A Gazing of Blast Domination Number Pro-Jump Graph

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Abstract: The extensive inquiries of the Blast domination number, this article dissects upsetting the parameter over Jump graph. The theoretical properties of the Blast domination number of a jump graph plus its accurate values for some customary graphs are derived. Relations between the Blast domination number of a jump graph and the Blast domination number of the analogous graphs are additionally perceived.
Keywords: TheBlast domination Number of a graph, The Triple connected graph

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1. Introduction

Here we consider only simple, connected, undirected graphs. For standardgraph theoretic notations, refer to **(West.2003)** while for terminology related todomination in graphs, refer to Haynes et al. **(G.Chatrand, H.Hevia.1997)**. Lately, enormous number of triple connected domination parameters was familiarized. Aggravated by such new triple connected domination parameters, recently the concept of the Blast DominationNumber was introduced by **(A. Ahila.2017)**, which got its delicate applications in blasting mines, quarries and sensor areas **(A. Ahila.2017).** A subset S of V of a non-trivial connected graph G is called aBlast dominating set (or) BD-set(or) $\gamma_c^{tc} - set$, if S is a connected dominating set and the induced sub graph $\langle V - S \rangle$ is triple connected. The least cardinality taken over all such Blast Dominating sets is called the Blast Domination Number (BDN) of G and is denoted by $\boldsymbol{\gamma_c^{tc}(G)}$. Now combined with the applications of jump graphs **(T.Haynes,S. Hedetneimi.1998),** the study narrowed towards higher applications. A set $S \subset V[J(G)]$ is a BlastDominating set of Jump Graphs, if 'S' is a connected dominating set and the induced sub-graph $\langle V - S \rangle$ is triple connected. The Blast domination number of a jump graph is the minimum cardinality of all blast dominating sets of jump graph,$J(G)$. Largely, the least number of edges in edge cover of G plus the least number of edges in independent set of edges of G is symbolized by $\alpha_1(G)$ and $\beta_1(G)$.

2.Preliminary Definitions

The $\theta - obrazom$graph **(A. Ahila.2018)**,the edges of G are the vertices in$L(G)$, that are adjacent in $L(G)$, iff the respective edges are neighbours in G. The complement of line graph, $\overline{L(G)}$ is the jump graph, $J(G)$. The jump graph, $J(G)$ is the graph defined on $E(G)$ and in which two vertices are adjacent, iff they are not adjacent in G. That is, iff the edges are connected in $\bar{\bar{G}}$. As both $L(G)$ and $J(G)$ are defined on the edge set of G, it is obvious that theisolated vertices got no role. So, in this related search, we consider only the non-empty graphs devoid of isolated vertices. Here we consider only the connected jump graph, with $p > 4$. Let us recall the following theorems, to prove our forthcoming results. It is obvious that for any graph, G with cardinality less thanor equal to 4, the jump graph, $J(G)$, is disconnected and so are not considered for the analysis here, which barely on connected graphs.

The reader is insinuated to **[1,2,7]** for basic definitions and terminologies not portrayed here.

Theorem 2.1 **[7]:** Let graph Ghas no isolated vertices. If S is a least dominating set, then $\langle V - S \rangle$ is a dominating set.

Theorem 2.2 **[7]:** If graphG is devoid of zeroδ(v)vertices, then $\gamma(G) \leq \frac{p}{2}$

Theorem 2.3 **[7]:** In a simple graph$G(V, E)$, $|p^2| - |p| \geq 2|q|$.

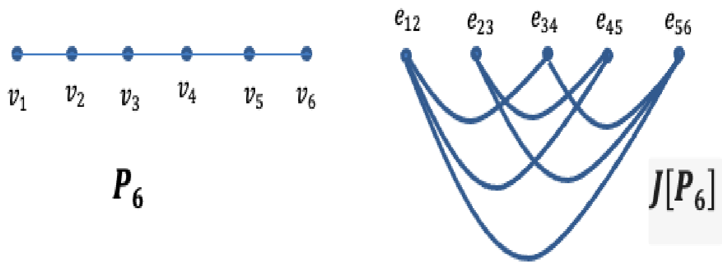
3.Core Results

Blast Domination Number for jump graph of certain basic graphs:

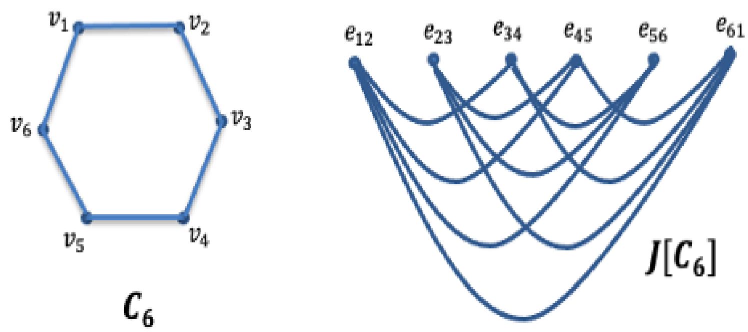
Let us observe some preliminary results for some standard graphs. Let us notate $V(G) = \{v_1, v_2, \dots v_n\}$ and $E(G) = \{e_{ij}, such\ that\ v_i\ adjacent\ to\ v_j\ in\ G, \forall\ i \neq j.$

Hence, $V[J(G)] = \{e_{ij}, such\ that\ v_i\ adjacent\ to\ v_j\ in\ G, \forall\ i \neq j\}$ and e_{ij} adjacent to e_{mn} , if i and j both neither equal to m nor m in $J(G)$.

***Theorem 3.1*:** In a path graph, $P_p, p \geq 6, \gamma_c^{tc}[J(P_p)] = 2$.
The Proof is obvious that the set $\{e_{12}, e_{(p-1)p}\}$ scraps the blast dominating set perpetually, the case.



***Theorem 3.2*:** For any cycle, $C_p \geq 5, \gamma_c^{tc}[J(C_p)] = 3$.
Proof: Obviously the set $\{e_{12}, e_{34}, e_{56}\}$ is the minimal cardinal set of $J(C_p)$, which satisfies the definition of blast domination number.



***Theorem 3.3*:** For any complete graph, $K_p, p \geq 6, \gamma_c^{tc}[J(K_p)] = 2$.
Proof:

For $p = 4$, $J(K_p)$ will be disconnected; when $p = 5$, $J(K_p)$ get 5 pendent vertices and blast domination set never exist. Thus, for other greater values of p, any set of the form $\{e_{ij}, e_{mn}, such\ that\ i, j, m, n\ are\ all\ distinct\}$ will be a blast dominating set. Thus, the proof is highly intuited and trivial.

***Theorem 3.4*:** For any Fan, $W_p \geq 7, \gamma_c^{tc}[J(W_p)] = 4$.
Proof:

Every set of the form $\{e_{ij}, e_{(j+1)(j+2)}, e_{i(j+3)}, e_{(j+4)(i+1)}\}$ will satisfy the least blast dominating set. Hence the result is evident.

***Theorem 3.5*:** Blast domination number seldom subsist for the jump graph of a Star,
Proof: As all edges are adjacent to each other $J(K_{1,p})$ contains all isolated vertices.

***Theorem 3.6*:** For any Complete bipartite graph, $K_{m,n}, \forall m \geq 2, n \geq 4, \gamma_c^{tc}[J(W_p)] = 4$.
Proof: It is evident that the set $\{e_{1(m+1)}, e_{1(m+3)}, e_{2(n-1)}, e_{2n}\}$ satisfies, for all m, n considered.

***Theorem 3.7*:** Considering all connected graphs, G, Blast domination number of the jump graph of G is greater than or equal to 2.

Proof:

Suppose, $\gamma_c^{tc}[J(G)] = 1$. Let the Blast dominating set be $\{v\}$. Then, it means that v is a full vertex in $J(G)$. Which implies v is isolated in G. Negates our connectedness of G. The rest of the cases hold as ever. Hence, $\gamma_c^{tc}[J(G)] \geq 2$.

***Theorem 3.8*:** Supposing S is a blast dominating set of $J(G)$, conditioned that $|S| = \gamma_c^{tc}[J(G)]$, then $|J(G)| - |S| \leq \sum \deg(v_i)$.

Proof:

Here S is a blast dominating set, every one vertex of $V[J(G)] - V[S]$ is adjacent to at least one vertex in S. There will be an influence from each vertex of $V[J(G)] - V[S]$ by one to the degrees of vertices of S. Thus the sum of degrees of S exceeds.

***Observation 3.9*:** The relations between Blast domination numbers of a jump graph with the Blast domination number of the corresponding graphs are also observed as follows:

a) Blast domination numbers of a jump graph exists for graphs such as Paths, cycles, Stars.
b) Probably, Blast domination numbers of a jump graph is greater than Blast domination number of the corresponding graphs, with exemptions like complete bipartite graph.
c) $\gamma_c^{tc}[J(G)] \geq 1$ and it exists for all non-connected graphs G.

***Theorem 3.10*:** For some blast dominating set S of $J(G)$, condition $|S| = \gamma_c^{tc}[J(G)]$, then $|V[J(G)]| - |S| \leq \sum deg(v_i)$.

Proof:

Whilst S is a blast dominating set, every vertex in $V[J(G)] - S$ is adjacent to at least one vertex in S, subsidizing 1 to the sum of degrees of vertices of S.

***Theorem 3.11*:** For every connected (p,q) graph G, $\gamma_c^{tc}[J(G)] \leq q - \beta_1(G) + 1$.

Proof:

Let $V_1[J(G)] = \{v_i, i = 1,2, \dots n\}$ corresponding to the set of independent edges, $\{e_i, i = 1,2, \dots n\}$ of G. By definition of $J(G)$, the elements of V form an induced subgraph $\langle K_n \rangle$ in $J(G)$. Foster, let $S \cup \{V_1\}$, where $S \subset [|V[J(G)]| - |V_1|]$ being a blast dominating set in $J(G)$. Hence, $\gamma_c^{tc}[J(G)] \leq q - \beta_1(G) + 1$.

***Theorem 3.12*:** For every connected (p,q) graph G, $\gamma_c^{tc}(G) + \gamma_c^{tc}[J(G)] < \left(\frac{p+1}{2}\right)^2$

Proof:

For every connected (p,q) graph G, $\gamma_c^{tc}(G) \leq m\{|S|, |V - S|\} \leq \frac{p}{2}$ is known. Moreover, by the definition of Jump graphs, we have $|V[J(G)]| = q$.

Subsequently, $\gamma_c^{tc}[J(G)] \leq \frac{q}{2}$. But for every simple graph G, $q \leq \frac{p(p-1)}{2}$.

Consequently, $\gamma_c^{tc}[J(G)] \leq \frac{p(p-1)}{4}$.

Thus, we get $\gamma_c^{tc}(G) + \gamma_c^{tc}[J(G)] \leq \frac{p}{2} + \frac{p(p-1)}{4} \leq \frac{p(p+1)}{4} < \left(\frac{p+1}{2}\right)^2$.

4.Conclusion

Thedomination and susceptibility of linkage are the 2 significant aspectson behalf of the web system. We have headed as anoteworthydegree of vulnerability termedtheBlast domination number. Hither we have investigated the Blast domination number for jump graphs. The outcomesrecounted here are surefire to toss some spark in the trajectory to labor the same in larger graphs attained from the specified graphs, with more down-to-earth remunerations. In practical situations, the parameter over this investigation, optimize the cost, installing processes as well.

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