otimes h_{v2}}{T\theta^2} \left\{ p\left\{ e^{-\theta T_1} + \theta T_1 - 1 \right\} + \sum_{i=1}^{N} d_i \left\{ e^{\theta T_2} - e^{-\theta T_1} - \theta T \right\} \right.$$

$$\left. - \sum_{i=1}^{N} \tfrac{n_i d_i}{\theta^2} \left\{ e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right\} \right\} \oplus \frac{p_{v2}}{T} \otimes \left\{ pT_1 - \sum_{i=1}^{N} \left\{ n_i \frac{d_i}{\theta} \left(e^{\theta T / n_i} - 1 \right) \right\} \right\}, \frac{c_{v3}}{T} \oplus \frac{c_{b3}}{T} \otimes$$

$$\sum_{i=1}^{N} n_i \oplus \frac{p_{b3} \otimes (h_{b3} + \theta)}{T\theta^2} \sum_{i=1}^{N} n_i d_i \left(e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right) \oplus \frac{p_{v3} \otimes h_{v3}}{T\theta^2} \left\{ p\left\{ e^{-\theta T_1} + \theta T_1 - 1 \right\} + \right.$$

$$\sum_{i=1}^{N} d_i \left\{ e^{\theta T_2} - e^{-\theta T_1} - \theta T \right\} - \sum_{i=1}^{N} \tfrac{n_i d_i}{\theta^2} \left\{ e^{\frac{\theta T}{n_i}} - \tfrac{\theta T}{n_i} - 1 \right\} \right\} \oplus \frac{p_{v3}}{T} \otimes \left\{ pT_1 - \sum_{i=1}^{N} \left\{ n_i \frac{d_i}{\theta} \right. \right.$$

$$\left(e^{\theta T/n_i}-1\right)\Big\}\Big\},\frac{c_{v4}}{T}\oplus\frac{c_{b4}}{T}\otimes\sum_{i=1}^{N}n_i\oplus\frac{p_{b4}\otimes(h_{b4}+\theta)}{T\theta^2}\sum_{i=1}^{N}n_id_i\left(e^{\frac{\theta T}{n_i}}-\tfrac{\theta T}{n_i}-1\right)$$

$$\oplus\frac{p_{v4}\otimes h_{v4}}{T\theta^2}\left\{\begin{array}{l}p\left\{e^{-\theta T_1}+\theta T_1-1\right\}+\sum_{i=1}^{N}d_i\left\{e^{\theta T_2}-e^{-\theta T_1}-\theta T\right\}-\\ \sum_{i=1}^{N}\frac{n_id_i}{\theta^2}\left\{e^{\frac{\theta T}{n_i}}-\frac{\theta T}{n_i}-1\right\}\end{array}\right\}$$

$$\oplus\frac{p_{v4}}{T}\otimes\left\{pT_1-\sum_{i=1}^{N}\left\{n_i\frac{d_i}{\theta}\left(e^{\theta T/n_i}-1\right)\right\}\right\}\Big]\qquad(21)$$

Now we want to determine the value of n_i that minimize TC , where i=1,2,3,……….,N. since the number of deliveries per period, since n_i is a discrete variable, so the value of n_i can be determined by the following procedure:

(I) For a range of n_i values, determine the derivative of TC w.r.t. T_2 and set it to zero. For each n_i denote the value of T_2 for minimum of TC by $T_2(n_i)$ where i = 1, 2, 3… N.

(II) The optimal value of n_i is derived by satisfying the following condition

$$TC(T_2(n_i^*-1),n_i^*-1)\geq TC(T_2(n_i^*,n_i^*)\leq TC(T_2(n_i^*+1),n_i^*+1)\qquad(22)$$

Secondly, we defuzzify the fuzzy integrated total cost using graded mean integration representation method. i.e.

$$F(TC)=\frac{1}{6T}\Bigg[\Bigg(C_{v1}+2C_{v2}+2C_{v3}+C_{v4}+(C_{b1}+2C_{b2}+2C_{b3}+C_{b4})\sum_{i=1}^{N}n_i\Bigg)$$

$$+\frac{1}{\theta^2}\left\{p_{b1}(h_{b1}+\theta)+2p_{b2}(h_{b2}+\theta)+2p_{b3}(h_{b3}+\theta)+p_{b4}(h_{b4}+\theta)\right\}\sum_{i=1}^{N}\Big\{n_id_i$$

$$\left(e^{\frac{\theta T}{n_i}}-\tfrac{\theta T}{n_i}-1\right)\Big\}+\frac{\left\{p_{v1}h_{v1}+2p_{v2}h_{v2}+2p_{v3}h_{v3}+p_{v4}h_{v4}\right\}}{\theta^2}\Big\{p\left(e^{-\theta T_1}+\theta T_1-1\right)+$$

$$\sum_{i=1}^{N}d_i\left(e^{\theta T_2}-e^{-\theta T_1}-\theta T\right)-\sum_{i=1}^{N}\frac{n_id_i}{\theta^2}\left\{e^{\frac{\theta T}{n_i}}-\frac{\theta T}{n_i}-1\right\}\Big\}+\left(p_{v1}+2p_{v2}+2p_{v3}+p_{v4}\right)$$

$$\left\{pT_1-\sum_{i=1}^{N}\left\{n_i\frac{d_i}{\theta}\left(e^{\theta T/n_i}-1\right)\right\}\right\}\Bigg]\qquad(24)$$

Thirdly we can get the optimal production quantity Q^* when $F(TC)$ is minimization. In order to minimize $F(TC)$, the necessary conditions is $\frac{d}{dT_2}F(TC)=0$,

And solving for T_2 we will get the optimal time value of T_2 and then from equations (13) and (14), we will find the optimal values of T_1 and T.

TABLE 1. Optimal solution of n_1 and n_2

n_1	n_2	T_1	T_3	BC	VC	TC
1	2	0.0448	0.3638	4867.67	10738.40	15606.07
2	3	0.0466	0.2926	5648.60	10724.60	16373.20
3	4	0.0471	0.2362	7056.40	10723.90	17780.30
4*	3*	0.0502	0.2840	5205.91	10196.03	15401.94*
$5^{\#}$	$4^{\#}$	0.0519	0.2448	4818.63#	10775.76	15594.39
6	3	0.0515	0.2576	5256.26	10767.74	16024.00
7	4	0.0547	0.2355	6052.51	10848.29	16900.80

5. RESULTS AND DISCUSSION

The preceding theory can be illustrated by considering two buyers, i.e. N=2. The capacity of production is 200000 units per year: the annual demand rate of the first and the second buyers are 4000 and 8000 units, respectively; the yearly percentage of holding cost per dollar for the vendor and the buyers are $(0.12, 0.15, 0.18, 0.20) and (0.15, 0.17, 0.19, 0.21) respectively. The other related factors are as follows the ordering cost is $(160, 180, 210, 240) for the buyers, the production setup cost is $(4000, 4600, 5000, 5500) the unit production cost is $(8, 10, 12, 15) the unit price for buyer is $(10, 11, 12, 14) and the deterioration rate is 0.1 per year. By applying the above solution procedure, obtained results are shown in Tables 1.

From the above tables it is clear that the total cost is reduced in both cases when compared to the costs obtained in buyers and vendor's separate policies. We see that the total cost is minimum when the number of deliveries are $n_1^*= 3$ and $n_2^*= 3$. The optimal cost in integrated policies is $15401.94 and the total cost in their separate policies is obtained $15594.39. Thus it is concluded that integrated policy results in an impressive cost reduction.

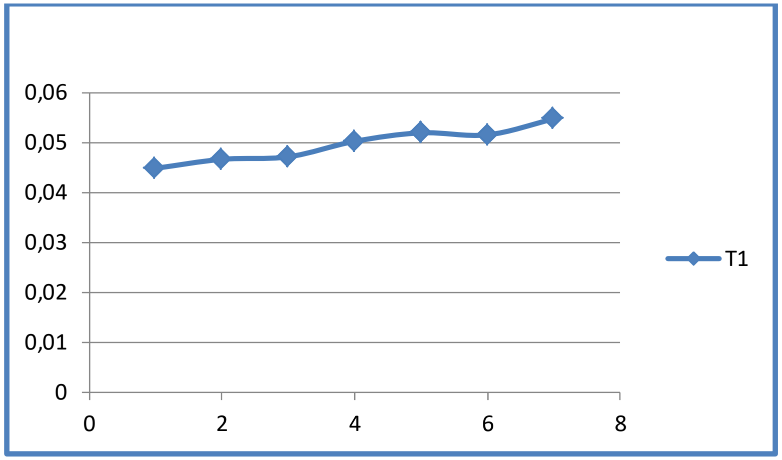


Figure3. Variation in T_1

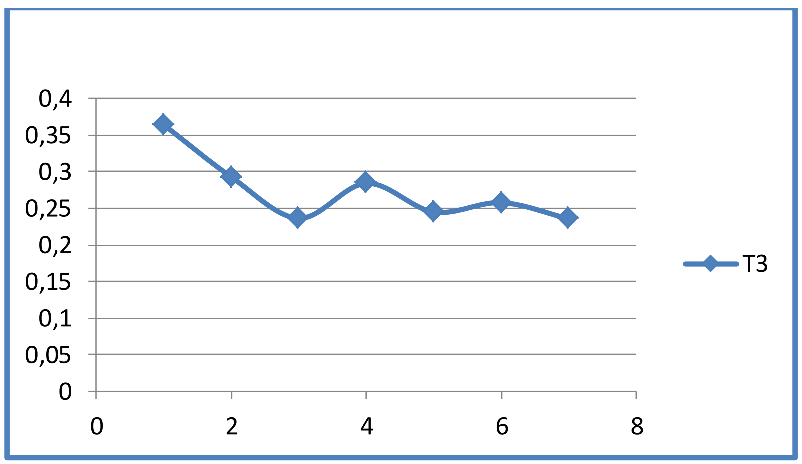


Figure 4. Variation in T_3

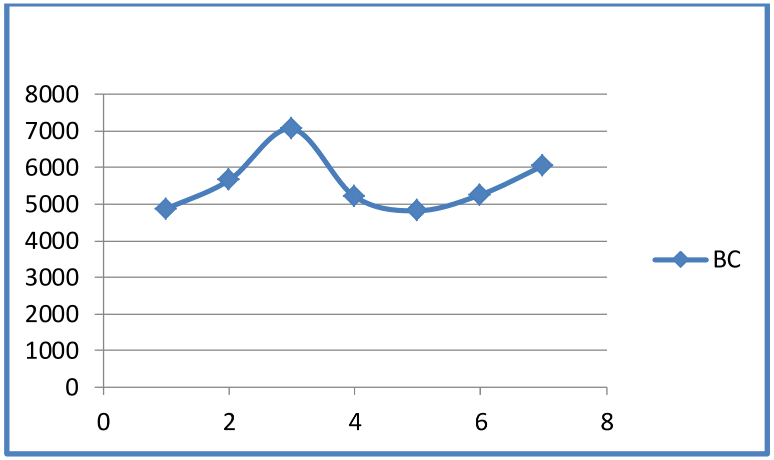


Figure 5. Variation in VC

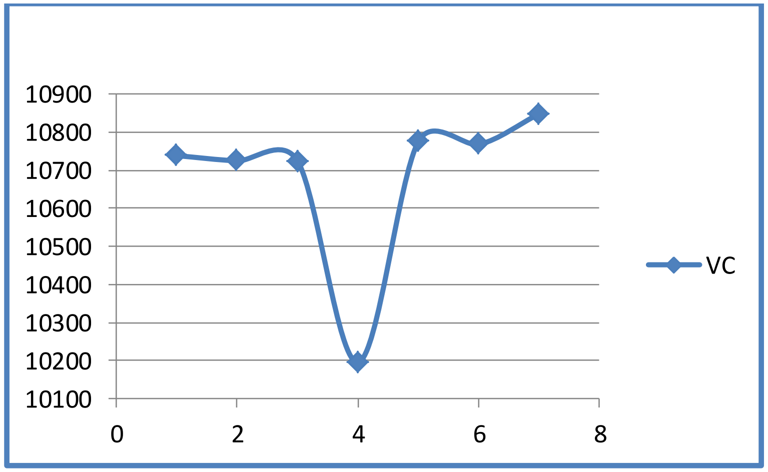


Figure 6. Variation in BC

6. CONCLUSION

A single vendor multi-buyers inventory model with fuzzy costs has been developed. We used integrated inventory policy instead of independent decisions made by the vendor and buyers. From the above observations it is concluded that the integrated inventory policy results in the impressive cost reduction as compared to the vendor's and buyers independent decisions. Fuzzy model provides more realistic situations with market uncertainties and to be prepared to best deal with them.

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