FUZZY TRANPORTATION MODEL USING L-R FLAT TRAPEZOIDAL APPROXIMATIONS

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ABSTRACT

Transportation problem is applied for many differentcircumstances being scheduling,production, investment, plant location, inventory control, employment scheduling, and several methods. The aim of the fuzzy transportation problem is to resolve the transport schedule that minimizes the total fuzzy transportation cost while delightful the fuzzy supply and fuzzy demand limits. In view of this, a novel approach to optimize fuzzy transportation modelby using trapezoidal approximations of L-R flat fuzzy numbers is proposed. A few generalized and modern properties of the trapezoidal and triangular approximations of L-R flat fuzzy numbers forparameters in fuzzy transportation problem is suggested. Numerical example is illustrated for the novel fuzzy transportation model.

Keywords:L-R flat fuzzy numbers; Approximation of L-R flat fuzzy numbers; Fuzzy transportation problem; L-R type fuzzy ranking

1. INTRODUCTION

The transportation problem is an important application of Linear Programming Problem (LPP) models. The main view ofmentioned concept is obtaining the minimal transportation cost of a product for gratifying the demands at destination, using the available supply at origin. The transportation problem can be used in different type of positions, such as scheduling, production, investment, plant location & inventory control. The transportation models are explainedfrequently with the inference that the transportation costs, supply and demandsare stated accurately. However,for the transportation problem, the policy maker does not give correct data about the coefficientsin most of the situations.If the data has ambiguity which means that,the corresponding coefficientscan be developed by using fuzzy sets to determine the problem due to lack of precision and which approaches to fuzzy transportation problems.

Zadeh [1]developed the fuzzy set theory to discuss with the ambiguities. And in that uncertain information is transportation problem may be characterized by triangular and trapezoidal fuzzy numbers.Two examples are shown in trapezoidal approximation of fuzzy number to clarify such positions[2]. It developed the operator which gives some new properties.The concept of an approximation rangeof a fuzzy number was recommended [3]. Which fulfils two conditions and the opinion was correlated with an expected range of known fuzzy number.Two parameters, value and ambiguity were introduced to attain an authorized representation and to discuss with fuzzy numbers in policy – making problems[4]. The general arithmetic operations on real numbers areinvestigated using fuzzy fundamentals and these algorithms to be fitted for fuzzy settheory[5].Fuzzy numbers on intervals was defined andshown to be additive with respect to addition of fuzzy range while only inequalities are satisfied and defined that the theory of interval – valued mean fuzzy numbers [6]. In [6], how the problem to take measure- theoretical assumptions into the branch of possibility theory was discussed. In [7], two metrics in space of fuzzy numbers are introduced and they are used torank fuzzy numbers.Grzegorzewski [8] has explained the problem of an interval approximation of a fuzzy numbers and suggested to measure of distance between two different fuzzy numbers for a relevant new approximation period. The closest triangular approximation operator defending the expected range from the closest trapezoidal approximation operator preserving the expected interval is proposed [9]. The theory of trapezoidal approximations of a fuzzy numbers and the set of principles for nearest operators are developed[10], which are defending the expected interval. In [11], they revisiteddedicated to trapezoidal approximations of a fuzzy number and they shown a correct explanation for the approximation operator perpetuating the expected interval. This operator maintains several properties are showed in [10]. After corrections they suggested all these parameters are firmly connected with the expected interval invariance norms.[12] was introduced the concept of an expected interval and an expected value of a fuzzy numbers. The expected value of this number is explained as the middle of the expected period and the expected range is defined as the expected value of irregular set of fuzzy number. In [13], proposed a process to determine the interrelated coefficient of fuzzy number by the virtue of an expected interval. Also the strength of the relationship between fuzzy numbers is computed. The ranking fuzzy numbers over the resemblance of its expected values are presented in [14]. These are certify that rationality and virtual qualities and also compared non normal fuzzy numbers. The basic of the kudo- Aumann integral of a set- valued mapping is defined and some properties of average level of a fuzzy set for some special cases are analyzed in [15].A few generalized and modern properties of the trapezoidal approximations of fuzzy numbers, some relations of distance betwixt a fuzzy number and its trapezoidal approximations are proved in [16].'Mehars Method' is an advanced method to deal with the problems on fuzzy sensitivity analysis with LR flat fuzzy numbers are discussed in [17], in which either few or all specifications are characterized from unregulated L-R flat fuzzy numbers, which are may or may not be explained by existing methods are presented in [17]. The fuzzy transportation problem is determined with a new procedure is given in [18], in which the cost, origin and destinations are described as non-negative L-R flat fuzzy numbers.

We focus on a particular type of L-R fuzzy numbers, where L and R represent left and right shaped functions. These are involves triangular fuzzy number, trapezoidal fuzzy number, Gaussian fuzzy number, Cauchy fuzzy number as special cases called regular L-R flat fuzzy numbers. In view of this paper constraints of existing works [10], [11] are referred. To get the better of these limitations we introduced a triangular and trapezoidal approximation of L-R flat fuzzy numbersforsolving transportation cost, origin and destinations, supply and demands of the product in which all the parameters are characterized with unrestricted L-R flat fuzzy numbers. For illustration of the proposed method a numerical example has been given.

This paper formulated as follows:Section2, discussed preliminaries like primary definitions and Yager's ranking approach for unrestricted L-R flat fuzzy numbers . Section3, discussed the properties of triangular and trapezoidal approximations of L-R flat fuzzy numbers are introduced in which all the variables are described with unrestricted L-R flat fuzzy numbers.Section4 discussed the Transportation model ,Section5 explained the proposed work ,Section 6 discussed the numerical example is determined., Section 7,explained the conclusions.

2. Preliminaries

In this segment, a few fundamentaldefinitions of LR- flat fuzzynumbers are given.

Definition2.1 [17]: Let $\tilde{P} = (m, n, \alpha, \beta)$ is L-R flat fuzzy number anditsmembership function is defined as

$$\mu_{\tilde{P}} = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right) & x \le m \quad \alpha > 0 \\ 1 & m \le x \le n \\ R\left(\dfrac{x-n}{\beta}\right) & x \ge n \quad \beta > 0 \end{cases}$$

Let $P_\lambda = \{x \in \Re : \mu_P(x) \ge \lambda\}$ denote a $\alpha -$ cut of L-R flat fuzzy number and λ be a realnumber in the closed interval. Then α - cut of a fuzzy number is $P_\lambda = [P_L(\lambda) \, , \, P_U(\lambda)]$, where $P_L(\lambda) = \inf \{x \in R : \mu_{\tilde{P}}(x) \ge \lambda\}$ and $P_U(\lambda) = \sup\{x \in R : \mu_{\tilde{P}}(x) \ge \lambda\}$.

Definition2.2 [17]: If $m - \alpha \ge 0$, a L-R flat fuzzy number $\tilde{P} = (m, n, \alpha, \beta)$ is known as a non- negative.And alsoif $n + \beta \le 0$, $\tilde{P}$ is known as s a non – positive L-R flat fuzzy number.

Definition2.3 [17]:A L-R flat fuzzy number $\tilde{P} = (m, n, \alpha, \beta)$ and $\lambda \in \Re$ in the period[0, 1] then the crisp set is

$P_\lambda = \{x \in X : \mu_{\tilde{P}}(x) \ge \lambda\} = [m - \alpha L^{-1}(\lambda) \, , \, n + \beta R^{-1}(\lambda)]$ is called α - cut of $\tilde{P}$.

Definition2.4[17]: Yager's ranking function is determined for L-R flat fuzzy number $\tilde{P} = (m, n, \alpha, \beta)$ from it's α - cut $P_\lambda = [m - \alpha L^{-1}(\lambda) \, , \, n + \beta R^{-1}(\lambda)]$as stated by the following formula

$$\Re(\tilde{P}) = \int_0^1 \left(m - \alpha L^{-1}(\lambda)\right) d\lambda + \int_0^1 \left(n + \beta R^{-1}(\lambda)\right) d\lambda$$

i.e; $\Re(\tilde{P}) = m + n - \frac{2\alpha}{3} + \frac{\beta}{2}$

Let us suppose an expected interval $EI(\tilde{P})$ of an L-R flat fuzzy number $\tilde{P}$ is given by([6],[12])

$$EI(\tilde{P}) = \left[\int_0^1 P_L(\lambda)\, d\lambda \,,\, \int_0^1 P_U(\lambda)\, d\lambda \right] \quad (1)$$

Trapezoidal approximations of Fuzzy numbers were discussed in [11] and theyhaveexhibited to replace irrational fuzzy number by the Trapezoidal fuzzy number. Similarly we express thetrapezoidalL-R flat fuzzy number by linear parts and having the membership function is given below

$$\mu_{\tilde{P}} = \begin{cases} 0 & x \le m-\alpha \\ \dfrac{x-(m-\alpha)}{\alpha} & m-\alpha \le x < m \\ 1 & m \le x \le n \\ \dfrac{(n+\beta)-x}{\beta} & n < x < n+\beta \\ 0 & x \ge n+\beta \end{cases} \quad (2)$$

Since the trapezoidal L-R flat fuzzy number is fully describedby the real numbers (m, n, α, β) and they are denoted as $\tilde{P}(m, n, \alpha, \beta)$. A group of all trapezoidal L-R flat fuzzy number will be indicated as $(LR^T(\Re))$. From Eq. (1) and Eq.(2) an expected range of the trapezoidal L-R flat fuzzy number is presented as

$$EI(\tilde{Q}) = \left[\frac{2m-\alpha}{2} \,,\, \frac{2n+\beta}{2} \right] (3)$$

For two arbitrary L-R flat fuzzy numbers $\tilde{P}$ and $\tilde{Q}$ with $\alpha-$cuts$[P_L(\lambda),\ P_U(\lambda)]and[Q_L(\lambda),\ Q_U(\lambda)]$ respectively, then the distance between $\tilde{P}$ and $\tilde{Q}$.

$$(4)\ d(\tilde{P},\tilde{Q}) = \sqrt{\int_0^1 (P_L(\lambda) - Q_L(\lambda))^2\ d\lambda + \int_0^1 (P_U(\lambda) - Q_U(\lambda))^2\ d\lambda}$$

3. Trapezoidal approximation:

In this part we introduce an approximate operator $T : (LR(\Re)) \to (LR^T(\Re))$ which develops trapezoidal L-R flat fuzzy number that is nearest to given actual L-R flat fuzzy number between the trapezoidal L-R flat fuzzy numbers are all having indistinguishableexpected interval by the actual thing [8]. Then this operator can be known as the nearest trapezoidal approximation operator perpetuating an expected interval.

Let us take $\tilde{P}$ as L-R flat fuzzy number and its $\alpha-$cut is$[P_L(\lambda),\ P_U(\lambda)]$. Given $\tilde{P}$ we seek to determine a trapezoidal L-R flat fuzzy number T ($\tilde{A}$) which is the closest to $\tilde{P}$ with respect to metric d from Eq.(4). Suppose $[T_L(\lambda),\ T_U(\lambda)]$ denote the $\alpha-$cutof$T(\tilde{P})$. Then we need to reduce

$$d(\tilde{P}, T(\tilde{P})) = \sqrt{\int_0^1 (P_L(\lambda) - T_L(\lambda))^2\ d\lambda + \int_0^1 (P_U(\lambda) - T_U(\lambda))^2\ d\lambda} \quad (5)$$

Subject to $T_L(\lambda)$ and $T_U(\lambda)$. In spite of that, since a trapezoidal L-R flat fuzzy number is fully characterized by suchfour real numbers (m, n, α, β). Our proposal isminimizes to finding trapezoidal L-R flat fuzzy number is represented as $T(\tilde{P}) = T(m, n, \alpha, \beta)$.

It is assuming that the $\alpha-$cut of $T(\tilde{P})$ is equal to $[(m-\alpha)+\alpha\lambda \ ,\ (n+\beta)-\beta\lambda]$

Therefore (5) reduces to

(6)

$$d(\tilde{P}, T(\tilde{P})) = \sqrt{\int_0^1 (P_L(\lambda) - ((m-\alpha)+\alpha\lambda))^2\ d\lambda + \int_0^1 (P_U(\lambda) - ((n+\beta)-\beta\lambda)^2\ d\lambda}$$

And we will seek to reduce (6) with respect to m, n, α, β .

Yet, we need to solve a trapezoidal L-R flat fuzzy numberand which is not nearest to given L-R flat fuzzy number but it is perpetuates the expected interval of that L-R flat fuzzy number. We confirm this further necessity by the powerful role of the expected interval in several positions and applications([3], [6]-[8], [12]-[15]). Thus our issue is to identify such real numbers m, n, α, β that reduce (6) with respect to condition

(7) $$EI(T(\tilde{P})) = EI(\tilde{P})$$

From (1) and (3) we can write (7) as below

(8) $$\left[\frac{2m-\alpha}{2} , \frac{2n+\beta}{2}\right] = \left[\int_0^1 P_L(\lambda)\, d\lambda , \int_0^1 P_U(\lambda)\, d\lambda\right]$$

It is simply observable that one mayreduce $d(\tilde{P}, T(\tilde{P}))$ it satisfies to diminish the function $f(m,n,\alpha,\beta) = d^2(\tilde{P}, T(\tilde{P}))$ with respect to following conditions:

$$\frac{2m-\alpha}{2} - \int_0^1 P_L(\lambda) \quad d\lambda = 0 \qquad (9)$$

$$\frac{2n+\beta}{2} - \int_0^1 P_U(\lambda) \quad d\lambda = 0 \quad (10)$$

Moreover, For $m \le n$.

We havetodiminishing the function

(11)

$$\phi(x) = \int_0^1 \left(P_L(\lambda) - ((m-\alpha)+\alpha\lambda)\right)^2 \, d\lambda + \int_0^1 \left(P_U(\lambda) - ((n+\beta)-\beta\lambda)\right)^2 \, d\lambda$$

Subject to

(12)

(13) $$\psi(x) = \left[2m-\alpha-2\int_0^1 P_L(\lambda)\, d\lambda, \quad 2n+\beta-2\int_0^1 P_U(\lambda)\, d\lambda\right]$$

Where $x \in R^4$.

$$\xi(x) = m - n \le 0$$

FromKarush- Kuhn- Tucker theorem (KKT), if x^* is a provincialminimiser to the problem of reducing the function ϕ with respect to $\psi(x) = 0$, $\xi(x) \le 0$, then $\exists\ \lambda$ is Lagrange multiplier vector and μ is KKT multiplier such that,

$$D\phi(x^*) + \lambda D\psi(x^*) + \mu D\xi(x^*) = 0^T \quad (14)$$

$$\mu\xi(x^*) = 0 \quad (15)$$

$$\mu \ge 0 \quad (16)$$

In our case, after some calculations, we get

$$D\phi(x^*) = \begin{cases} 2m-\alpha-2\int_0^1 P_L(\lambda)\, d\lambda \\ 2n+\beta-2\int_0^1 P_U(\lambda)\, d\lambda \\ \frac{2\alpha}{3} - m + 2\int_0^1 P_L(\lambda)\, d\lambda - 2\int_0^1 \lambda P_L(\lambda)\, d\lambda \\ n + \frac{4\beta}{3} + 2\int_0^1 \lambda P_U(\lambda)\, d\lambda - 2\int_0^1 p_U(\lambda)\, d\lambda \end{cases} \qquad (17)$$

$$D\psi(x^*) = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix} \quad (18)$$

$D\xi(x^*) = [1 \quad -1 \quad 0 \quad 0]$ (19)

Then, we will takethe following KKT conditions

$$2m - \alpha - 2\int_0^1 P_L(\lambda)\, d\lambda + 2\lambda_1 + \mu = 0 \quad (20)$$

$$2n + \beta - 2\int_0^1 P_U(\lambda)\, d\lambda + 2\lambda_2 - \mu = 0 \quad (21)$$

$$\frac{2\alpha}{3} - m + 2\int_0^1 P_L(\lambda)\, d\lambda - 2\int_0^1 \lambda P_L(\lambda)\, d\lambda - \lambda_1 = 0 \quad (22)$$

$$n + \frac{2\beta}{3} + 2\int_0^1 \lambda P_U(\lambda)\, d\lambda - 2\int_0^1 P_U(\lambda)\, d\lambda + \lambda_2 = 0 \quad (23)$$

$$2m - \alpha - 2\int_0^1 P_L(\lambda)\, d\lambda = 0 \quad (24)$$

$$2n + \beta - 2\int_0^1 P_U(\lambda)\, d\lambda = 0 \quad (25)$$

$\mu(m-n) = 0$, (26)

$\mu \geq 0$ (27)

To evaluate the values of m, n, α, β for satisfying the above equations and initially, we use $\mu > 0$, i.e; $m - n = 0$. So we deal with a linear system of equations.

$$2m - \alpha - 2\int_0^1 P_L(\lambda)\, d\lambda + 2\lambda_1 + \mu = 0 \quad (28)$$

$$2n + \beta - 2\int_0^1 P_U(\lambda)\, d\lambda + 2\lambda_2 + \mu = 0 \quad (29)$$

$$\frac{2\alpha}{3} - m + 2\int_0^1 P_L(\lambda)\, d\lambda - 2\int_0^1 \lambda P_L(\lambda)\, d\lambda - \lambda_1 = 0 \quad (30)$$

$$n + \frac{2\beta}{3} + 2\int_0^1 \lambda P_U(\lambda)\, d\lambda - 2\int_0^1 P_U(\lambda)\, d\lambda + \lambda_2 = 0 \quad (31)$$

$$2m - \alpha - 2\int_0^1 P_L(\lambda)\, d\lambda = 0 \quad (32)$$

$$2n + \beta - 2\int_0^1 P_U(\lambda)\, d\lambda = 0 \quad (33)$$

Finding the above linear equations, we get

$$m = n = -\int_0^1 P_L(\lambda)\, d\lambda + 3\int_0^1 \lambda P_L(\lambda)\, d\lambda - \int_0^1 P_U(\lambda)\, d\lambda + 3\int_0^1 \lambda P_U(\lambda)\, d\lambda \quad (34)$$

$$\alpha = -4\int_0^1 P_L(\lambda)\, d\lambda + 6\int_0^1 \lambda P_L(\lambda)\, d\lambda - 2\int_0^1 P_U(\lambda)\, d\lambda + 6\int_0^1 \lambda P_U(\lambda)\, d\lambda \quad (35)$$

$$\beta = 2\int_0^1 P_L(\lambda)\,d\lambda - 6\int_0^1 \lambda P_L(\lambda)\,d\lambda + 4\int_0^1 P_U(\lambda)\,d\lambda - 6\int_0^1 \lambda P_U(\lambda)\,d\lambda \quad (36)$$

$$\lambda_1 = -\int_0^1 \lambda P_L(\lambda)\,d\lambda + \frac{1}{3}\int_0^1 P_L(\lambda)\,d\lambda - \frac{-1}{3}\int_0^1 P_U(\lambda)\,d\lambda + \int_0^1 \lambda P_U(\lambda)\,d\lambda \quad (37)$$

$$\lambda_2 = \frac{-1}{3}\int_0^1 P_L(\lambda)\,d\lambda + \int_0^1 \lambda P_L(\lambda)\,d\lambda + \frac{1}{3}\int_0^1 P_U(\lambda)\,d\lambda - \int_0^1 \lambda P_U(\lambda)\,d\lambda \quad (38)$$

$$\mu = 2\int_0^1 \lambda P_L(\lambda)\,d\lambda - \frac{2}{3}\int_0^1 P_L(\lambda)\,d\lambda + \frac{2}{3}\int_0^1 P_U(\lambda)\,d\lambda - 2\int_0^1 \lambda P_U(\lambda)\,d\lambda \quad (39)$$

In any manner, by the assume that $\mu > 0$, then we get appropriate results for the KKT equationsiff

$$\left(\int_0^1 \lambda P_L(\lambda)\,d\lambda - \frac{1}{3}\int_0^1 P_L(\lambda)\,d\lambda\right) - \left(\int_0^1 \lambda P_U(\lambda)\,d\lambda - \frac{1}{3}\int_0^1 P_U(\lambda)\,d\lambda\right) > 0$$

$$\int_0^1 \lambda P_L(\lambda)\,d\lambda - \frac{1}{3}\int_0^1 P_L(\lambda)\,d\lambda > \int_0^1 \lambda P_U(\lambda)\,d\lambda - \frac{1}{3}\int_0^1 P_U(\lambda)\,d\lambda$$

$$\int_0^1 [P_L(\lambda) - P_U(\lambda)]\left(\lambda - \frac{1}{3}\right) d\lambda > 0 \quad (40)$$

And then we get a solution $x^* = (m, n, \alpha, \beta)$ is provided from (34)-(36)

We consider $\mu = 0$ in the second try. So we have to determine the below system of equations

$$2m - \alpha - 2\int_0^1 P_L(\lambda)\,d\lambda + 2\lambda_1 + \mu = 0 \quad (41)$$

$$2n + \beta - 2\int_0^1 P_U(\lambda)\,d\lambda + 2\lambda_2 - \mu = 0 \quad (42)$$

$$\frac{2\alpha}{3} - m + 2\int_0^1 P_L(\lambda)\,d\lambda - 2\int_0^1 \lambda P_L(\lambda)\,d\lambda - \lambda_1 = 0 \quad (43)$$

$$2m - \alpha - 2\int_0^1 P_L(\lambda)\,d\lambda = 0 \quad (44)$$

$$2n + \beta - 2\int_0^1 P_U(\lambda)\,d\lambda = 0 \quad (45)$$

Obtain the above equations, and then we have

$$m = 6\int_0^1 \lambda P_L(\lambda)\,d\lambda - 2\int_0^1 P_L(\lambda)\,d\lambda \quad (46)$$

$$n = -2\int_0^1 P_U(\lambda)\,d\lambda + 6\int_0^1 \lambda P_U(\lambda)\,d\lambda \quad (47)$$

$$\alpha = 12\int_0^1 \lambda P_L(\lambda)\,d\lambda - 6\int_0^1 P_L(\lambda)\,d\lambda \quad (48)$$

$$\beta = 6\int_0^1 P_U(\lambda)\,d\lambda - 12\int_0^1 \lambda P_U(\lambda)\,d\lambda \quad (49)$$

$\lambda_1 = 0$ (50)

$\lambda_2 = 0$ (51)

Hencewe get trapezoidal approximation of L-R flat fuzzy numbers.
For further calculations,

$$\int_0^1 P_L(\lambda)\ d\lambda = \frac{2m-\alpha}{2} \quad (52)$$

$$\int_0^1 \lambda P_L(\lambda)\ d\lambda = \frac{3m-\alpha}{6} \quad (53)$$

$$\int_0^1 P_U(\lambda)\ d\lambda = \frac{2n+\beta}{2} \quad (54)$$

$$\int_0^1 \lambda P_U(\lambda)\ d\lambda = \frac{3n+\beta}{6} \quad (55)$$

4. **Fuzzy Transportation Model**

The transportation model is considered that the decision maker has particular information about the specific values of cost, availability and demand of the commodity in regular transportation models. The parameters are all of the transportation problem may or may not be known accurately in real life situations due to wild components. For e.g: in general life applications the following position may appear: consider a commodity is to be shipped at a destination and skilled person have no idea about the cost then there exist ambiguity data about the shipping cost. In such cases for obtaining transportation problems, the costs are considered as L-R flat fuzzy numbers. In the fuzzy transportation model, the decision maker has confusion about the definite values of the shipping cost from ith source to jth destination but if it has correct data about the supply and demand of the commodity, then the mathematical expression of the transportation model is given by

$$\text{Min} \quad Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, t_{ij}$$

Subject to

$$\sum_{j=1}^{n} t_{ij} \le \alpha_i \qquad i = 1,2,3,....$$

$$\sum_{i=1}^{m} t_{ij} \ge \beta_j \qquad j = 1,2,3,....$$

$$t_{ij} \ge 0 \quad \forall\, i, j$$

Where α_i be the total availability of the commodity at source (i) and β_j be the total demand of the product at destination (j), c_{ij} be the unit rough shipping cost for a unit quantity from source(i) to destination (j) and $Z = c_{ij}\, t_{ij}$ s the total least fuzzy transportation cost.

If $\sum_{i=1}^{m}\alpha_i = \sum_{j=1}^{n}\beta_j$ is called balance fuzzy transportation problem and it is not equal fuzzy transportation problem.

5. Proposed Methodology:

There are several methods for finding Fuzzy transportation problem with L-R flat fuzzy numbers [17][18] etc. In which methods are finding initial basic feasible solution (IBFS) by using Mehar's method and linear programming with non-negative L-R flat fuzzy numbers.

In this part, a few new properties are introduced for solving IBFS and fuzzy optimal solution by using Yager's ranking function with unrestricted trapezoidal and triangular approximations of L-R flat fuzzy numbers. The process of proposed method is given below and also numerical example.

The steps are given below for finding IBFS and optimal solution:

Step1: Transform L-R flat fuzzy numbers to trapezoidal and triangular approximation of L-R flat fuzzy numbers.

Step2: Form the transportation table using Yager's ranking function with L-R flat fuzzy Numbers.

Step3: Finding initial basic feasible solution (IBFS) using **VAM**

The process for finding IBFS using VAM as follows:

Case1: Calculate the fuzzy penalty cost for every row and column by finding the negative mean of least cost and immediate next to the least cost of every row and column. i.e; the difference between least cost and next to least cost by every row and column is divided.

Case2: If least cost appears higher than one time in a row or column then select same shipping cost as lowest cost and next to lowest cost and penalty becomes zero.

Case3: Take the rows or columns with the largest penalty costs (i.e; select the least cost cell). If there is bind appears in largest penalty cost, then select that row or column where he cost is least.

Case4: Determine shipping costs for chosen rows or columns in step3 by assigning approximately feasible value to the feasible cell with the smallest shipping cost.

Case5: At present balanced all the rows and columns also delete the gratified row or column. And allocate zero if satisfied at the same time to persist rows and columns.

Case6: Continue this process step1 – 5 up to all demands have been reached.

Case7: Calculate the total transportation cost for the feasible allotments by using fuzzy transportation problem matrix.

Step4: Finding optimal solution by using modified distribution method. This method provides a least cost solution to the fuzzy transportation problem.

The process for finding optimal solution by MODI method as follows:

Case1: Determine IBFS of fuzzy transportation problem using VAM

Case2: Find the twin variables α_i and β_j for the very row and columns respectively. Write α_i ahead of every row and β_j at the ground of every column. Choose any α_i or β_j is to be zero trapezoidal or triangular L-R flat fuzzy number.

Case3: To find α_i and β_j, using the formula $\alpha_i + \beta_j = c_{ij}$ where c_{ij} is the cost amount for the assigned chamber.

Case4: Calculate the penalties using $P_{ij} = \alpha_i + \beta_j - c_{ij}$ for unassigned chambers. And write them in area of concerned chamber. It may appear some following rules

(i) If all the penalties $P_{ij} \geq 0$ then the optimality is reached. So find the optimal solution.

(ii) If at least one penalty P_{ij} is negative then optimality is not reached. So go to case5.

Case5: Select that P_{ij}, whose penalty is largest negative then developed the solution can be determined by entering a vacant chamber (i, j) into the solution mix. A vacant chamber having highest negative value of P_{ij} is selected for entering into the new transportation programme.

Case6: when P_{ij} is highest negative then make a loop and create the loop with chosen vacant chamber and highlight a plus sign (+) in their chamber. Mark a loop along the rows or columns to a vacant chamber, and highlight the corner with a minus sign (-). Repeat this process the row or column to an inhabited chamber. Then the corner with plus sign (+) or minus sign (-) is traced alternatively.

Case7: Choose the lowest value amongst the chambers with minus sign is denoted on the corners of the closed path. Assign this value to the chosen vacant chamber and add or subtract it to inhabited chambers are traced with plus sign and minus sign.

Case8: According to case5 determine a new developed solution by assigning units to the vacant chambers and compute the new total transportation cost.

Case9: Continue this process until $P_{ij} \geq 0$ for case1 to case8.

Case10: Solve the fuzzy optimal solution and optimal costs are t_{ij} and $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, t_{ij}$ respectively.

6. **Numerical Examples**

In this section, two examples are given to solve fuzzy transportation problem of the proposed method.

Example: 1

The table 1 represents LR- flat fuzzy numbers for Trapezoidal fuzzy transportation cost of the product between different sources and different destinations.

Table 1: Transportation costs

	D_1	D_2	D_3	Supply
S_1	4,9,3,10	2,5,1,4	5,8,3,10	5,7,4,2
S_2	9,12,1,14	5,8,2,4	9,13,2,15	7,8,3,2
S_3	12,20,1,7	5,10,5,5	5,8,1,3	5,8,1,3`
Demand	5,8,2,4	8,9,4,1	4,6,2,2	

Trapezoidal approximations of LR- flat fuzzy numbers for obtaining fuzzy transportation cost are represented in table 2.

Table 2:Trapezoidal Approximations of Transportation Costs

	D_1	D_2	D_3	Supply
S_1	4,9,3,10	2,5,1,4	5,8,3,10	5,7,4,2
S_2	9,12,1,14	5,8,2,4	9,13,2,15	7,8,3,2
S_3	12,20,1,7	5,10,5,5	5,8,1,3	5,8,1,3`
Demand	5,8,2,4	8,9,4,1	4,6,2,2	

$\Re(\tilde{A})$iscalculated for the fuzzy costs in table 2using the formula
$\boldsymbol{\Re(\tilde{A}) = m + n - \frac{2\alpha}{3} + \frac{\beta}{2}}$.

Fuzzy transportation problem after implementing the ranking technique is presented in
Table 3

Table 3: Fuzzy transportation problem after fuzzy ranking

	D_1	D_2	D_3	Supply
S_1	16	8.3	16	10
S_2	27.3	13.7	28.2	14
S_3	34.8	20.8	13.8	14
Demand	14	15	10	

After applying fuzzy ranking function in Table 5, the optimal cost or the total minimum fuzzy transportation cost – 613.8 is obtained by using modified Vogel's approximation method.

Example: 2

From Table1, the triangular approximations of LR – flat fuzzy numbers are calculated and presented in Table-4

Table 4: Triangular approximations of Transportation costs

	D_1	D_2	D_3	Supply
S_1	6.5,8,15	3.5,4,7	6.5,6,13	6,6,4
S_2	10.5,4,17	6.5,5,7	11,6,19	7.5,4,3
S_3	16,9,15	7.5,10,10	7.5,4,6	7.5,4,6
Demand	6.5,5,7	8.5,5,2	5,4,4	

By applying ranking method $\Re(\tilde{A})$ is calculated for fuzzy costs in table 4 using the formula.

$$\Re(\tilde{A}) = m + n - \frac{2\alpha}{3} + \frac{\beta}{2}$$

Fuzzy transportation problem after implement the ranking technique is shown in table 5.

Table 5: Fuzzy Transportation problem after Fuzzy ranking

	D_1	D_2	D_3	Supply
S_1	10.2	7.8	15.5	10
S_2	26.8	13.2	29.5	14
S_3	33.5	13.3	15.3	16
Demand	13	15	9	

After applying fuzzy ranking function in Table 5,the optimal cost or thetotal minimum fuzzy transportation cost – 518.5 is obtained by using modified Vogel's approximation method.

7. Conclusion

In view of this paper, a new proposed method is to get the initial basic feasible solution and optimal solution of the fuzzy transportation problem. The transportation cost, supply and demands are taken as unrestrictedLR – flat fuzzy trapezoidal and triangular numbers, which are real and general in nature. The fuzzy transportation problem of trapezoidal and triangular approximations of LR– flat fuzzy numbers have been changed to crisp transportation problem by using Yager's ranking function. A numerical example is pageant that the proposed method gives the better results. The advantage of the proposed method is obtained thefuzzy optimal solution by unrestricted LR – flat fuzzy numbers. This method is simple to gain and apply for solving fuzzy transportation cost in the real life circumstances.

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