Bernoulli and Riccati fractional differential equations can be solved analytically by using conformable derivatives

Salim S.Mahmooda, KamaranJ.Hamad[b],SarwarA.Hamad[c],Dlvan Omar[d]

[a,b,c]Department of Mathematic, Faculty of Science, Soran University, Kurdistan Regional Government, Iraq

[d]Department of Mathematic, Faculty of Science, Zakho University, Kurdistan Regional Government, Iraq.

Article History: Do not touch during review process(xxxx)

Abstract: The Bernoulli and Riccati fractional differential equations are solved analytically using conformable derivatives in this paper. And To demonstrate the proposed solution, numerical examples of each equation are given.

Keywords: Fractional integral, fractional derivative, fractional deferential equation,Bernoulli Equation,Riccati Equation_

1. Introduction (Times New Roman 10 Bold)

The history of fractional computation is popularly believed to have arisen from an issue raised in the year 1695 by the Hospital at Leibniz on September 30 for the first time, l'Hôpital questioned Leibniz about the possibility and significance of a start date in this letter derived from the order.$\frac{1}{2}$, that is a derivative of fraction order. The follow-up survey resulted in the first results of what we now call fractional computing. **(A. A. H. M. S. a. J. J. T. Kilbas, 2006)**

During this decade, fractional computing has played a major role in various fields such as physics., chemical, mechanical, electrical, biology, economics....

Some of these fractionated derivatives are (Caputo, Riemann-Liouville) which are more popular

i. the Caputo Fractional Derivatives for $p > 0, t > a, a, t \in R$

$$ {}_{a}^{C}D_{t}^{p}h(t) = \frac{1}{\Gamma(n-p)}\int_{a}^{t}(t-\tau)^{n-p-1}\,h^{n}(\tau)d\tau,\ \ n-1 < p < n $$

ii. The Riemann-Liouville fractional Derivatives defined by

$$ {}_{a}^{RL}D_{t}^{p}h(t) = \frac{1}{\Gamma(n-p)}\frac{d^{n}}{dt^{n}}\left[\int_{a}^{t}(t-\tau)^{n-p-1}\,h^{n}(\tau)d\tau\right],\ \ n-1 < p < n $$

But all definition above have some setbacks like **(Khalil, 2014)**

1. A derivative of Riemann-liouville. does not satisfy${}_{a}^{RL}D_{t}^{\alpha}l \neq 0$, and for the Caupto derivative${}_{a}^{C}D_{t}^{\alpha}l = 0$ when α is not natural number.
2. Not all fractional derivatives are in compliance with the known product rule:
$$ D_{a}^{\alpha}(hk) = hD_{a}^{\alpha}(k) + kD_{a}^{\alpha}(h) $$
3. Not all fractional derivatives fulfil the known quotient rule.:
$$ D_{a}^{\alpha}\left(\frac{h}{k}\right) = \frac{kD_{a}^{\alpha}(h) - hD_{a}^{\alpha}(k)}{k^{2}} $$
4. All fractional derivatives fail the chain rule:
$$ D_{a}^{\alpha}(h \circ k) = h^{\alpha}(k(t))k^{\alpha}(\mathrm{t}) $$
5. All fractional derivatives are not satisfactory $D^{\alpha}D^{\beta} = D^{\alpha+\beta}$ in general.

In last few years a new definition was found by (R.khalil), which is simpler and more efficient. The new definition reflects a nature extension of normal derivative which is called "conformable fractional derivative "which is this definition removes this issue and very useful for solving deferential equation.

The purpose of this article is to introduce a new way of solving the linear fractional equation of the fractional differential equations Bernoulli and Riccati using the conformable fractional derivative.**(Iyiola, 2016)**

2.MATERIALS AND METHODS

Definition 2.1: Function given $\boldsymbol{h}: [\mathbf{0}, \infty) \to \boldsymbol{R}$ and then the conformable fractional derivative. $\boldsymbol{h}$ of order is determined by

$$D_\alpha h(t) = \lim_{\varepsilon \to 0} \frac{h(t + \varepsilon t^{1-\alpha}) - h(t)}{\varepsilon}, \qquad \forall t > 0, \alpha \in (0,1) \tag{1}$$

Sometimes, write $\boldsymbol{h^\alpha(t)}$ to indicate $\boldsymbol{D_\alpha h(t)}$ for designating the conformable derivative of $\boldsymbol{f}$ of order $\boldsymbol{\alpha}$.

Theorem 2.1: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{h}, \boldsymbol{k}$ be $\boldsymbol{\alpha}$ time differentiable at a point $\boldsymbol{t} > \mathbf{0}$ then

$$D_\alpha(ah + bk) = aD_\alpha(h) + bD_\alpha(k), \forall a, b \in R \tag{2}$$

Proof:

$$\begin{aligned} D_\alpha(ah + bk) &= \lim_{\varepsilon \to 0} \frac{ah(t + \varepsilon t^{1-\alpha}) + bk(t + \varepsilon t^{1-\alpha}) - (ah(t) + bk(t)}{\varepsilon} \\ &= \lim_{\varepsilon \to 0} \frac{a\left(h(t + \varepsilon t^{1-\alpha}) - h(t)\right) + b\left(k(t + \varepsilon t^{1-\alpha}) - k(t)\right)}{\varepsilon} \\ &= aD_\alpha(h) + bD_\alpha(k) \end{aligned}$$

Theorem 2.2: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{h}, \boldsymbol{k}$ be $\boldsymbol{\alpha}$ time differentiable at a point $\boldsymbol{t} > \mathbf{0}$ then

$$D_\alpha(x^p) = px^{p-\alpha}, \forall p \in R \tag{3}$$

Proof: the fractional derivative for power function

$$D_\alpha(x^p) = \lim_{\varepsilon \to 0} \frac{(x + \varepsilon x^{1-\alpha})^p - (x^p)}{\varepsilon}$$

hence ,$(x + \varepsilon x^{1-\alpha})^p = x^p + px^{p-1}\varepsilon x^{1-\alpha} + \cdots + (\varepsilon x^{1-\alpha})^p$

$$= px^{p-\alpha}$$

Theorem 2.3: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ then

$$D_\alpha(\mu) = 0, \forall \mu \in R \tag{4}$$

Proof: for any constant function the fractional derivative is zero

$$\begin{aligned} D_\alpha(\mu) &= \lim_{\varepsilon \to 0} \frac{\mu - (\mu)}{\varepsilon} \\ &= \lim_{\varepsilon \to 0} \frac{0}{\varepsilon} \\ &= 0 \end{aligned}$$

Theorem 2.4: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{h}, \boldsymbol{k}$ be $\boldsymbol{\alpha}$ time differentiable at a point $\boldsymbol{t} > \mathbf{0}$ then

$$D_\alpha(hk) = hD_\alpha(k) + kD_\alpha(h) \tag{5}$$

Proof: a fractional derivative to multiply two functions

$$\begin{aligned} D_\alpha(hk) &= \lim_{\varepsilon \to 0} \frac{h(t + \varepsilon t^{1-\alpha})k(t + \varepsilon t^{1-\alpha}) - h(t)k(t)}{\varepsilon} \\ &= \lim_{\varepsilon \to 0} \frac{h(t + \varepsilon t^{1-\alpha})\left(k(t + \varepsilon t^{1-\alpha}) - k(t)\right) + k(t)\left(h(t + \varepsilon t^{1-\alpha}) - h(t)\right)}{\varepsilon} \\ &= hD_\alpha(k) + kD_\alpha(h) \end{aligned}$$

Theorem 2.5: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{h}, \boldsymbol{k}$ be $\boldsymbol{\alpha}$ time differentiable at a point $\boldsymbol{t} > \mathbf{0}$ then

$$D_\alpha\left(\frac{h}{k}\right) = \frac{kD_\alpha(h) - hD_\alpha(k)}{k^2} \tag{6}$$

Proof: the fractional derivative of fractional function

$$\begin{aligned} D_\alpha\left(\frac{h}{k}\right) &= \lim_{\varepsilon\to 0} \frac{\frac{h(t+\varepsilon t^{1-\alpha})}{k(t+\varepsilon t^{1-\alpha})} - \frac{h(t)}{k(t)}}{\varepsilon} \\ &= \lim_{\varepsilon\to 0} \frac{1}{\varepsilon} \frac{h(t+\varepsilon t^{1-\alpha})k(t) - h(t)\left(k(t+\varepsilon t^{1-\alpha})\right)}{k(t+\varepsilon t^{1-\alpha})k(t)} \\ &= \lim_{\varepsilon\to 0} \frac{1}{k(t+\varepsilon t^{1-\alpha})k(t)}\Big(\frac{k(t)h(t+\varepsilon t^{1-\alpha}) - h(t)}{\varepsilon} \\ &\quad -h(t)\ \frac{k(t+\varepsilon t^{1-\alpha}) - k(t)}{\varepsilon}\Big) \\ &= \frac{1}{k(t)k(t)}(k(t)D_\alpha h(t) - h(t)D_\alpha k(t) \\ &= \frac{kD_\alpha(h) - hD_\alpha(k)}{k^2} \end{aligned}$$

Theorem 2.6: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{h}$ be $\boldsymbol{\alpha}$ time differentiable at a point $\boldsymbol{t} > \mathbf{0}$ then

$$D_\alpha(h) = x^{1-\alpha}\frac{dh}{dx}(x) \tag{7}$$

Proof:

$$D_\alpha(h) = \lim_{\varepsilon\to 0} \frac{h(x+\varepsilon x^{1-\alpha}) - h(x)}{\varepsilon}$$

By definition, let $\boldsymbol{r} = \boldsymbol{\varepsilon x^{1-\alpha}} \rightarrow \boldsymbol{\varepsilon} = \boldsymbol{x^{\alpha-1} r}$

$$\begin{aligned} &= \lim_{r\to 0} \frac{h(x+r) - h(x)}{x^{\alpha-1}r} \\ &= x^{1-\alpha} \lim_{r\to 0} \frac{h(x+r) - h(x)}{r} \\ &= x^{1-\alpha}\frac{dh}{dx}(x) \end{aligned}$$

Theorem 2.7: Let $\boldsymbol{\alpha} \in (\mathbf{0}, \mathbf{1}]$ and $\boldsymbol{h}, \boldsymbol{k}$ be $\boldsymbol{\alpha}$ time differentiable at a point $\boldsymbol{t} > \mathbf{0}$ then

$$D_\alpha(h \circ k) = x^{1-\alpha}h^\alpha(k(x))k^\alpha(x) \tag{8}$$

Proof:

$$\begin{aligned} D_\alpha(h\circ k)(x) &= \lim_{\varepsilon\to 0} \frac{h\left(k(x+\varepsilon x^{1-\alpha})\right) - h\left(k(x)\right)}{\varepsilon} \\ &= \lim_{\varepsilon\to 0} \frac{h\left(k(x+\varepsilon x^{1-\alpha})\right) - h\left(k(x)\right)}{\varepsilon} \times \frac{k(x+\varepsilon x^{1-\alpha}) - k(x)}{k(x+\varepsilon x^{1-\alpha}) - k(x)} \\ &= \lim_{\varepsilon\to 0} \frac{h\left(k(x+\varepsilon x^{1-\alpha})\right) - h\left(k(x)\right)}{k(x+\varepsilon x^{1-\alpha}) - k(x)} \times \frac{k(x+\varepsilon x^{1-\alpha}) - k(x)}{\varepsilon} \end{aligned}$$

We have, $\boldsymbol{r} = \boldsymbol{\varepsilon x^{1-\alpha}} \rightarrow \boldsymbol{\varepsilon} = \boldsymbol{x^{\alpha-1} r}$

$$\begin{aligned} &= \lim_{r\to 0} \frac{h\left(k(x+r)\right) - h\left(k(x)\right)}{k(x+r) - k(x)} \times \frac{k(x+r) - k(x)}{x^{\alpha-1}r} \\ &= x^{1-\alpha} \lim_{r\to 0} \frac{h\left(k(x+r)\right) - h\left(k(x)\right)}{k(x+r) - k(x)} \times \frac{k(x+r) - k(x)}{r} \\ &= x^{1-\alpha}h^\alpha(k(x))k^\alpha(x) \end{aligned}$$

Theorem 2.8: conformable fractional derivative of known functions.

1. $\boldsymbol{D_\alpha(e^{bx)} = bx^{1-\alpha}e^{(bx)}}$
2. $\boldsymbol{D_\alpha(\sin(bx)) = bx^{1-\alpha}\cos(bx)\,,\ b \in R}$
3. $\boldsymbol{D_\alpha(\cos(bx)) = -bx^{1-\alpha}\sin(bx),\ b \in R}$
4. $\boldsymbol{D_\alpha(\tan(bx)) = bx^{1-\alpha}\,sec^2(bx),\ b \in R}$
5. $\boldsymbol{D_\alpha(\cot(bx)) = -bx^{1-\alpha}\,csc^2(bx),\ b \in R}$
6. $\boldsymbol{D_\alpha(\sec(bx)) = bx^{1-\alpha}\sec(bx)\tan(bx),\ b \in R}$
7. $\boldsymbol{D_\alpha(\csc(bx)) = -bx^{1-\alpha}\csc(bx)\cot(bx),\ b \in R}$
8. $\boldsymbol{D_\alpha(\frac{1}{\alpha}x^\alpha)}$
9. $\boldsymbol{D_\alpha\left(\sin\left(\frac{1}{\alpha}x^\alpha\right)\right) = \cos\left(\frac{1}{\alpha}x^\alpha\right)}$
10. $\boldsymbol{D_\alpha\left(\cos\left(\frac{1}{\alpha}x^\alpha\right)\right) = -\sin\left(\frac{1}{\alpha}x^\alpha\right)}$
11. $\boldsymbol{D_\alpha\left(e^{\frac{1}{\alpha}x^\alpha}\right) = e^{\frac{1}{\alpha}x^\alpha}}$

Definition 2.2: Let h be a continuous function Then $\boldsymbol{\alpha}$ time integrable of h is defined as:

$$\boldsymbol{j_\alpha^a h(t) = \int_a^t \frac{h(x)}{x^{1-\alpha}}dx} \tag{9}$$

When $\boldsymbol{\alpha} \in [\boldsymbol{0},\boldsymbol{1})$ and the integral is the usual integral.

Example 2.1: find the following integral $\boldsymbol{j_{\frac{1}{2}}^0(\cos 2\sqrt{t})}$

Solution:

$$\boldsymbol{j_{\frac{1}{2}}^0(cos\,2\sqrt{t}) = \int_0^t \frac{cos\,2\sqrt{t})}{x^{1-\frac{1}{2}}}dx}$$

Let u=2$\sqrt{\boldsymbol{x}} \rightarrow \boldsymbol{du} = \frac{\boldsymbol{1}}{\sqrt{x}}$

$$\boldsymbol{\int_0^t \cos u\,du = \sin(2\sqrt{t})}$$

Theorem 2.9: Let h be any continuous function in the domain of $\boldsymbol{j_\alpha}$.then

$$\boldsymbol{D_\alpha j_\alpha^a\big(h(t)\big) = h(t), \forall\, t \geq 0} \tag{10}$$

Proof: since f continues, so$\boldsymbol{j_\alpha^a h(t)}$ is differentiable then

$$\begin{aligned}\boldsymbol{D_\alpha j_\alpha^a\big(h(t)\big)} &= \boldsymbol{t^{1-\alpha}\frac{d}{dt}j_\alpha^a h(t)}\\ &= \boldsymbol{t^{1-\alpha}\frac{d}{dt}\int_a^t \frac{h(t)}{x^{1-\alpha}}dx}\\ &= \boldsymbol{h(t)}\end{aligned}$$

1. Theorem 2.10: Let $\boldsymbol{\alpha} \in (\boldsymbol{0},\boldsymbol{1}]$ and h can be any continuous function in domain$\boldsymbol{j_\alpha}$, for $\boldsymbol{t > \alpha}$ since we have

$$\boldsymbol{\frac{d}{dt}[j_\alpha^a h(t)] = \frac{h(t)}{t^{1-\alpha}}} \tag{11}$$

This theory is fundamental to obtaining the analytical solution of conformable differential equations.

3.RESULTS and DISCUSSION

3.1 (Fractional differential equation)

In this section, we have presented the definition and methods for solving the fractional differential equation for the two systems.

Definition 3.1: typically the differential equations of order $\boldsymbol{\alpha}$which we remember mathematically represented by way of the following form:

$$\boldsymbol{D_\alpha(y) + h(x)y = k(x)} \tag{12}$$

When $\boldsymbol{0 < \alpha < 1}$, $\boldsymbol{y \in R^n}$ and $\boldsymbol{D_\alpha(y)}$describe the conformable derivative of y and $\boldsymbol{h, k}: \boldsymbol{R \to R}$ are

$\boldsymbol{\alpha}$ time differentiable Functions, if $\boldsymbol{\alpha = 1}$we get better the classical differential equations of first order expressed as$\boldsymbol{y' + h(x)(y) = k(x)}$We first take the case in which $\boldsymbol{k(x) = 0}$, then

$$\boldsymbol{D_\alpha(y) + h(x)y = 0} \tag{13}$$

Is called the homogeneous, If $\boldsymbol{k(x) \neq 0}$is called the non-homogeneous.

Theorem 3.1: The homogeneous solution of the conformable differential equation (13) is indicated by

$$\boldsymbol{y_h(x) = ce^{-j_\alpha^0 h(x)} = ce^{r\frac{1}{\alpha}x^\alpha}} \tag{14}$$

Where $\boldsymbol{y_h}$ is homogeneous solution and h is any continuous function in the domain of $\boldsymbol{j_\alpha^0}$.

Proof: we have just verified that equation (13) is fulfilled by obtaining the function

$$\boldsymbol{y(x) = ce^{-j_\alpha^0 h(x)}}$$

We get by substituting into the above equation and applying theorem (2.10)

$$\begin{aligned}
&\boldsymbol{D_\alpha(y) + h(x)y = ct^{1-\alpha}\frac{d}{dx}\left[e^{-j_\alpha^0 h(x)}\right] + ch(x)e^{-j_\alpha^0 h(x)}}\\
&\boldsymbol{= -cx^{1-\alpha}\frac{d}{dx}[j_\alpha^0 h(x)]e^{-j_\alpha^0 h(x)} + ch(x)e^{-j_\alpha^0 h(x)}}\\
&\boldsymbol{= -cx^{1-\alpha}\frac{h(x)}{x^{1-\alpha}}e^{-j_\alpha^0 h(x)} + ch(x)e^{-j_\alpha^0 h(x)}}\\
&\boldsymbol{= 0}
\end{aligned}$$

Theorem 3.2: The particular solution of the conformable differential equation (12) is shown by

$$\boldsymbol{y_p(t) = \lambda(t)e^{-j_\alpha^0 h(t)}} \tag{15}$$

Where$\boldsymbol{h}$ is any continuing function in the field of$\boldsymbol{j_\alpha^0}$ and the function $\boldsymbol{\lambda}: \boldsymbol{R \to R}$ is obtained through the following condition,

$$\boldsymbol{\lambda(t) = j_\alpha^0(k(x)e^{j_\alpha^0 k(x)}} \tag{16}$$

Proof: By obtaining the function, we have confirmed that equation (15) is satisfied

$$\boldsymbol{y_p(x) = \lambda(x)e^{-j_\alpha^0 h(x)}}$$

Substituting in the latter equation and applying the theorem (2.9) We've got

$$\begin{aligned}
&\boldsymbol{D_\alpha(y) + h(x)y = D_\alpha\left(j_\alpha^0\left(k(x)e^{j_\alpha^0 h(x)}\right)\right)e^{-j_\alpha^0 h(x)} + h(x)(j_\alpha^0\left(k(x)e^{j_\alpha^0 h(x)}\right)e^{-j_\alpha^0 h(x)}}\\
&\boldsymbol{= k(x)e^0 - j_\alpha^0\left(k(x)e^{j_\alpha^0 h(x)}\right)h(x)e^{-j_\alpha^0 h(x)} + h(x)(j_\alpha^0\left(k(x)e^{j_\alpha^0 h(x)}\right)e^{-j_\alpha^0 h(x)}}\\
&\boldsymbol{= k(x)}
\end{aligned}$$

Remark: The following is a general candidate solution for the differential equations defined by (12):

$$\boldsymbol{y(x) = y_h(x) + y_p(x)}$$

Example 3.1: find the particular and homogeneous solution to the following differential equation

1. $\boldsymbol{D_{\frac{1}{2}}(y) + y = t^2 + 2t^{\frac{3}{2}}}$
2. $\boldsymbol{D_{\frac{1}{2}}(y) - y = 5e^{2\sqrt{t}}}$

Solution 1: homogeneous solution is

$$\boldsymbol{y_h = ce^{-j_{\frac{1}{2}}^{0}(1)} = ce^{-2\sqrt{t}}}$$

Particular solution is

$$\boldsymbol{y_p = at^2 + bt^{\frac{3}{2}} + ct + d\sqrt{t} + e}$$
$$\boldsymbol{D_{\frac{1}{2}}(y_p) = 2at^{\frac{3}{2}} + \frac{3}{2}bt + c\sqrt{t} + \frac{1}{2}d}$$

When we substitute $\boldsymbol{D_{\frac{1}{2}}(y_p)}$ and $\boldsymbol{y_p}$ in basic equation we find a,b,c,d

$$\boldsymbol{y_p = t^2 - \frac{4}{3}t^{\frac{3}{2}} + 2t - 2\sqrt{t} + 1}$$

Solution 2: homogeneous solution is

$$\boldsymbol{y_h = ce^{-j_{\frac{1}{2}}^{0}(-1)} = ce^{2\sqrt{t}}}$$

Particular solution is

$$\boldsymbol{y_p = \lambda(t)e^{-j_{\frac{1}{2}}^{0}(-1)} = 10\sqrt{t}e^{2\sqrt{t}}}$$

3.2 (Fractional differential equations Bernoulli and Riccati)

By modifying the dependent variable, Bernoulli and Riccati equations can be translated to first order linear equations.

The solution of first order linear fractional equations is explained in detail, The general solutions to the fractional equations of Bernoulli and Riccati are provided.

The linear fractional differential equation is the most common form of fractional differential equation, where the higher order conformable fractional derivative is a linear function of the lower order conformable fractional derivative consequently, the first order in general the conformable fractional derivative of fractional differential equation is defined as

$$D_\alpha(y) + h(x)y = k(x) \tag{17}$$

Where $h(x)$ and $k(x)$ are α time differentiable functions, and y is a function that we don't know about. Equation (17) will be written as using theorm(2.10)

$$x^{1-\alpha}y' + h(x)y = k(x)$$
$$y' + \frac{h(x)}{x^{1-\alpha}}y = \frac{k(x)}{x^{1-\alpha}} \tag{18}$$

Equation (18) is a general solution to a first order linear ordinary differential equation

$$y = e^{-\int \frac{h(x)}{x^{1-\alpha}}dx}\left[\int \frac{k(x)}{x^{1-\alpha}} e^{\int \frac{h(x)}{x^{1-\alpha}}dx} dx + c\right] \tag{19}$$

Wherever C is a fixed value and arbitrary. Using (9) and replacement in equation (19), we can get

$$y = e^{-j_\alpha h(x)}\left[j_\alpha(k(x)e^{j_\alpha h(x)}) + c\right] \tag{20}$$

As a consequence, equation (20) has a general solution (18).

Example 3.2: solve the first-order linear fractional differential equation

$$D_{\frac{1}{2}}(y) + y = x^2 + 2x^{\frac{3}{2}}$$

Solution: where $h(x) = 1$ and $k(x) = x^2 + 2x^{\frac{3}{2}}$

$$y = e^{-j_\alpha h(x)}\left[j_\alpha k(x)e^{-j_\alpha h(x)} + c\right]$$

$$y = e^{-2x^{\frac{1}{2}}}\left[x^2 e^{2x^{\frac{1}{2}}} + c\right]$$
$$y = x^2 + ce^{-2x^{\frac{1}{2}}}$$

Definition 3.2: Bernoulli fractional differential equations can be expressed in the following way.

$$D_\alpha(y) + h(x)y = k(x)(y)^n \tag{21}$$

When $h(x)$ and $k(x)$ are α time differentiable function, and y is a function that is unknown. Equation (21) is obtained by applying theorem

$$x^{1-\alpha}(y) + h(x)y = k(x)(y)^n$$
$$y' + \frac{h(x)}{x^{1-\alpha}}y = \frac{k(x)}{x^{1-\alpha}}(y)^n \tag{22}$$

The Bernoulli equation is equation (22).
We know it will be linear for$n = 0$ or $n = 1$ and by changing the dependent variable, it can be reduced to a linear ordinary equation for every other value of n

$$y'y^{-n} + \frac{h(x)}{x^{1-\alpha}}y^{1-n} = \frac{k(x)}{x^{1-\alpha}}$$

Let $z = y^{1-n}$ then $z' = (1-n)y^{-n}y'$

$$z' + (1+n)\frac{h(x)}{x^{1-\alpha}}z = (1-n)\frac{k(x)}{x^{1-\alpha}}$$

According to the results the general solution is as follows

$$y = \left(e^{-j_\alpha((1-n)h(x))}\left[j_\alpha((1-n)k(x)e^{j_\alpha((1-n)h(x))}) + c\right]\right)^{\frac{1}{1-n}} \tag{23}$$

Example 3.3: find the answer to the following questions Centered on conformable fractional derivatives, Bernoulli fractional differential equations.

$$D_{\frac{1}{2}}(y) + \sqrt{x}y = (xe^{-2x})(y)^{-1}$$

Solution: where $h(x) = \sqrt{x}$ and$k(x) = xe^{-2x}$

$$y = \left(e^{-j_\alpha((1-n)h(x))}\left[j_\alpha((1-n)k(x)e^{j_\alpha((1-n)h(x))}) + c\right]\right)^{\frac{1}{1-n}}$$
$$y = (e^{-2x}\left(\frac{4}{3}x^{\frac{3}{2}} + c\right))^{\frac{1}{2}}$$

Definition 3.3: The Riccati fractional differential equations are a natural extension of a first order fractional differential equation.

$$D_\alpha(y) = h(x) + k(x)y + u(x)(y)^2 \tag{24}$$

Where $h(x), k(x)$ and $u(x)$ are α time differentiable functions, and y is a function that isn't recognized.
If a specific solution $y_1$is known, Then there's the general solution, which comes in the form of $y = y_1 + z$ where z is an all-encompassing solution to the following Fractional differential equation Bernoulli

$$D_\alpha(z) + (-k(x) - 2u(x)y_1)z = u(x)(z)^2 \tag{25}$$

Example 3.4: To solve the Riccati fractional differential equations, find the general solution

$$D_{\frac{1}{2}}(y) = -x\sqrt{x} + \frac{1}{2\sqrt{x}}y + \sqrt{x}(y)^2, y_1 = \sqrt{x}$$

Solution:

$$D_\alpha(z) + \left(-\frac{1}{2\sqrt{x}} - 2\sqrt{x}\sqrt{x}\right)z = \sqrt{x}(z)^2$$
$$z = \left(e^{-j_\alpha((-1)(-\frac{1}{2\sqrt{x}}-2x))}\left[j_{\frac{1}{2}}((-1)\sqrt{x}e^{-j_\alpha((-1)(-\frac{1}{2\sqrt{x}}-2x))}) + c\right]\right)^{-1}$$
$$z = \frac{2\sqrt{x}e^{\frac{4}{3}x^{\frac{1}{2}}}}{c - e^{\frac{4}{3}x^{\frac{3}{2}}}}$$

8.Conclusion

We will briefly discuss several methods for solving fractional deferential equations in this article.
Finding the form of the general solution of conformable differential equations is difficult. The conformable differential equation has a large number of solutions, as we know. The specific form of the candidate solution of the conformable differential equation is given in this article.
The general solution of first order linear fractional differential equations based on conformable fractional derivative has been solved for Bernoulli and Riccati fractional differential equations.
There was no need to use a numerical method since this method yielded the same solution.

References

A. A. H. M. S. a. J. J. T. Kilbas, „. ,. (2006). Theory and applications of fractional differential equations. elsevier,, Vol. 204.

Al-Tarawneh, S. A. (2016). Solving Fractional Differential Equations by Using Conformable Fractional Derivatives Definition. Zarqa University, Jordan.

Bali, N. P. (2005). Golden real analysis. Firewall Media.

D. Y. Abdulwahab, „. ,. (2019). Properties and Application of the Caputo Fractional Operator. Baghdad: Baghdad University.

Guzman, .. M. (2018.). A new definition of a fractional derivative of local type. Journal of Mathematical Analysis, pp. 88-98.

Hammad, M. A. (2014). Abel's formula and wronskian for conformable fractional differential equations. International Journal of Differential Equations and Applications, p. 13.3,.

Haubold, H. J. (2011). Mittag-Leffler functions and their applications. Journal of Applied Mathematics.

Horani, M. A. (2016). Variation of parameters for local fractional nonhomogenous linear differential equations. J. Math. Comput. Sci , pp. 147-153.

Ishteva, M. (2005). Properties and applications of the Caputo fractional operator,. University of Karlsruhe.

Iyiola, O. S. (2016). Some new results on the new conformable fractional calculus with application using D'Alambert approach,. Progr. Fract. Differ. Appl,, pp. 115-122.

J. L. Schiff, T. (2013). he Laplace transform: theory and applications. Business Media.

Kareem, A. M. (2017). Conformable fractional derivatives and it is applications for solving fractional differential equations. J. Math, pp. 81-87.

Khalil, R. e. (2014). A new definition of fractional derivative. Journal of Computational and Applied Mathematics, pp. 65-70.

Khalil, R. M. (2016). Undetermined coefficients for local fractional differential equations. J. Math. Comput, pp. 140-146.

Kimeu, J. M. (2009). Fractional calculus: Definitions and applications.

Koning, D. E. (2015). Fractional Calculus. Diss.

M. J. B. a. Z. A. Ilie. (2018). General solution of second order fractional differential equations. Int. J. Appl. Math., pp. 56-61.

Matlob, M. a. (2017). he Concepts and Applications of Fractional Order Differential Calculus in Modelling of Viscoelastic Systems,.

Matlob, M. A. (2019). „The concepts and applications of fractional order differential calculus in modeling of viscoelastic systems. Critical Reviews.

Milici, C. G. (2019). Introduction to fractional differential equations. Springer.

Oliveira, .. C. (2014.). A review of definitions for fractional derivatives and integral,. Mathematical Problems in Engineering.

Oliveira, E. C. (2019). Solved Exercises in Fractional Calculus. Springer International Publishing.

Omar, A. A. (2017). The Numerical Solution of Linear Variable Order Fractional Differential Equations Using Bernstein Polynomials.

Ortigueira, M. D. (2015). What is a fractional derivative? Journal of computational Physics, pp. 4-13,.

Ouyang, Y. a. (2016). Comparison of Definition of Several Fractional Derivatives,. International Conference on Education.

Podlubny, I. (1998.). Fractional differential equations: an introduction to fractional derivatives. Elsevier.

Sene, N. (2018). Solutions for some conformable differential equations. Progr. Fract. Differ Appl , pp. 493-501.

T.Abdeljawad. (pp. 57-66). On conformable fractional calculus. Journal of computational and Applied Mathematics,, 2015.

Z. H. O. U. W. J. a. Z. L. Yong. (2016). Basic theory of fractional differential equations. World Scientific,.